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Pulsed transparency of anisotropic media with Stark level splitting

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Abstract. **The propagation of few-cycle electromagnetic pulses through an anisotropic medium is considered. The effect of a permanent dipole moment of the resonant transition on the formation of self-induced transparency pulses and their spectral composition is studied.**

Keywords: *nonlinear coherent processes, optical solitons, resonant media, optical anisotropy, harmonic generation.*

1. Introduction

In the last years, the investigation of coherent nonlinear effects in anisotropic media attracts great attention [1 – 11]. The practical interest in these effects stems from the development of technologies for the production of low-dimensional quantum structures (wells, wires, and dots) in semiconductors [12]. An important feature of anisotropic media is that the stationary states of the quantum particles located in them do not possess a specific parity. That is why the diagonal matrix elements of the dipole-moment operator and their difference, which is called the permanent dipole moment (PDM) of the transition, are nonzero.

The effect of a PDM on the second harmonic generation in asymmetric semiconductor quantum wells was studied, in particular, in paper [1], where the effect was shown to become stronger with increasing the pump wavelength. The propagation of polarisation-single-component, extremely short, femtosecond electromagnetic pulses through the media with a PDM was considered in papers [4 – 7, 11]. The distinct property of these extremely short pulses (ESPs) is the absence of a high-frequency carrier wave. It was shown [4] that the generalised truncated Maxwell – Bloch equations for a two-level medium with a nonzero PDM are a completely integrable system [13 – 15]. Pulse solutions of this system were constructed on a constant background, which was so selected that the problem under consideration reduced to the case of an isotropic medium. The dynamics of these pulses in the presence of a pump was considered in Ref. [5]. The complete system of the Maxwell – Bloch equations was investigated in papers [6, 7], where an algebraic stationary ESP was found and the stationary ESPs were found to exhibit PDM-induced asymmetry in polarity. The propagation through resonant optically uniaxial media of two-component radiation pulses consisting of short-wavelength ordinary and long-wavelength extraordinary components was studied in papers [8 – 10]. It was shown that the regimes of pulse propagation through the medium may differ from the regime of self-induced transparency upon a strong interaction between the components.

The effects of propagation of single-component ESPs with a duration down to few electromagnetic field cycles through a medium with a PDM were numerically investigated in [11]. Stable bipolar solitary pulses with a nonzero area were shown to exist. This makes significant the theoretical consideration of the role of a PDM in the formation of single-component self-induced transparency pulses with a filling (including the high-frequency one). In this work, we studied the exact breather-type solution of the system of truncated Maxwell – Bloch equations.

2. Formulation of the problem and basic equations

Consider an optically uniaxial medium whose anisotropy is induced by the electric field. This field splits the energy levels due to the Stark effect, retaining the degeneracy of electronic levels in the modulus of the projection M of the total angular momentum. In this case, the π transition ($\Delta M = 0$) and doubly degenerate σ transitions ($|\Delta M| = 1$) occur in the electron subsystem. An anisotropic medium can also contain asymmetric quantum wells or quantum wires [12].

Consider an electromagnetic pulse propagating in the positive direction of the y axis of a Cartesian coordinate system perpendicular to the optical axis z of the medium. Only the extraordinary component $E_{\rm e}$ of the electric field parallel to the z axis is assumed to be nonzero. It can be shown that this pulse will interact only with the π transition. The system of truncated Maxwell – Bloch equations describing this process in the unidirectional propagation approximation [16] has the form:

$$\frac{\partial\sigma_3}{\partial t} = {\rm i}\frac{d}{\hbar}E_{\rm e}(\sigma - \sigma^*), \qquad (1)$$

$$\frac{\partial\sigma}{\partial t} = {\rm i}\left(\omega_0 + \frac{DE_{\rm e}}{\hbar}\right)\sigma + 2{\rm i}\frac{d}{\hbar}E_{\rm e}\sigma_3, \qquad (2)$$

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$$\frac{\partial E_e}{\partial y} + \frac{n_e}{c}\frac{\partial E_e}{\partial t} = -2\pi i \frac{Nd\omega_0}{n_e c}(\sigma - \sigma^*), \qquad (3)$$

where $\sigma_3 = (\rho_{22} - \rho_{11})/2$ is the population inversion; $\sigma = \rho_{12}$; ρ_{jk} $(k = 1, 2)$ are the elements of the density matrix; d, D, and ω_0 are the dipole moment, the PDM, and the frequency of the π transition; n_e is the extraordinary refractive index of the medium; and N is the density of the π transition. In these equations we neglected the relaxation terms and the inhomogeneous broadening of the resonant absorption line.

For $D = 0$, system (1)–(3) coincides with the truncated Maxwell–Bloch equations for an isotropic medium [16]. It is easy to see that the extraordinary component of the electric field fulfils two functions here: it gives rise to quantum transitions and is responsible for the their dynamic frequency shift. In the case of a two-component radiation pulse, these functions were fulfilled respectively by its short-wavelength ordinary and long-wavelength extraordinary components [8–10]. In this case, the ordinary component generated the extraordinary one due to the PDM of the σ transitions.

It is convenient now to pass to the new variables

$$u = \frac{dE_e}{\hbar\omega_0}, \quad \tau = \omega_0 t - n_e\omega_0 y/c, \quad \eta = 2\frac{\pi N d^2}{n_e c\hbar}y. \qquad (4)$$

In this case, Eqns (1)–(3) take the form

$$\frac{\partial\sigma_3}{\partial\tau} = iu(\sigma - \sigma^*), \qquad (5)$$

$$\frac{\partial\sigma}{\partial\tau} = i(1 + 2ku)\sigma + 2iu\sigma_3, \qquad (6)$$

$$\frac{\partial u}{\partial\eta} = i(\sigma^* - \sigma), \qquad (7)$$

where $k = D/2d$.

3. Breather-like pulse

The stationary pulse solution of Eqns (5)–(7) and its algebraic limit can be obtained by direct integration as, for instance, for the complete system of the Maxwell–Bloch equations [6, 7]. The formulas arising in this case differ from those presented in [6, 7] only in the determination of pulse velocities. The two-parameter nonstationary breather-like pulse can be constructed with the help of a Darboux transformation [17]. By expanding the corresponding expressions in a Taylor series in the vicinity of $k = 0$ and keeping two first terms, we obtain

$$u = u_0 + ku_1,$$

$$u_0 = 4\Omega\frac{\Omega T\cosh B_R \sin B_I + \sinh B_R \cos B_I}{\Omega^2 T^2(\cosh 2B_R + 1) + \cos 2B_I + 1}, \qquad (8)$$

$$u_1 = -\frac{4\Omega^2 T^2(\cosh 2B_R + 1)(\cos 2B_I + 1)}{[\Omega^2 T^2(\cosh 2B_R + 1) + \cos 2B_I + 1]^2}, \qquad (9)$$

where

$$B_R = \frac{\tau}{T} + \frac{4T[(\Omega^2 + 1)T^2 + 1]\sigma_0\eta}{[(\Omega + 1)^2 T^2 + 1][(\Omega - 1)^2 T^2 + 1]};$$

$$B_I = \Omega\tau - \frac{4\Omega T^2[(\Omega^2 - 1)T^2 + 1]\sigma_0\eta}{[(\Omega + 1)^2 T^2 + 1][(\Omega - 1)^2 T^2 + 1]}.$$

The real constants Ω and T are the free parameters of the pulse. The quantity σ_0 is the initial population of the medium ($|\sigma_0| \leqslant 1/2$). We call this pulse breather-like, because for $\Omega T \gg 1$ it transforms to a breather (pulse with a higher-frequency filling, ultrashort pulse), and for $\Omega T < 1$ the variable u cannot change sign whatsoever, like for an ESP.

It follows from the above expressions that the modulus of the Fourier transform u_0 will have a maximum at the odd harmonics of the basic frequency Ω and the modulus of the Fourier transform u_1 – at the even harmonics, including the zero harmonic. Because the generation of secondary harmonics due to the PDM is a nonlinear effect, they are localised at the centre of the pulse. The PDM-induced asymmetry of breather-like pulses manifests itself in that the zero harmonic and k have opposite signs. For ESPs, this asymmetry was discovered in [6, 7].

The above-said is most evident for pulses with a higher-frequency filling. Indeed, when the condition $\Omega T \gg 1$ is fulfilled, expressions (8) and (9) are simplified:

$$u_0 = \frac{2\sin B_I}{T\cosh B_R}, \qquad (10)$$

$$u_1 = -\left(\frac{2\cos B_I}{\Omega T\cosh B_R}\right)^2. \qquad (11)$$

By subjecting expressions (10) and (11) to the Fourier transform

$$F(\nu, u_{0,1}) = \int_{-\infty}^{\infty} e^{i\nu\tau} u_{0,1}\, d\tau,$$

we obtain

$$F(\nu, u_0) = -i\pi\exp(-iT\theta_R\nu)$$

$$\times\left[\frac{\exp i(\theta_I - \Omega T\theta_R)}{\cosh\pi T(\nu + \Omega)/2} - \frac{\exp i(\Omega T\theta_R - \theta_I)}{\cosh\pi T(\nu - \Omega)/2}\right],$$

$$F(\nu, u_1) = -\pi\frac{\exp(-iT\theta_R\nu)}{\Omega^2}\left[\frac{2\nu}{\sinh\pi T\nu/2} + (\nu + 2\Omega)\right.$$

$$\left.\times\frac{\exp 2i(\theta_I - \Omega T\theta_R)}{\sinh\pi T(\nu + 2\Omega)/2} + (\nu - 2\Omega)\frac{\exp 2i(\Omega T\theta_R - \theta_I)}{\sinh\pi T(\nu - 2\Omega)/2}\right],$$

where θ_R and θ_I are the values of B_R and B_I for $\tau = 0$.

The moduli of $F(\nu, u_0)$ and $F(\nu, u_1)$ have maxima at $\nu = \Omega$ and $\nu = 0, 2\Omega$, respectively. The width of spectral lines is equal to T^{-1} and the maximum values of $|F(\nu, u_1)|$ are proportional to $\Omega^{-2}T^{-1}$. The efficiency of secondary harmonic generation increases with decreasing the carrier frequency of the pulse, which is consistent with the conclusions of Ref. [1]. The effect of PDM on the pulses having a high-frequency filling will be weak. This is due to the fact that the average frequency shift $2ku$ [see Eqn (6)] during such a pulse is equal to zero.

Consider now a two-parameter breather-like pulse for arbitrary k. The dependences of u and σ_3 on τ for $k = 1$ are plotted in Fig. 1. The pulse parameters $\tilde{\Omega} = \Omega/(1 + k^2)^{1/2}$ and $\tilde{T} = (1 + k^2)^{1/2}T$, which characterise its frequency and duration, are so selected that it will strongly excite the medium. The modulus of the Fourier transform $F(\nu, u)$ is

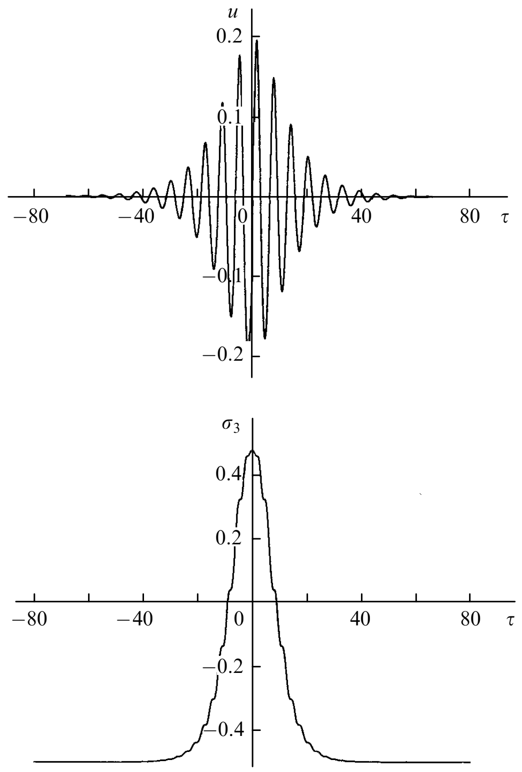

Figure 1. Dependences of u and σ_3 on τ for $k = 1$, $\sigma_0 = -0.5$, $\tilde{\Omega} = 1$ and $T = 10$.

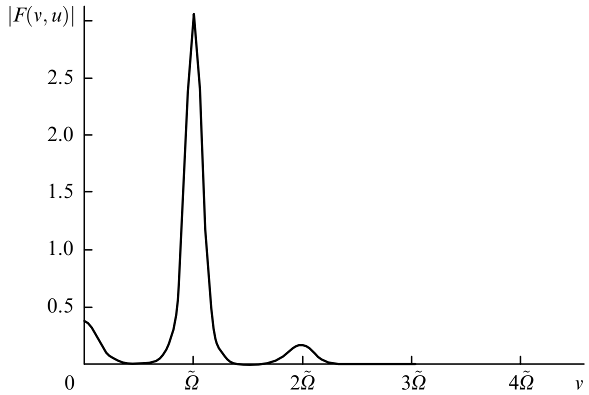

Figure 2. Modulus of the Fourier transform $F(v, u)$ of the pulse in Fig. 1.

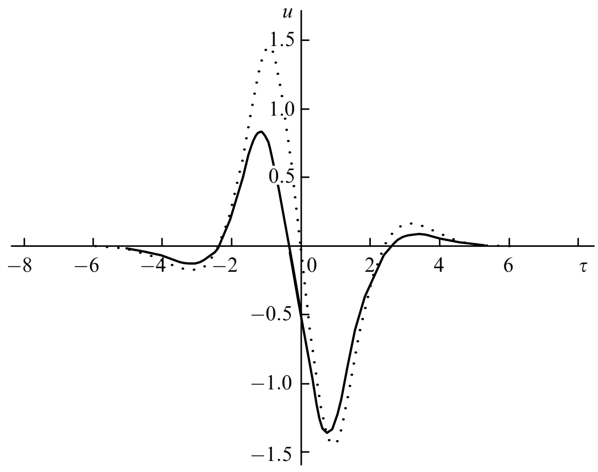

Figure 3. Dependences of u on τ for $k = 1$, $\sigma_0 = -0.5$, $\tilde{\Omega} = 1$, $T = 1$ (solid curve) and $k = 0$ (dotted line).

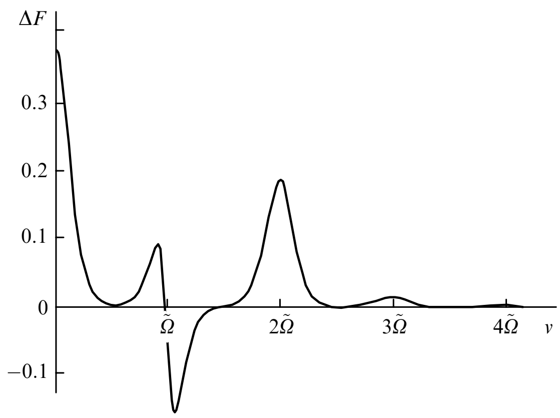

Figure 4. Difference ΔF of the moduli of the Fourier transform $F(v, u)$ for $k = 1$ and $k = 0$ (the remaining parameters are the same as for the pulse in Fig. 1).

shown in Fig. 2. Note that the breather-like pulse area is nonzero due to the presence of the zero-order harmonic in the spectrum. For pulses with $\Omega T \approx 1$ this fact was established in Ref. [11] (see also Fig. 3).

It can be shown that the position of the central peak on the v axis of the Fourier spectrum shifts to the red with increasing $|k|$. This is the reason why the pulses whose basic carrier frequency at the input of the anisotropic medium is lower than the resonance frequency will generate secondary harmonics with a higher efficiency. The same effect will take place as the input pulse shortens due to spectral line broadening. The curve in Fig. 4 shows the difference in spectral composition for u with the same $\tilde{\Omega}$ and $\tilde{T}$ for anisotropic and isotropic media. The dip to the right of $\tilde{\Omega}$ is a consequence of the asymmetry of the principal peak of the Fourier spectrum in the medium with a PDM.

In the limiting case $T \to \infty$, we obtain from the formulas for a two-parameter breather-like pulse

$$u = -2\,\frac{\Omega \cot\varphi}{(1+k^2)^{1/2}}$$

$$\times\,\frac{\zeta_1 \sin^2\varphi\, \cos\zeta_2 + (\cos^2\varphi - 2)\sin\zeta_2 + 2\sin\varphi}{(\zeta_1 \sin\varphi + \cos\zeta_2)^2 + \cot^2\varphi\,(1 - \sin\varphi\,\sin\zeta_2)^2},$$

where

$$\zeta_1 = \Omega\tau + \frac{4\Omega(1+\Omega^2)\sigma_0\eta}{(1-\Omega^2)^2}; \quad \zeta_2 = \Omega\tau + \frac{4\Omega\sigma_0\eta}{1-\Omega^2};$$

$$\Omega = \frac{k\,\cos\varphi}{(1+k^2)^{1/2}}.$$

Here, the variable u decreases rationally. The arbitrary parameter of the pulse is a real constant φ, which defines its frequency, the frequency of such pulses always being lower than the resonant frequency.

The existence of rationally decreasing pulses is a distinguishing feature of anisotropic media. In particular, an algebraic single-component ESP was found in Refs [6, 7] and two-component one-parameter pulses were constructed in Ref. [18]. u, σ_3 and the modulus of the Fourier transform $F(\nu, u)$ are plotted in Figs 5 and 6. A comparison of the curves in Figs 2 and 6 shows that the position of secondary harmonic peaks on the ν axis is stronger red-shifted for rationally decreasing pulses. Furthermore, a distinct asymmetry is inherent not only in the principal peak, but in secondary peaks of the Fourier spectrum as well.

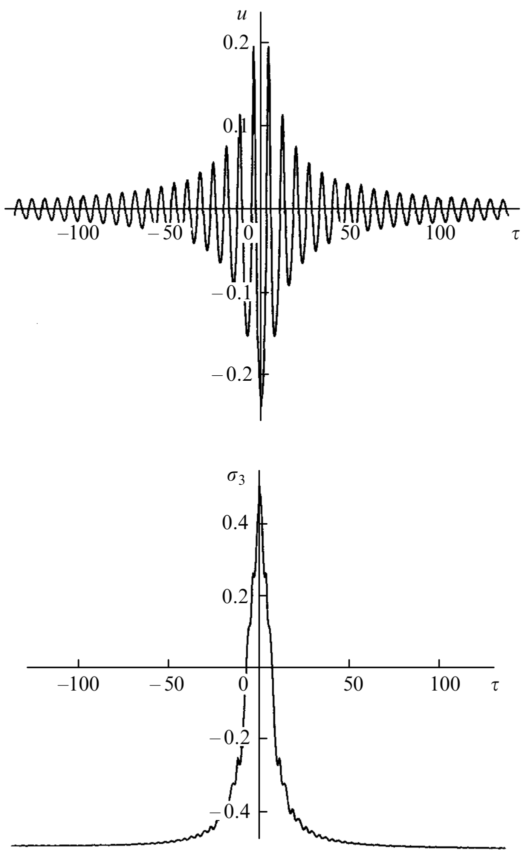

Figure 5. Dependences of u and σ_3 of a rationally decreasing pulse on τ for $k = 3$, $\sigma_0 = -0.5$ and $\varphi = \pi/8$ ($\Omega \approx 0.88$).

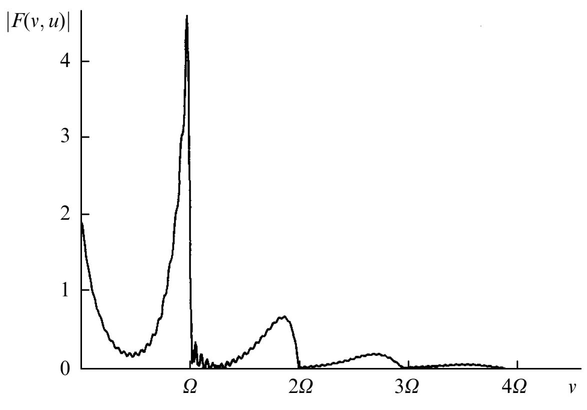

Figure 6. Modulus of the Fourier transform $F(\nu, u)$ of a rationally decreasing pulse (the parameters are the same as for the pulse in Fig. 5).

4. Conclusions

We have considered the propagation of few-cycle electromagnetic pulses through a medium possessing a PDM. State-of-the-art technologies make it possible to fabricate semiconductor crystals with properties varying over a wide range. In particular, the authors of Ref. [1] considered a medium in which the modulus of the ratio between the PDM and the corresponding off-diagonal matrix element of the dipole moment lies in the $0.15-7.1$ range for different quantum transition frequencies. It is therefore of value to elucidate the part played by the PDM in the formation of extremely short and ultrashort pulses. Like for ESPs, for breather-like pulses there occurs asymmetry in signal polarity: the zero harmonic and the PDM are opposite in sign. The effect of PDM on secondary harmonic generation is enhanced with lowering the basic carrier frequency of the pulse and with shortening its duration. In this case, secondary harmonics will be most efficiently generated by the pulses whose carrier frequency at the input of the medium is lower than the resonant frequency. Among these pulses are rationally decreasing pulses, which exist only in the case of anisotropic media.

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