

# Two-photon electron transfer between quantum dots

A.M. Basharov, S.A. Dubovis

**Abstract.** The effective Hamiltonian of a system describing electrons of two quantum dots in a two-frequency electromagnetic field upon Raman resonance of the field with a pair of the electronic levels of different dots is obtained by the method of equivalent transformation. The states of the continuous spectrum (conduction bands) are correctly taken into account. The role of the terms describing the dynamic Stark effect in a two-body system is analysed. A new mechanism of particle transfer between quantum wells in a non-resonance electromagnetic field is proposed, which is free from disadvantages of previous models.

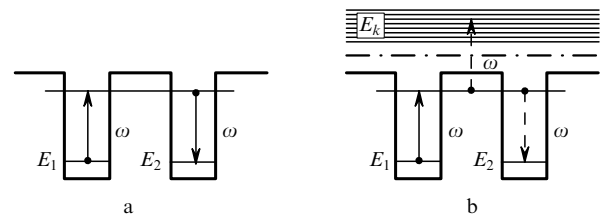
**Keywords:** quantum dot, optical electron transfer, two-photon and Raman resonances, effective Hamiltonian.

## 1. Introduction

Electron transfer between two widely spaced quantum dots induced by an electromagnetic pulse was considered in Refs [1–8]. If a pair of quantum dots is treated as a qubit, the optical electron transfer between them can be used to realise quantum logical elements [1, 2].

The author of paper [1] initiated investigations of three-level models of quantum dots, in which optical electron transfer between the low-lying states of two identical quantum dots proceeds through the third energy level located near the upper boundary of a barrier separating the dots (Fig. 1a). The electromagnetic-pulse frequency  $\omega$  is selected so that to provide the resonance between a pair of the lower energy levels and the upper delocalised state. The experimental implementation of this mechanism involves considerable difficulties: along with the technological problems of manufacturing two almost identical quantum dots with initially specified properties, there exist rigid restrictions caused by the one-photon transition of an electron from the upper level to a state with a continuous spectrum (the dashed arrow in Fig. 1b) [7].

The electron transfer mechanism based on the Raman resonance does not require the use of identical quantum dots [8–11]. Within the framework of this mechanism,



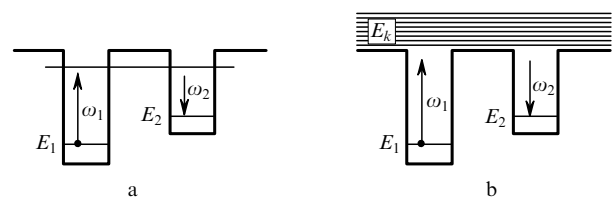
**Figure 1.** Schemes of the electron transfer between two identical quantum dots through the third bound state delocalised between them by neglecting continuous-spectrum states (a) and taking into account the single-photon electronic transition the continuous-spectrum state (transition to the continuum) (b).

periodic transitions between two stationary states with energies  $E_1$  and  $E_2$  appear under the action of two monochromatic waves (Fig. 2a) with frequencies selected to provide the Raman resonance condition

$$E_2 - E_1 \approx \omega_1 - \omega_2 \quad (1)$$

(hereafter  $\hbar = 1$ ). Nevertheless, the method of electron transfer proposed in [8] has the same drawback as three-level systems [1–7] – it requires the presence of a quasi-resonance bound state delocalised between widely spaced quantum dots, i.e., lying in fact near the upper boundary of a barrier separating the dots. The presence of such a level also imposes additional restrictions on the parameters of the problem caused by direct transitions to a state with a continuous spectrum [7].

We show in this paper that to provide electron transfer between quantum wells, it is sufficient to have continuous-spectrum states in the potential relief of any type, which were not considered in Refs [1–6, 8]. Because of their delocalisation, the energy levels of the continuum can play the role of quasi-resonance states required for Raman



**Figure 2.** Schemes of the Raman mechanism of the electron transfer when the bound state delocalised between quantum dots is quasi-resonant (a) and in the general case with the use of continuous-spectrum states (b).

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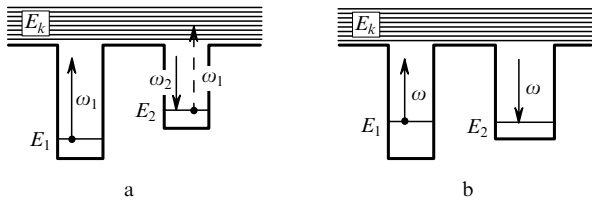
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transitions between widely spaced quantum dots (Fig. 2b). As shown below, the closer the quasi-resonance energy level lies to the continuous spectrum

$$E_{\text{res}} = E_1 + \omega_1 \approx E_2 + \omega_2. \quad (2)$$

the most efficient will be the electron transfer. However, if the energies  $E_1$  and  $E_2$  are substantially different, the choice of the frequencies  $\omega_1$  and  $\omega_2$  at which the energy  $E_{\text{res}}$  is close to the continuum boundary leads to the irreversible electronic transition to the continuous-spectrum state from the state with a lower binding energy (the dashed arrow in Fig. 3a).



**Figure 3.** Schemes of the single-photon transition to the continuum (a) and the electron transfer between the states with close energies ( $\omega_1 \approx \omega_2 \equiv \omega$ ) (b).

To obtain intense electronic transitions, avoiding a direct ionisation, we propose in this paper to use the parameters of quantum dots with the electronic states with close energies (Fig. 3b), i.e., with energies  $E_1 \approx E_2 \equiv E$ . In this case, it follows from the Raman resonance condition (1) that  $\omega_1 \approx \omega_2$ , i.e., to induce electron transfer between the two quantum dots, it is sufficient to irradiate the system by an electromagnetic field at one frequency only. Generally speaking, this frequency can be arbitrary, but if it higher than the binding energy  $E$ , then along with periodic transitions between localised states, the irreversible electron transition will occur due to a direct ionisation to the continuous-spectrum state.

## 2. Effective Raman resonance Hamiltonian

Consider the dynamics of the system in Fig. 2b subjected to the action of the electromagnetic field with the electric component strength

$$\mathbf{E} = \sum_{j=1}^2 \boldsymbol{\varepsilon}_j(t) e^{-i\omega_j t} + \text{c.c.},$$

where  $\boldsymbol{\varepsilon}_j(t)$  are slowly varying (compared to  $\omega_j$ ) wave amplitudes. We neglect the spatial dependence of the field assuming that the linear dimensions of the system are much smaller than the wavelength of the incident electromagnetic waves. In addition, to simplify expressions, we assume that the electric strength vectors  $\boldsymbol{\varepsilon}_1$  and  $\boldsymbol{\varepsilon}_2$  have the same direction or the fields  $\boldsymbol{\varepsilon}_1$  and  $\boldsymbol{\varepsilon}_2$  have the same polarisation. Note that subscripts 1 and 2 correspond to the localised states between which electron transfer should be performed, while the subscript  $k$  corresponds to other stationary states, including the continuous-spectrum state. For definiteness, we will measure the energy from the lower boundary of the continuum (conduction band).

The Schrödinger equation describing the dynamics of the

wave function  $\Psi$  of an electron in the dipole approximation in this system has the form

$$i \frac{\partial \Psi}{\partial t} = (\hat{H}_0 - E \hat{d}) \Psi, \quad (3)$$

where  $\hat{H}_0$  is the Hamiltonian in the absence of the electromagnetic field and  $\hat{d}$  is the operator of the dipole moment projection on the direction of the vector  $\mathbf{E}$ .

To obtain the closed system of equations describing transitions only between the bound states  $|1\rangle$  and  $|2\rangle$ , we perform with the help of the operator  $\hat{S}$  the equivalent transformation of a state vector of the system, similar to that presented in monographs [10, 11]:

$$\Psi = e^{i\hat{S}} \tilde{\Psi}, \quad (4)$$

Then, Eqn (3) will take the form

$$i \frac{\partial \tilde{\Psi}}{\partial t} = \hat{H} \tilde{\Psi} \quad (5)$$

with the effective Hamiltonian

$$\hat{H} = e^{-i\hat{S}} \hat{H}_0 e^{i\hat{S}} - e^{-i\hat{S}} E \hat{d} e^{i\hat{S}} - i e^{-i\hat{S}} \frac{d e^{i\hat{S}}}{dt}. \quad (6)$$

Let us expand  $\hat{S}$  and  $\hat{H}$  in power series in the electric field strengths:

$$\hat{S} = \hat{S}^{(1)} + \hat{S}^{(2)} + \dots, \quad \hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} + \hat{H}^{(2)} + \dots, \quad (7)$$

where  $\hat{S}^{(n)}$ ,  $\hat{H}^{(n)}$  are terms of the  $n$ th order in the field. It follows from (6) that

$$\hat{H}^{(0)} = \hat{H}_0,$$

$$\hat{H}^{(1)} = -E \hat{d} - i[\hat{S}^{(1)}, \hat{H}_0] + \frac{\partial \hat{S}^{(1)}}{\partial t}, \quad (8)$$

$$\begin{aligned} \hat{H}^{(2)} = & \frac{i}{2} [\hat{S}^{(1)}, E \hat{d}] - \frac{i}{2} [\hat{S}^{(1)}, \hat{H}^{(1)}] \\ & - i[\hat{S}^{(2)}, \hat{H}_0] + \frac{\partial \hat{S}^{(2)}}{\partial t}. \end{aligned} \quad (9)$$

To obtain the closed system of equations, we will require that all the matrix elements of the term  $\hat{H}^{(1)}$  of the effective Hamiltonian would be zero (see details in [10, 11]), while among the matrix elements of the term  $\hat{H}^{(2)}$  only the diagonal elements and matrix elements corresponding to the transitions between the states  $|1\rangle$  and  $|2\rangle$  would be nonzero. Mathematically, these requirements are represented by the expressions

$$\tilde{H}_{\alpha\beta}^{(1)} = 0, \quad \alpha, \beta = 1, 2, k, \quad (10)$$

$$\tilde{H}_{1k}^{(2)} = \tilde{H}_{2k}^{(2)} = 0.$$

Conditions (10), taking (8) into account, lead to the following equation for the matrix elements of the operator  $\hat{S}^{(1)}$ :

$$S_{\alpha\alpha}^{(1)} = 0,$$

$$\frac{\partial}{\partial t} S_{\alpha\beta}^{(1)} + i\omega_{\alpha\beta} S_{\alpha\beta}^{(1)} = d_{\alpha\beta} \sum_{j=1}^2 [\varepsilon_j(t) e^{-i\omega_j t} + \varepsilon_j^*(t) e^{i\omega_j t}],$$

where  $\omega_{\alpha\beta} = E_\alpha - E_\beta$ .

The solution of this equation is the matrix elements

$$\begin{aligned} S_{\alpha\beta}^{(1)} &= d_{\alpha\beta} \sum_{j=1}^2 \int_{-\infty}^t e^{i\omega_{\alpha\beta}(\tau-t)} [\varepsilon_j(t) e^{-i\omega_j t} + \varepsilon_j^*(t) e^{i\omega_j t}] d\tau \\ &= d_{\alpha\beta} \sum_{j=1}^2 \left\{ \left[ \frac{\varepsilon_j e^{-i\omega_j t}}{i(\omega_{\alpha\beta} - \omega_j)} + \frac{\varepsilon_j^* e^{i\omega_j t}}{i(\omega_{\alpha\beta} + \omega_j)} \right] \right. \\ &\quad \left. + \left[ \frac{\partial \varepsilon_j}{\partial t} \frac{e^{-i\omega_j t}}{(\omega_{\alpha\beta} - \omega_j)^2} + \frac{\partial \varepsilon_j^*}{\partial t} \frac{e^{i\omega_j t}}{(\omega_{\alpha\beta} + \omega_j)^2} \right] + \dots \right\}. \end{aligned} \quad (11)$$

Because we assume that the amplitudes  $\varepsilon_{1,2}(t)$  of the electromagnetic waves change slowly compared to their frequencies  $\omega_{1,2}$ , we will take into account only the first term in (11):

$$S_{\alpha\beta}^{(1)} = d_{\alpha\beta} \sum_{j=1}^2 \left[ \frac{\varepsilon_j e^{-i\omega_j t}}{i(\omega_{\alpha\beta} - \omega_j)} + \frac{\varepsilon_j^* e^{i\omega_j t}}{i(\omega_{\alpha\beta} + \omega_j)} \right].$$

By using this expression in the calculation of the matrix elements of the operator  $\tilde{H}^{(2)}$ , we obtain

$$\begin{aligned} \tilde{H}_{21}^{(2)} &= e^{-i\omega_{21}t} \frac{\partial}{\partial t} [S_{21}^{(2)} e^{i\omega_{21}t}] \\ &+ \frac{1}{2} \sum_{q=1}^2 [\varepsilon_q(t) e^{-i\omega_q t} + \varepsilon_q^*(t) e^{i\omega_q t}] \\ &\times \sum_k d_{2k} d_{k1} \sum_{j=1}^2 \left[ \varepsilon_j e^{-i\omega_j t} \left( \frac{1}{\omega_{2k} - \omega_j} - \frac{1}{\omega_{k1} - \omega_j} \right) \right. \\ &\quad \left. + \varepsilon_j^* e^{i\omega_j t} \left( \frac{1}{\omega_{2k} + \omega_j} - \frac{1}{\omega_{k1} + \omega_j} \right) \right]. \end{aligned} \quad (12)$$

We select the matrix elements so that  $\tilde{H}_{21}^{(2)} \sim \exp(-i\omega_{21}t) \sim \exp[i(\omega_2 - \omega_1)t]$ , which corresponds physically to oscillations at the Raman resonance frequency. In this case, expression (12) takes the form

$$\begin{aligned} \tilde{H}_{21}^{(2)} &= -\frac{1}{2} \varepsilon_1 \varepsilon_2^* e^{i(\omega_2 - \omega_1)t} \sum_k d_{2k} d_{k1} \left( \frac{1}{\omega_{k2} + \omega_1} \right. \\ &\quad \left. + \frac{1}{\omega_{k1} - \omega_1} + \frac{1}{\omega_{k2} - \omega_2} + \frac{1}{\omega_{k1} + \omega_2} \right). \end{aligned}$$

Therefore, the term of the effective Hamiltonian of the second order in the field strength, which describes two-photon processes, has the nonzero nondiagonal matrix elements corresponding to transitions between the states  $|1\rangle$  and  $|2\rangle$ . To describe such transitions, we can restrict expansion (7) to the second-order terms. The expressions for matrix elements of the effective Hamiltonian  $\tilde{H} = \tilde{H}^{(0)} + \tilde{H}^{(1)} + \tilde{H}^{(2)}$  of two-photon transitions take the final form

$$H_{11} = E_1 + E_1^{\text{St}}, \quad H_{22} = E_2 + E_2^{\text{St}}, \quad (13)$$

$$H_{21} = e^{i(\omega_2 - \omega_1)t} A, \quad H_{12} = e^{-i(\omega_2 - \omega_1)t} A^*, \quad (14)$$

$$H_{1k} = H_{2k} = 0, \quad (15)$$

where

$$E_i^{\text{St}} = - \sum_{j=1}^2 |\varepsilon_j|^2 \sum_k |d_{ki}|^2 \left( \frac{1}{\omega_{ki} + \omega_j} + \frac{1}{\omega_{ki} - \omega_j} \right) \quad (16)$$

are the Stark level shifts and

$$A = -\varepsilon_1 \varepsilon_2^* \sum_k d_{2k} d_{k1} \left( \frac{1}{\omega_{k2} + \omega_1} + \frac{1}{\omega_{k1} - \omega_1} \right) \quad (17)$$

is the Rabi frequency.

### 3. Electron transfer probability

To describe the electron transfer between the states  $|1\rangle$  and  $|2\rangle$  analytically, we will neglect the possible relaxation and electron–electron correlation processes and will solve transformed Schrödinger equation (5) by representing its solution as a superposition of all the states:

$$\tilde{\Psi} = \sum_\alpha a_\alpha(t) e^{-i(E_\alpha + E_\alpha^{\text{St}})t} |\alpha\rangle.$$

By using expressions (13)–(15), we can obtain the closed system of equations describing transitions between the states  $|1\rangle$  and  $|2\rangle$ :

$$\begin{aligned} \dot{a}_1 &= -ia_2 e^{-i\Delta t} A^*, \\ \dot{a}_2 &= -ia_1 e^{i\Delta t} A, \end{aligned} \quad (18)$$

where  $\Delta \equiv (E_2 + E_2^{\text{St}} - E_1 - E_1^{\text{St}}) - (\omega_1 - \omega_2)$  is the detuning of the system from the Raman resonance.

If the time dependence of  $A$  is neglected and it is assumed that  $a_1(t=0) = 1$ ,  $a_2(t=0) = 0$  (i.e., an electron is located initially in the first quantum dot), the solution of the system of equations (18) has the form

$$a_1 = e^{-i(A/2)t} \left( \cos \Omega t + i \frac{A}{2\Omega} \sin \Omega t \right),$$

$$a_2 = -i \frac{A}{\Omega} e^{-i(A/2)t} \sin \Omega t,$$

which is standard for two-level systems, where

$$\Omega = \left( |A|^2 + \frac{1}{4} A^2 \right)^{1/2}.$$

The square of the modulus of the coefficient  $a_2$ , which determines the probability of electron transfer from the first to the second quantum dot, is described by the expression

$$|a_2|^2 = \frac{|A|^2}{2\Omega^2} (1 - \cos 2\Omega t).$$

Therefore, if the detuning from the resonance is much

smaller than the Rabi frequency ( $\Delta \ll |A|$ ), the electron transfer between two quantum dots can occur with the probability  $|a_2|_{\max}^2 \approx 1 - \Delta^2/(4|A|^2)$  close to unity.

#### 4. Choice of the optimal frequency of an electromagnetic pulse

The efficiency of electron transfer between quantum dots is determined by the Rabi frequency  $A$ , which can be varied not only by changing the strength of electromagnetic pulses but also by the choice of their frequencies.

By analysing expression (17), note first of all that it contains contributions only from the states that are not localised in one of the quantum dots, because otherwise the product  $d_{2k}d_{k1}$  will be zero. Therefore, it is necessary to take into account only the stage of the continuous spectrum and probably discrete states with energies near the upper boundary of a barrier separating the quantum dots.

We will seek for the conditions of increasing  $A$  considering only the states of the continuous spectrum because they are present in the potential relief of any type, whereas the presence of discrete delocalised states is a rather particular case and their influence on two-photon processes can be taken into account as a correction if necessary.

By going in (17) from summation to integration over the continuous-spectrum states, we obtain

$$A = -\varepsilon_1 \varepsilon_2^* \frac{V}{(2\pi)^3} \int d_{2k} d_{k1} \left( \frac{1}{E_k - E_{\text{res}}} + \frac{1}{E_k - 2E_1 + E_{\text{res}}} \right) dk, \quad (19)$$

where  $V$  is the system volume and  $\mathbf{k}$  is the quasi-wave vector.

We will consider only the situations when  $E_{\text{res}} < 0$ , because otherwise an electron will undergo the transition to a state with a continuous spectrum due to one-photon ionisation. One can see from expression (19) that for any  $E_k > 0$  and  $E_2 < 0$ , the sum in parentheses increases monotonically in the interval  $E_1 < E_{\text{res}} < 0$ . Therefore, it is reasonable to assume that the optimal conditions for electron transfer are achieved when the quasi-resonance region of energies  $E_{\text{res}}$  lies near the lower boundary of the continuous spectrum, which is a predictable result in a certain sense.

Such a choice of frequencies aimed for increasing the Rabi frequency is also based on more substantial grounds. Upon integration over the quasi-wave vector  $\mathbf{k}$ , the complex factor  $d_{2k}d_{k1}$  in expression (19) oscillates in a complicated way and is 'self-quenched' to a great extent, resulting in a substantial decrease in the Rabi frequency. This effect will be suppressed in fact if the frequencies of external fields are chosen so that  $E_{\text{res}} \sim 0$ , because in this case the states near the lower boundary of the continuous spectrum are separated. The terms in dominators in expression (19) corresponding to these energy levels are close to zero, and the phase of the factor  $d_{2k}d_{k1}$  can be considered identical for all the states near the lower boundary of the continuum. Therefore, it is the states lying close to the continuum boundary that will make the main contribution to the integral, not resulting in their mutual weakening.

#### 5. Single-frequency electron transfer

Thus, the choice of energy  $E_{\text{res}}$  near the lower boundary of the continuous spectrum results in a drastic increase in the Rabi frequency. However, such a choice does not provide a reliable electron transfer because it leads to the single-photon electronic transition to the continuous-spectrum state from the state with a lower binding energy (the dashed arrow in Fig. 3a). Formally, this is manifested in the appearance of zero denominators in terms in expression (16), which correspond to the levels of the continuum that are resonant with the  $|2\rangle$  level at the frequency  $\omega_1$ . From the point of view of mathematical description of such processes, the zero denominators can be eliminated in a standard way, by replacing  $\omega_1$  by  $\omega_1 + i\delta$ , where  $\delta = +0$ . This results in the appearance of the imaginary addition in the diagonal matrix element  $H_{22}$ , which describes the irreversible electronic transition to the continuous-spectrum state [7]:

$$H_{22} = E_2 + E_2^{\text{St}} + i\Gamma_2, \quad (20)$$

where

$$E_2^{\text{St}} = -|\varepsilon_2|^2 \sum_k |d_{k2}|^2 \left( \frac{1}{\omega_{k2} + \omega_2} + \frac{1}{\omega_{k2} - \omega_2} \right) - |\varepsilon_1|^2 \left( \sum_k \frac{|d_{k2}|^2}{\omega_{k2} + \omega_1} + \sum_k P \frac{|d_{k2}|^2}{\omega_{k2} - \omega_1} \right), \quad (21)$$

$$\Gamma_2 = -\pi |\varepsilon_1|^2 \sum_k |d_{k2}|^2 \delta(\omega_{k2} - \omega_1).$$

Generally speaking, for a particular form of the potential the conditions can be found when the ionisation efficiency will be negligible compared to other processes involved. However, analysis of the situation in the general case showed that we can avoid the consideration of unfavourable processes of the irreversible transition by changing somewhat the initial model.

We will consider the pairs of quantum dots for which the energy of stationary states localised in them only slightly differ from each other, i.e.,  $E_1 \approx E_2 \equiv E$ . It follows from the Raman resonance condition (1) that in this case  $\omega_1 \approx \omega_2 \equiv \omega$ , i.e., to realise electron transfer between such quantum dots, it is sufficient to irradiate the system by the electromagnetic field only at one frequency. The equivalent transformation (3)–(9) of the wave function of this system leads to the following expressions for the matrix elements of the effective Hamiltonian:

$$H_{1,2} = E_{1,2} + E_{1,2}^{\text{St}}, \quad H_{21} = A = H_{12}^*, \quad (22)$$

where

$$E_{\alpha}^{\text{St}} = -|\varepsilon|^2 \sum_k |d_{k\alpha}|^2 \left( \frac{1}{\omega_{k\alpha} + \omega} + \frac{1}{\omega_{k\alpha} - \omega} \right);$$

$$A = -|\varepsilon|^2 \sum_k d_{2k} d_{k1} \left( \frac{1}{\omega_{k\alpha} + \omega} + \frac{1}{\omega_{k\alpha} - \omega} \right).$$

In this case, the electron dynamics is described by a system of equations equivalent to (18) with accuracy to renaming  $A$  and  $\Delta$ . By repeating the above considerations or simply

passing to the limit, we can show that the Rabi frequency (22) will be maximal when the external electromagnetic field frequency is chosen so that the quasi-resonance energy region  $E_{\text{res}} = E + \omega$  lies close to the lower boundary of the continuum.

## 6. Quantitative estimate of the Rabi frequency

After a standard passage from summation to integration in (20), we obtain

$$A \sim |\varepsilon|^2 \frac{V}{(2\pi)^3} \int \frac{d_{2k} d_{k1}}{E_k + |E_{\text{res}}|} dk. \quad (23)$$

We will make estimates by using the dispersion law for free electrons. Let us introduce the notation  $q = (2m^* |E_{\text{res}}|)^{1/2}$ , where  $m^*$  is the effective electron mass. Then, expression (23) will take the form

$$A \sim |\varepsilon|^2 \frac{V}{(2\pi)^3} 2m^* \int \frac{d_{2k} d_{k1}}{k^2 + q^2} dk. \quad (24)$$

The states with the energy close to  $E_{\text{res}}$ , i.e., to the boundary of the continuous spectrum make the main contribution to the integral in (24). The corresponding wave functions are plane waves (strictly speaking, the waves distorted near quantum wells), whose wavelengths are large directly at the continuum boundary and decrease with increasing energy. Accordingly, the integrand in expression (24) oscillates for two reasons: first, the modulus of each of the factors  $d_{2k}$  and  $d_{k1}$  changes in a complicated way (depending on  $k$ ) and, second, the phase difference periodically changes, which can be approximately written as  $d_{2k} d_{k1} = |d_{2k} d_{k1}| e^{ikL}$ , where  $L$  is the characteristic distance between quantum dots. Because it is initially assumed in the model that the states  $|1\rangle$  and  $|2\rangle$  are well isolated from each other, we assume that the distance between quantum dots noticeably exceeds their size, which corresponds to real systems. Therefore, we can believe that the second of the above-mentioned reasons for oscillations is dominant.

To estimate roughly the value of  $A$  (24), we can restrict the integration to the region of  $k$  where the first oscillation occurs. Therefore, we will restrict the integration region to the sphere of radius  $k_{\text{max}} = 1/L$  and will assume that each of the matrix elements  $d_{2k}$  and  $d_{k1}$  within this sphere is constant and  $d_{2k} \sim d_{k1} \sim ea(a^3/V)^{1/2}$ , where  $a$  is the characteristic linear size of quantum dots and  $e$  is the electron charge. The root dependence appears because upon multiplying the wave function  $\Psi_k$  normalised to the volume  $V$  by one of the functions  $\Psi_{1,2}$  localised near the corresponding quantum dot, the 'effective' integration region decreases to the volume  $\sim a^3$ . Taking all the above assumptions into account, expression (24) takes the form

$$A \sim |\varepsilon|^2 \frac{V}{(2\pi)^3} 2m^* \frac{e^2 a^5}{V} 4\pi \int_0^{k_{\text{max}}} \frac{k^2 dk}{k^2 + q^2}. \quad (25)$$

One can see from this expression that it is reasonable to consider that the energy  $E_{\text{res}}$  is close to the continuum boundary when the value of  $q^2$  is comparable to or even much smaller than  $k_{\text{max}}^2$ . In this case, we obtain from expression (25), taking Planck's constant into account,  $A \sim |\varepsilon|^2 e^2 m^* a^5 / (\hbar^2 L)$ . If we assume for estimate that  $a \sim 10$  nm,  $L \sim 100$  nm,  $m^* \sim 0.1m_e \sim 10^{-28}$  g (where  $m_e$

is the electron mass), which corresponds to the unfavourable case from the point of view of obtaining a large transition energy, then the Rabi frequency becomes equal to  $\sim 10^{-5}$  eV already at moderate wave amplitudes  $\varepsilon \sim 10^3$  V cm $^{-1}$ . Note for comparison that the range of optimal values of  $A$  in [1] is  $\sim 10^{-5} - 10^{-4}$  eV.

## 7. Conclusions

The effect of nonresonance two-photon electron transfer predicted in this paper can be also important in the confinement of ultracold atoms in magneto-optical traps, where the energy level diagram is the same as in Fig. 1b.

The transfer of atoms from the  $E_1$  energy level of one trap to the  $E_2$  energy level of another trap induced by a pulsed two-frequency electromagnetic field under conditions of the Raman resonance (1) was discussed in papers [12–14]. However, the continuous-spectrum states have not been correctly considered so far in the models of ultracold atoms in magneto-optical traps. Expressions (20) and (21) are also valid for ultracold atoms in traps, taking the continuous-spectrum states into account. The non-resonance transfer mechanism proposed in the paper can be used for atoms in magneto-optical traps for the states with energies  $E_1 \approx E_2$  because the main expressions presented in the paper are valid within the framework of a simple model.

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