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Spectral perturbations in a semiconductor laser: II. Nonlinear interaction of modes

P.G. Eliseev

***Abstract*.** **The spectral perturbation is considered in a semiconductor active medium in the vicinity of the frequency of a strong electromagnetic wave. The nonlinear interaction of waves via the dynamic interference grating, or nonlinear scattering by population waves leads to the shift of resonance frequencies, which is manifested, in particular, in a change of the mode-beating spectrum of the laser. The spectral profile of this perturbation depends on the amplitude – phase coupling factor α. For large α, the profile becomes symmetrical in detuning from the strong-mode frequency. A change in the group refractive index corresponding to the 'slowing' of light is also calculated.**

Keywords: *semiconductor lasers, nonlinear interaction of modes, mode-beating spectrum.*

1. Introduction

Intense electromagnetic waves in a nonlinear medium can exchange the energy due to their interaction with dynamic inhomogeneities ('gratings') induced by the interference of the waves in the medium. This mechanism was discovered and theoretically interpreted in the case of the asymmetric interaction of spectral modes in a semiconductor laser [1 – 3]. It is a particular case of the more general mechanism of scattering by waves or population oscillations [4]. The latter mechanism also describes self-mode stabilisation [5, 6]. These processes play an important role in semiconductor lasers, where the carrier concentration N changes in a broad range and affects the refractive index n (the differential refractive index $\mathrm{d}n/\mathrm{d}N$ is of the order of 10^{-20} cm^3). The corresponding nonlinearity is called 'giant'; the effects related to this nonlinearity in semiconductor lasers are considered in [7].

The measurements of mode-beating frequencies in lasers allow one to observe comparatively small variations in the cavity mode frequencies. Recently, the splitting and shift of the lines was observed in the longitudinal-mode-beating spectrum, which depended on the optical power [8, 9]. For the laser cavity length of 5 mm, the relative change in the beat frequency was 0.4 GHz [8]. This can be explained by the fact that the beat lines are produced by a pair of modes perturbed due to the nonlinear interaction.

The spectral splitting of this type was earlier observed in the radio-frequency noise spectra of a semiconductor laser with an external cavity [10 – 12]. Analysis performed in papers [3, 11 – 13] showed that the nonlinear interaction of modes causes spectral perturbations of the complex refractive index in the vicinity of a strong mode, which include the pulling of adjacent modes, thereby resulting in the shift of the difference frequencies. The theoretical calculation of the beating spectra for the three modes in a semiconductor laser [12] was compared with experiments performed with a laser with an external cavity.

The model of the nonlinear interaction of modes in a semiconductor laser considered below allows one to calculate the perturbation of the refractive index at the adjacent-mode frequencies in the approximation of two modes, one of which is strong and the other is weak. The spectral shift of weak modes located on both sides of the strong mode is considered in this approximation. It is shown that the perturbation spectrum depends on the amplitude – phase coupling factor α. This approach qualitatively explains the splitting observed in the mode-beating spectra of InGaAs quantum well lasers [8, 9].

2. Nonlinear interaction of modes in semiconductor lasers

2.1 General remarks

In [1], the spectrally asymmetric interaction of spectral modes was discovered in a semiconductor laser (in a special external cavity). This nonlinear nonorthogonality of modes was theoretically considered in [2, 3], where it was shown that electromagnetic waves in a nonlinear medium exchange the energy by interacting with a dynamic grating produced due to mode beating. This interaction depends on the optical power, mode detuning, and the amplitude – phase coupling factor $\alpha = (\mathrm{dRe}\varepsilon/\mathrm{d}N)/(\mathrm{dIm}\varepsilon/\mathrm{d}N)$, where ε is the dielectric constant. The factor α is also called the line-broadening factor (introduced in [2]). If $\alpha = 0$, the interaction suppresses modes in the vicinity of a strong mode with a symmetric spectral profile. If $\alpha > 0$, the profile becomes asymmetric, and under lasing conditions the additional amplification appears at lower frequencies and suppression at higher frequencies. This effect can be considered as a variant of stimulated scattering by population oscillations [4]. This interaction is one of the

P.G. Eliseev P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninskii prosp. 53, 119991 Moscow, Russia; present address: Center for High Technology Materials, University of New Mexico, Albuquerque, NM, USA; e-mail: eliseev@chtm.unm.edu

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reasons for nonlinear amplification and related dynamic effects. It is taken into account in the nonlinear gain characterising the self-action of a strong field. In addition, the amplification at a detuned frequency causes spectral instability, multimode lasing, and additional noise due to 'hopping' of lasing between adjacent modes. Under certain conditions, such a noise quenches single-mode lasing.

Experiments with InGaAs/InP lasers [14] confirmed the asymmetric interaction between modes described by the model [2]. However, the characteristic time determining the frequency scale of the mode interaction proved to be of the order of the intraband relaxation time (0.3 ps) rather than the recombination time. This result was confirmed in experiments with AlGaAs lasers in paper [15], where the expression for the spectral profile identical to that in [2] was derived and also the contribution of relaxation, faster than interband relaxation, was discussed. The detailed theoretical study performed in [16] took into account two nonlinearity sources leading to the interaction of modes: population oscillations with the characteristic decay time ~ 1 ns and oscillations of the energy distribution of carriers with the characteristic time approximately equal to the intraband relaxation time (0.1 – 1 ps). In this case, the spectral profiles differ only in the frequency scale, having the same shape, and contributions to nonlinear amplification at high intensities have the same order.

In this paper, the interaction of modes is analysed from the point of view of its possible influence on the frequency shift of the interacting modes, which is manifested in the anomalous splitting of the mode-beating spectrum and can affect the operation of a laser used as a gyroscopic sensor, upon heterodyne reception, and in the mode-locking regime.

2.2 Theoretical model

Consider the interference of waves represented by the expression

$$E_j(t,x) = E_{i0}\exp[\mathrm{i}(-2\pi\nu_j t + k_j x)], \tag{1}$$

where ν_j and k_j are the optical frequencies and wave numbers, respectively. The waves propagate in the positive direction along the x axis, and the positive sign of $\mathrm{Im}\varepsilon$ in this representation corresponds to the absorption of light. The wave (E_0, ν_0) in the wave pair (1) is assumed strong, while the wave (E_1, ν_1) is assumed weak, i.e., $E_0 \gg E_1$. No other meaning to the definition of a strong wave is implied here. The interference field has the total intensity

$$\begin{aligned}|E(t,r)|^2 &= |E_0\exp[\mathrm{i}(-2\pi\nu_0 t + k_0 r)]\\ &\quad + E_1\exp[\mathrm{i}(-2\pi\nu_1 t + k_1 r)]|^2 = |E_0|^2 + |E_1|^2\\ &\quad + 2\mathrm{Re}\{E_0E_1\exp[\mathrm{i}(\Omega t - \Delta k r)]\},\end{aligned} \tag{2}$$

where $\Omega = 2\pi(\nu_0 - \nu_1)$ is the frequency detuning and $\Delta k = k_0 - k_1$. The cross terms describe the intensity inhomogeneity proportional to the product E_0E_1 oscillating at the beat frequency Ω. Because the total intensity determines the local rate of stimulated recombination, the inhomogeneous field of the carrier concentration $\delta N(r,t)$ is formed in the active medium. This field is calculated in the general case by considering recombination and diffusion from the equation [2, 3]

$$\frac{\partial N}{\partial t} = \frac{J}{ed}D_{\mathrm{dif}}\nabla^2 N + \frac{N}{\tau} + B(N - N_0)|E(r,t)|^2, \tag{3}$$

where J is the pump current density; d is the active layer thickness; D_{dif} is the ambipolar diffusion coefficient; τ is the carrier lifetime; B is the stimulated recombination coefficient; N_0 is the carrier concentration at the inversion threshold; $E(r,t)$ is the strength of the field containing interference waves. The simplified solution of this equation (by neglecting diffusion and linearising the concentration dependence of the gain) was obtained in [3]

$$\delta N(r,t) = \frac{B(N - N_0)E_0E_1\exp(\mathrm{i}\Omega t - \mathrm{i}\Delta k r)}{\mathrm{i}\Omega + (1/\tau) + BE_0^2}. \tag{4}$$

Expression (4) describes the dynamic grating of the carrier concentration produced by the wave beating. It oscillates at the difference frequency and its value is proportional to the cross product E_0E_1. The relaxation rate entering the denominator affects the population oscillations in such a way that this effect should disappear in the limit of very rapid relaxation.

2.3 Dynamic grating of the dielectric constant

The variations of the dielectric constant are described in the linearised form by the equation

$$\delta\varepsilon(r,t) = \frac{\mathrm{d}\varepsilon}{\mathrm{d}N}\,\delta N(r,t). \tag{5}$$

The real part gives the refractive index grating and the imaginary part – the gain-absorption grating. The wave E_0 propagating over the dynamic grating is characterised by the electric induction

$$\delta D(r,t) = \delta\varepsilon(r,t)E_0\exp[\mathrm{i}(-2\pi\nu_0 t + k_0 r)]. \tag{6}$$

This induction contains a component at the weak-wave frequency, which means the energy exchange between the waves. The value of D is proportional to the product $E_0^2E_1$. Therefore, the component at the weak-wave frequency is proportional to E_0^2, i.e., to the strong-wave intensity. The increment of the dielectric constant at the frequency ν_1 is

$$\delta\varepsilon = -|E_0|^2B(N - N_0)\frac{\mathrm{d}\varepsilon''}{\mathrm{d}N}\frac{\alpha + \mathrm{i}}{\gamma + 2\pi\mathrm{i}f}, \tag{7}$$

where γ is the total recombination probability; and $f = \nu_0 - \nu_1$ is the intermode frequency. By separating the real part, we obtain the refractive index

$$\begin{aligned}\delta n = \frac{\mathrm{Re}(\delta\varepsilon)}{2n} &= -|E_0|^2B(N - N_0)\\ &\times\frac{\mathrm{d}\varepsilon''}{\mathrm{d}N}\frac{\alpha + 2\pi f/\gamma}{2n\gamma[1 + (2\pi f/\gamma)^2]}.\end{aligned} \tag{8}$$

This expression shows that the increment of the refractive index is proportional to the strong-wave intensity and its spectral dependence (on f) includes the recombination probability and the factor α. Therefore, this effect is directly related to the radiation power.

Consider a numerical example with the parameters $\gamma = 10^{10}$ s^{-1}, $f = 8\times10^9$ s^{-1}, $\mathrm{d}g/\mathrm{d}N = (8.8 \pm 0.3)\times10^{-16}$ cm^2, $\mathrm{d}\varepsilon''/\mathrm{d}N = -(\lambda n/2\pi)\mathrm{d}g/\mathrm{d}N = -5.6\times10^{-21}$ cm^3, $\alpha = 2.7$ [17], $N - N_0 \approx 10^{18}$ cm^{-3}, $(B|E_0|^2)^{-1} = 0.2$ ns is the partial time of stimulated recombination caused by the strong mode. With these parameters, the nonlinear increment of the refractive index is $\sim 1.2 \times 10^{-4}$. When this increment is positive, the resonance frequency (of the weak mode) decreases, i.e., 'repulsion' occurs on the low-frequency side and 'attraction' on the high-frequency side. The low-frequency adjacent mode is additionally amplified due to the action of the strong mode and its beating with the strong mode gives rise to the line shifting to the higher beat frequency.

3. Nonlinear shift of resonances

The calculated perturbation of the dielectric constant propagates over the spectral distance from the strong mode, which is determined by the total rate of population relaxation of the corresponding levels in the semiconductor bands. In the case of high-power lasing, the recombination rate of level repopulation is approximately $10^{10} - 10^{11}$ s^{-1}, which is comparable with the intermode frequency in a long semiconductor cavity. The relaxation component related to intraband processes can be revealed in the influence of the strong mode at large spectral distances. As for processes characterised by a weak spectral dependence, for example, the frequency chirp in a transient lasing regime, they cover many modes and do not cause any changes in the intermode frequencies.

If the frequency detuning of the interacting modes decreases, the above consideration remains adequate until the wave are discernible, i.e., up to the distance comparable with the individual spectral width of these modes. As for the self-action of the strong mode, it proves to be small within the framework of the considered nonlinear mechanism for the following reasons. First, the increment of the dielectric constant at the strong-mode frequency is oscillatory, i.e., it vanishes at all after averaging over a large interval. Upon averaging over finite intervals (over the cavity length and signal detection time), this increment is substantially smaller than that appearing upon the action of the strong mode on the weak one. Second, the action of the weak mode E_1 on the strong one is proportional to $|E_0 E_1|$, i.e., it is certainly smaller than the action of the strong mode on the weak one, which is proportional to $|E_0|^2$. Therefore, self-action within the framework of our model is negligibly small. The contribution of other self-action mechanisms is described by the nonlinearity coefficient n_2:

$$n(I) = n(0) + n_2 I, \quad (9)$$

where I is the strong-wave intensity. The value of n_2 in semiconductors near absorption resonances can be very large (the so-called giant nonlinearity). The main contribution is made by free carriers; therefore, the same process occurs which we took into account in the calculation of the mode interaction. However, the action of processes described by expression (9) is nonselective, i.e., is related to many modes and does not change intermode frequencies. In an amplifier, processes described by expression (9) produce a strong nonlinear phase shift, which is used in modulation. In a laser, the stationary effect is substantially suppressed due to the fixation of the carrier concentration above the threshold concentration. As for transient regimes, expression (9) refers to the above-mentioned frequency chirp.

The analytic calculation performed above does not give the final value of the expected frequency shift because the stationary value of frequency pulling was found by solving the self-consistent problem. In other words, the steady value of the oscillating part of the concentration and dielectric constant should be determined taking saturation effects into account. This is a rather difficult nonlinear problem. Because the final increment is comparatively small, we will estimate here the effect only by the order of magnitude. According to Fig. 1, as the factor α increases, the nonlinear refraction curve becomes symmetrical. As a result, a part of weak modes in the vicinity of the strong mode, which are symmetric with respect to the latter, shift in the same direction (Fig. 2). In this case, one of them (the higher-frequency mode) approaches ν_0, while the other moves off from ν_0. Therefore, the difference frequencies change in the opposite directions, as was observed in experiments [7, 8] on the anomalous splitting in the mode-beating spectrum. In this case, the nonlinear gain/suppression also corresponds to experiments: the line of beating with the high-frequency adjacent mode is weakened due to nonlinear suppression, while the low-frequency adjacent-mode line, on the contrary, is enhanced (as shown by the arrows in Fig. 2). The line corresponding to the approaching mode was in the experiment weaker by 10–25 dB than the mode line moving off from the strong mode.

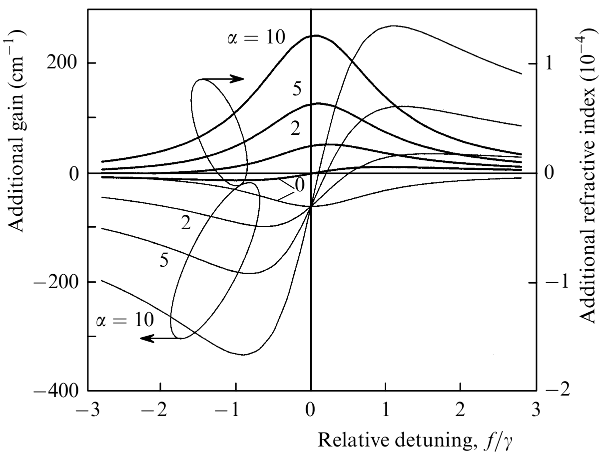

Figure 1. Dependences of the gain and refraction profiles on the relative detuning from the strong-mode frequency calculated for different values of the amplitude–phase coupling factor α.

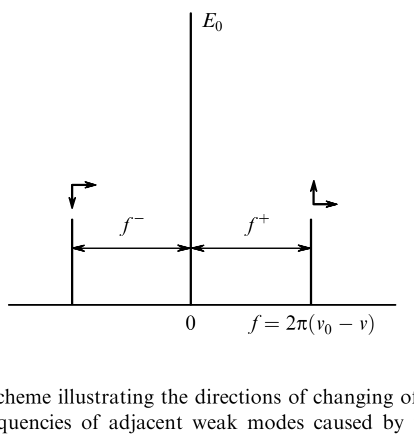

Figure 2. Scheme illustrating the directions of changing of the amplitudes and frequencies of adjacent weak modes caused by the nonlinear interaction.

4. Group refractive index

Even small variations in the absolute value of the refractive index in a narrow spectral range can noticeably affect the group velocity due to the high dispersion $dn/d\nu$. In connection with the slow light effect, when the group velocity v_g is much lower than the speed of light c, it is interesting to estimate to what extent this effect can be favoured by the above-described mode interaction. The group refractive index $n^* = c/v_g$ calculated formally is shown in Fig. 3. One can see that n^* has a peak located at a lower frequency with respect to the strong mode. Therefore, some slowing of light occurs in the laser or amplifying medium under comparatively simple conditions – in the oscillation regime of a semiconductor laser. The peak value is $n^* \approx 28$; however, because the peak is narrow, such a slowing of photons is possible in a virtually monochromatic wave. During the propagation of a signal carrying 'useful' information and, therefore, having a broad spectrum, it can be strongly distorted. In particular, the wave packets can spread and decompose due to the presence of spectral components lying outside the peak. Therefore, there are no conditions that would be sufficient for the construction of an optical delay device or other devices using *slow light*. At the same time, the frequency interval exists where the group refractive index is noticeably lower (*fast light*), which in principle can be used to increase the sensitivity of laser sensors, for example, gyroscopes.

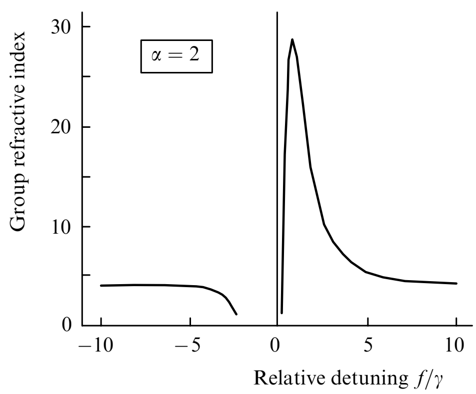

Figure 3. Calculated profile of the group refractive index in the vicinity of the strong mode with a maximum corresponding to the slowing of light caused by the nonlinear interaction of modes via population oscillations.

5. Conclusions

The nonlinear interaction of modes in a semiconductor laser is accompanied by frequency-local perturbations of the optical spectrum resulting in the shifts of mode frequencies of the laser cavity. Unlike the nonselective frequency chirp, these perturbations are characterised by a relatively narrow spectral interval comparable with inter-mode frequencies in long semiconductor lasers. Therefore, they change the relative positions of the adjacent resonances. It is these perturbations that can explain anomalous frequency shifts in the mode-beating spectra of semiconductor lasers, which depend on the optical power. The spectral symmetry of the refraction perturbation profile is determined by the amplitude–phase coupling factor α, the profile being more symmetric at large α. The theoretical model is in qualitative agreement with the anomalous splitting in the mode-beating spectrum and with the frequency shift sign and its value within the order of magnitude. The local perturbations of refraction are accompanied by considerable changes in the group refractive index in the vicinity of the strong mode.

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