

Transformation of the spatial coherence of pulsed laser radiation transmitted in the nonlinear regime through a multimode graded-index fibre

A.I. Kitsak, M.A. Kitsak

Abstract. A method is proposed for transformation of the spatial coherence of pulsed laser radiation upon nonlinear interaction in a multimode fibre. The specific features of the transmission of correlation properties of radiation in a graded-index fibre with regular and irregular profiles of the refractive index of the fibre core are analysed. A comparative analysis of the parameter of global degree of radiation coherence at the output of inhomogeneous waveguide and non-waveguide media is performed. It is shown that the most efficient mechanism of decorrelation of pulsed radiation in an optical fibre is fluctuations of the phase of radiation scattered by inhomogeneities of the refractive index of the fibre core induced due to nonlinear interaction with radiation with the spatially inhomogeneous intensity distribution.

Keywords: spatial coherence, graded-index fibre, speckle noise suppression, laser beams, induced incoherence.

1. Introduction

One of the main conditions for obtaining submicron resolution in the modern projection photolithography is the use of highly monochromatic ($\Delta\lambda = 0.5$ nm) electromagnetic radiation with a low spatial coherence. Nanosecond solid-state lasers allowing the conversion of the fundamental frequency to high harmonics can be promising sources of such monochromatic radiation. However, the problem in this case is a high spatial coherence of radiation resulting in the appearance of the speckle noise upon imaging.

One of the methods for decreasing the spatial coherence of pulsed laser radiation is based on self-phase modulation in a nonlinear medium [1]. The nonstationary modulation of the radiation phase appearing upon interaction of radiation with a medium with the cubic nonlinearity destroys the spatial coherence of the initially coherent field [2–5].

In this paper, we studied the transformation efficiency of the spatial coherence of a highly monochromatic pulsed (~ 10 ns) laser radiation upon stationary self-action in a multimode optical fibre with the parabolic refractive-index profile in the fibre core. An optical fibre, treated as a nonlinear medium, attracts attention because it allows the efficient transformation of the spatial coherence of low-power radiation due to accumulation of nonlinear phase perturbations at large interaction lengths [6]. In addition, SRS accompanied by the blue shift of the pump frequency can be excited in the fibre [7, 8], which is also important for photolithographic applications.

Analysis of the data from the literature shows that processes of transformation of the spatiotemporal structure of radiation upon its self-modulation in fibres still attract great attention. This is explained by the possibility to obtain the soliton propagation of radiation in certain regimes of nonlinear interaction, which is very convenient for optical data transfer over large distances [9, 10]. However, the dynamics of correlation characteristics of light beams in such media has not been adequately studied so far. Thus, this paper is also of interest as a study of the transfer of coherent properties of light beams through long multimode fibres.

2. Experiment

Figure 1 shows the optical scheme of the experimental setup for studying the efficiency of transformation of the spatial coherence of pulsed laser radiation upon self-action in a multimode fibre. The setup includes pulsed coherent radiation source (1) based on a doubled Nd:YAG laser emitting 10–15-ns, 532-nm pulses, microobjective (2) for radiation coupling into a fibre, short fibre (3) wound on a reel and forming guided modes, main fibre (4) of length ~ 200 m and diameter ~ 100 μm with the parabolic refractive-index profile of the core serving as a nonlinear

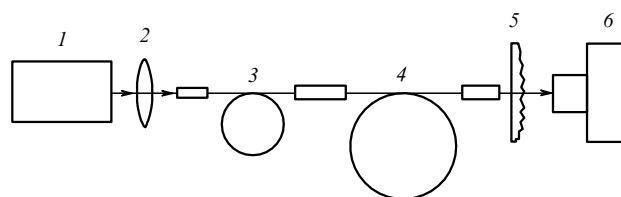


Figure 1. Optical scheme of the experimental setup.

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medium, mat plate (5), and CCD linear array detector (6) with 2048 pixels (with a spatial resolution of $\sim 14 \mu\text{m}$).

The transformation efficiency of coherent properties of the light field was estimated from the number of spatially coherent modes N_z produced during the propagation of radiation in a fibre [2]:

$$N_z = \left[\int B(\mathbf{r}, \mathbf{r}, z) d^2r \right]^2 / \iint |B(\mathbf{r}_1, \mathbf{r}_2, z)|^2 d^2r_1 d^2r_2. \quad (1)$$

Here,

$$B(\mathbf{r}_1, \mathbf{r}_2, z) = \int_{-\infty}^{\infty} \langle E(\mathbf{r}_1, z, t) E^*(\mathbf{r}_2, z, t) \rangle dt$$

is the spatial correlation function of the radiation field $E(\mathbf{r}, z, t)$ in the plane z perpendicular to the field propagation direction; \mathbf{r}_1 and \mathbf{r}_2 are the radius vectors in this plane; and t is the time coordinate.

For the Gaussian correlation function

$$B(\mathbf{r}_1, \mathbf{r}_2) = I_0 \exp \left[-\frac{(\mathbf{r}_1^2 + \mathbf{r}_2^2)}{a_0^2} - \frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{r_0^2} - i\alpha_0(\mathbf{r}_1^2 - \mathbf{r}_2^2) \right], \quad (2)$$

defined in a plane $z = 0$, the effective number of spatially coherent initial modes is

$$N_0 = 1 + 2a_0^2/r_0^2 = 1 + 2/C^2,$$

where I_0 , α_0 , r_0 and a_0 are the intensity, the phase of the spatial correlation function, the coherence radius of radiation and the radius of the light beam envelope in this plane, respectively; and $C = r_0/a_0$ is the radiation coherence coefficient [11].

The parameter N_z is directly related to the contrast K of speckles observed upon scattering of radiation by a fine-structure diffuser [12, 13]:

$$K \equiv \frac{\langle I^2 \rangle^{1/2}}{\langle I \rangle} \approx \frac{1}{\sqrt{N_z}}. \quad (3)$$

Here, $I(\mathbf{r}, z) = B(\mathbf{r}, \mathbf{r}, z)$ is the spatial distribution of the speckle pattern intensity.

We studied the dependence of N_z on the input radiation power P for different numbers N_0 of its initial modes and fixed z . For this purpose, we recorded the spatial distribution of the radiation intensity scattered by a mat plate in front of the fibre and at its output. The radiation intensity was recorded in the plane with a speckle structure (resolved by the detector) containing the number of speckles sufficient for obtaining correct statistical estimates. The speckle contrast K and related numbers N_0 and N_z were calculated from the recorded radiation intensity distributions. The mode composition of the input radiation was changed by placing apertures of different diameters into the resonator.

Figure 2 shows the experimental dependences of the effective number N_z of radiation modes on the power P for

$N_0 = 3$ and 25. One can see that, as the input radiation power increases, the degree of spatial coherence of radiation at the fibre output decreases (N_z increases). The maximum value of N_z is obtained when the radiation power is close to the SRS excitation threshold. This power was 700–1000 W for the fibre of length 200 m used in experiments. The spatial coherence of radiation was decreased approximately by an order of magnitude compared to the coherence of input radiation. Figure 3 presents the spatial distributions of the radiation intensity scattered from a mat plate in front of the fibre and at the fibre output for the input power close to the SRS excitation threshold. They clearly illustrate the efficient lowering of the spatial coherence of radiation by this method. The intensity distributions were recorded in the same plane at the distance from the mat plate greatly exceeding the transverse size of the secondary source formed on the plate by the recorded radiation.

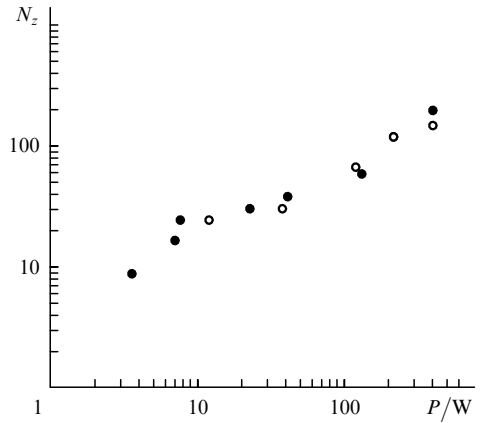


Figure 2. Dependences of the effective number N_z of radiation modes at the fibre output on the input power P for $N_0 = 3$ (●) and 25 (○).

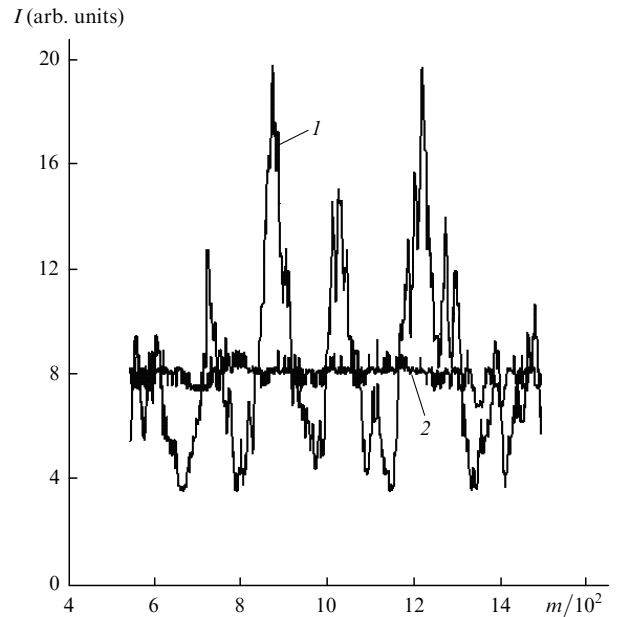


Figure 3. Spatial intensity distributions I behind a diffuser in the far-field zone in front of the fibre (1) and at the fibre output (2) for $P = 600$ W; m is the number of the detector channel.

3. Theoretical analysis

Consider the dynamics of the spatial coherence of quasi-monochromatic radiation pulses of duration $t_0 \geq 1$ ps interacting nonlinearly in a fibre with a parabolic distribution of the permittivity ε in the fibre core:

$$\varepsilon(\mathbf{r}) = \varepsilon_0(1 - q^2 r^2), \quad (4)$$

where \mathbf{r} is the radius vector in the plane perpendicular to the fibre axis; ε_0 is the permittivity at the fibre axis; $q^2 = \Delta/\rho^2$; $\Delta = (\varepsilon_0 - \varepsilon_c)/\varepsilon_0$ is the parameter of the parabolic profile height [14]; ε_c is the permittivity of the fibre cladding; and ρ is the fibre core radius.

The transfer equation for the spatial correlation radiation function $B(\mathbf{r}_1, \mathbf{r}_2, z)$ in such a fibre for narrow light beams with the Gaussian statistics of the input radiation field $E(\mathbf{r}, z) = A(\mathbf{r}) \exp[i(\omega t + kz)]$ in the case of the dominant Kerr nonlinearity has the form

$$\begin{aligned} & \left[\frac{\partial}{\partial z} - \frac{1}{2k}(\Delta_1 - \Delta_2) + \frac{i}{2}kq^2(r_1^2 - r_2^2) \right] B(\mathbf{r}_1, \mathbf{r}_2, z) \\ & = -i\beta[I_0(\mathbf{r}_2) - I_0(\mathbf{r}_1)]B(\mathbf{r}_1, \mathbf{r}_2, z). \end{aligned} \quad (5)$$

Here, $A(\mathbf{r})$ is the slowly varying complex amplitude of the light wave; $I_0 = |A(\mathbf{r}, z=0)|^2$ is the average radiation intensity at the fibre input; $k = 2\pi n/\lambda$; λ is the average radiation wavelength; n is the average refractive index of the fibre core; ω is the average radiation frequency; $\Delta_{1,2}$ are the transverse Laplacians; $\beta = 2k(n_2/n)$; and n_2 is the induced refractive index of the medium.

The interaction of radiation with a nonlinear medium is often described by a model in which the statistics of radiation propagating in the medium does not change during the interaction, and only the correlation parameters of the function of reciprocal coherence of radiation such as, for example, its phase, the size of the coherence region and the envelope of a light beam change. Such an approach can be applied in full measure to Gaussian beams with the Gaussian correlation function [11]. Assuming that these conditions are satisfied and the random field $E(\mathbf{r})$ is quasi-homogeneous, we will seek the solution of Eqn (5) in the form

$$\begin{aligned} B(\mathbf{R}, \mathbf{r}_{12}, z) = I_0 f(z) \exp \left[-\frac{2\mathbf{R}^2}{a^2(z)} - \frac{r_{12}^2}{r_{12}^2(z)} - \right. \\ \left. -i\alpha(z)\alpha_0 \mathbf{R} \mathbf{r}_{12} \right], \end{aligned} \quad (6)$$

where

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}; \quad \mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2; \quad a^2(z) = \frac{a_0^2(0)}{g(z)};$$

$$r_{12}^2(z) = \frac{r_{\text{com}}^2(0)}{h(z)}; \quad r_{\text{com}}^2(0) = \frac{2a_0^2 r_0^2}{2a_0^2 + r_0^2};$$

$f(z)$, $g(z)$, $h(z)$ and $\alpha(z)$ are the transformation coefficients

of the energy, spatial, and correlation characteristics of radiation, respectively, and $f(0) = g(0) = h(0) = \alpha(0) = 1$.

By substituting correlation function (6) into Eqn (5), grouping terms with the same powers of the product $R^m r_{12}^n$ ($m, n = 0, 1, 2$), and setting to zero the coefficients found after the grouping at all such products, we obtain a system of ordinary differential equations. By solving this system, we find the relations

$$g(z) = h(z) = f(z), \quad (7)$$

$$\begin{aligned} f(z) = 2q \{ (\alpha_0/k)^2 + F + q^2 - [(\alpha_0/k)^2 + F - q^2] \\ \times \cos(2qz) - 2\sqrt{\alpha_0}q \sin(2qz) \}^{-1}, \end{aligned} \quad (8)$$

for the axial region of the spatial distribution of the input radiation intensity, where

$$F = \frac{1}{l_d^2} - \frac{1}{l_n^2}; \quad l_d = \frac{ka_0 r_{\text{com}}}{2\sqrt{2}}$$

is the characteristic diffraction length of the beam spreading in a linear medium; and

$$l_n = a_0 \left(\frac{k}{8\beta I_0} \right)^{1/2}$$

is the characteristic length of nonlinear self-action in the medium.

It follows from expressions (7) that the radii of the envelope and correlation of the light beam change by the same law. The transformation law itself is determined by relation (8) and is periodic, which is typical for transmission of light beams through graded-index fibres. Thus, if the radius of the light-beam envelope in a fibre decreases by several times, the correlation radius of radiation decreases by the same factor. Therefore, the number $N_z(z)$ of spatially coherent radiation modes or the coherence coefficient $C(z) = r(z)/a(z)$ remains constant at any values of the input power, although the contrast of speckles formed by radiation transmitted through the fibre depends on the input radiation power. This result is obviously explained by the fact that we considered the propagation of radiation in an idealised straight fibre with a parabolic transverse distribution of the refractive index. However, the structure of real fibres is not perfect. Apart from various bendings, the distribution of the refractive index in fibres exhibits macroscopic deviations from the ideal distribution both in the transverse and longitudinal directions [15, 16]. All this gives rise to the interaction between modes excited in the fibre, their transformation and the appearance of spatial intensity fluctuations of a light beam. During the propagation of modulated radiation in the fibre and its nonlinear interaction with the fibre core, a random modulation of the permittivity of the waveguide medium is induced in the core. Radiation scattering by induced and stationary inhomogeneities and subsequent incoherent summation of scattered fields over the fibre length lead to the averaging of radiation fluctuations produced at the initial stage of radiation propagation in the fibre, which points to the transformation of the spatial coherence of radiation during the pulse action.

As an example, consider the influence of a random modulation of the spatial distribution of the permittivity of the fibre core

$$\varepsilon(z, \mathbf{r}) = \varepsilon_0[1 + \varepsilon_1(z, \mathbf{r}) - q^2 \mathbf{r}^2], \quad (9)$$

on the spatial coherence of radiation propagating in the fibre, where $\varepsilon_1(z, \mathbf{r})$ is the random component of the fibre core permittivity.

According to [17, 18], we will write the transfer equation for the second-order spatial coherence function $B(\mathbf{r}_1, \mathbf{r}_2, z)$ in the fibre taking into account the nonlinear interaction of radiation with the fibre core. For the near-axial part of a light beam in the approximations of the Markovian process of propagation of the light field in a random medium and the Gaussian statistics of the input radiation, this equation has the form

$$\left[\frac{\partial}{\partial z} - \frac{i}{2k}(\Delta_1 - \Delta_2) + \frac{i}{2}k\gamma^2(\mathbf{r}_1^2 - \mathbf{r}_2^2) + \frac{\pi k^2}{4}H(\mathbf{r}_1 - \mathbf{r}_2) \right] B(\mathbf{r}_1, \mathbf{r}_2, z) = 0, \quad (10)$$

where

$$H(\mathbf{r}_{12}) = \int_{-\infty}^{\infty} \Phi_{\varepsilon_1}(\mathbf{v}) [1 - \cos(\mathbf{v}\mathbf{r}_{12})] d^2v;$$

$\Phi_{\varepsilon_1}(\mathbf{v})$ is the three-dimensional fluctuation spectrum of ε_1 for $v_z = 0$; and

$$\gamma = \left(q^2 + \frac{1}{l_n^2} \right)^{1/2}.$$

Because the degree of transformation of the spatial coherence [i.e., the number $N_z(z)$ of radiation modes] of radiation with the Gaussian statistics can be estimated by determining the coherence coefficient $C(z) = r(z)/a(z)$, it is sufficient to find the quantities $r(z)$ and $a(z)$ entering this coefficient. By using the method of moments [18, 19] and results [16], we obtain for $\Phi_{\varepsilon_1}(\mathbf{v}) = \pi^{-3/2} \langle \varepsilon_1^2 \rangle l_0^3 \exp(-v^2/l_0^2)$ [18] the following expressions for the effective dimensions of the coherence region of radiation [$r_{\text{eff}}^2(z)$] and its envelope [$a_{\text{eff}}^2(z)$]:

$$\begin{aligned} r_{\text{eff}}^2(z) &= r_{\text{com}}^2(0) \frac{a_{\text{eff}}^2(z)}{a_0^2} \\ &\times \left\{ 1 + \left[\frac{1 + 4(l_d \gamma)^2}{\gamma^2 a_0^2} \right] \left[z - \frac{1}{2\gamma} \sin(2\gamma z) \right] \sigma + 4 \left(\frac{l_d}{\gamma a_0^2} \right)^2 \right. \\ &\quad \left. \times \left[z^2 - \frac{1}{\gamma^2} \sin^2(\gamma z) \right] \sigma^2 \right\}^{-1}, \end{aligned} \quad (11)$$

$$\begin{aligned} a_{\text{eff}}^2(z) &= a_0^2 \left\{ \cos^2(\gamma z) + \left(\frac{1}{l_d \gamma} \right)^2 \sin^2(\gamma z) \right. \\ &\quad \left. + \frac{4}{\gamma^2 a_0^2} \left[z - \frac{1}{2\gamma} \sin(2\gamma z) \right] \sigma \right\}, \end{aligned} \quad (12)$$

where

$$\sigma = \frac{\langle \varepsilon_1^2 \rangle}{\pi \sqrt{\pi} l_0};$$

l_0 and $\langle \varepsilon_1^2 \rangle$ are the correlation radius and dispersion of fluctuations of the permittivity ε_1 , respectively.

It can be shown after simple calculations that in the absence of permittivity fluctuations ($\sigma = 0$), the radius $r_{\text{eff}}^2(z)$ coincides with the radius $r_{12}^2(z)$ obtained earlier for a perfect fibre. By transforming (11) and (12), we can find the expressions for the relative numbers of modes N_w/N_0 and N_{nw}/N_0 in inhomogeneous waveguide and non-waveguide media ($\gamma = 0$), respectively. For distances $\gamma z \gg 1$ (passage to the limit $\gamma z \rightarrow \infty$) in a fibre, we have

$$\frac{N_w(z)}{N_0} = 1 + \left[\frac{1 + 4(l_d \gamma)^2}{\gamma^2 a_0^2} \right] \tau + 4 \left(\frac{l_d}{\gamma a_0^2} \right)^2 \tau^2. \quad (13)$$

Similarly, we obtain for a non-waveguide medium for the same distances:

$$\frac{N_{\text{nw}}(z)}{N_0} = 1 + 8 \left(\frac{l_d}{a_0} \right)^2 \tau + 4 \left(\frac{l}{a_0 \sigma} \right)^2 \tau^3 + \frac{4}{3} \left(\frac{l_d}{a_0^2 \sigma} \right)^2 \tau^4. \quad (14)$$

In (13) and (14), $N_{w,\text{nw}}(z) = 2a_{\text{eff}}^2(z)/r_{\text{eff}}^2(z)$; $N_0 = 2a_0^2 \times r_{\text{com}}^{-2}(0)$; and $\tau = \sigma z$ is the optical length. Note that the radius a_0 of the light-beam envelope at the fibre input can be considered approximately equal to the radius w of its fundamental mode [16], i.e.,

$$a_0 = w = \left(\frac{2\rho}{n_0 k \sqrt{2\Delta}} \right)^{1/2}. \quad (15)$$

It follows from relation (13) that the number of spatially coherent modes is no longer invariant during the propagation of radiation in an inhomogeneous waveguide medium. This number, and hence the degree of coherence transformation, depend on the propagation parameter τ , the relation between the diffraction characteristics of radiation and refraction properties of the fibre, which are determined by the product $l_d \gamma$, as well as on the parameter γ itself.

Figure 4 shows the contrast ratios for speckles formed by light beams transmitted through the fibre and non-waveguide medium with the same parameters of inhomogeneities as functions of the optical length τ for two propagation regimes: when the radiation divergence dominates over refraction and when the refraction of radiation exceeds diffraction spreading. The estimates were made using the following parameters of the fibre and radiation: $\Delta = 0.04$, $\rho = 50 \mu\text{m}$, $l_0 = 0.5 \mu\text{m}$, $n = 1.6$, $\lambda = 532 \text{ nm}$, $P = 600 \text{ W}$, $N_2 = 4\pi n_2/(nc) = 3.2 \times 10^{-16} \text{ cm}^2 \text{ W}^{-1}$, and $\langle \varepsilon_1^2 \rangle = 10^{-10}$.

The analysis of these dependences shows that for the model of distribution of the refractive index in the fibre core used here and the non-random spatial distribution of the input radiation amplitude, the radiation coherence is destroyed mainly due to the phase modulation upon radiation scattering by stationary inhomogeneities of the refractive index. Note that this process proceeds in the waveguide medium less efficiently than in the non-wave-

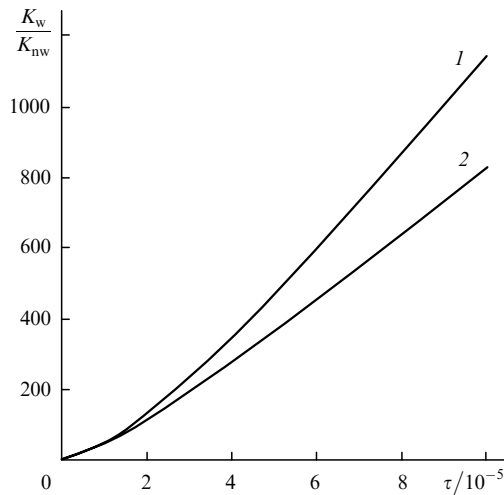


Figure 4. Contrast ratios of speckles formed by light beams transmitted through a fibre (K_w) and a non-waveguide medium (K_{nw}) with the same parameters of inhomogeneities as functions of the optical length τ for $l_d\gamma = 0.6$ (1) and 1.2 (2).

guide medium. This is probably explained by the escape of radiation scattered at angles exceeding the critical angle from the fibre, which is not involved in subsequent averaging events. Nonlinear interaction is manifested only as the addition $\Delta q = 1/l_n$ to the refraction parameter q of the fibre. Such an addition is approximately two orders of magnitude smaller than q even for the radiation power close to the excitation power of SRS in the fibre and virtually does not affect the efficiency of coherence destruction. In experiments (see Fig. 2), we observed the distinct dependence of the degree of radiation decorrelation in the fibre on the input radiation power. This dependence can be explained by the fact that a part of radiation in a real fibre always fluctuates. This radiation induces a random variation of the permittivity upon nonlinear interaction in the fibre.

As the input radiation intensity is increased, the dispersion of permittivity fluctuations also increases, resulting in the increase in the dispersion of phase fluctuations of radiation scattered by induced inhomogeneities of the refractive index of the waveguide medium, which finally leads to the destruction of radiation coherence. Unfortunately, the theoretical model used here neglects fluctuations of the radiation amplitude (it is assumed that the radiation statistics is mainly determined by fluctuations of the radiation phase) and therefore the real dynamics of the correlation properties of radiation in a fibre is described incompletely by this model.

4. Conclusions

It follows from our experimental results and theoretical consideration of the transformation of spatial coherence of radiation in an optical fibre that the radiation coherence in the fibre is destroyed upon stationary interaction mainly due to the radiation-phase modulation caused by scattering by the inhomogeneities of a waveguide medium followed by the incoherent summation of scattered fields during radiation propagation. Scattering can occur both from stationary and induced inhomogeneities of the refractive index of the fibre core caused by the nonlinear interaction

of radiation with the waveguide medium. The induced inhomogeneities are formed in the presence of spatial fluctuations of the radiation intensity, which can be caused by the lasing regime of the input radiation and interference effects in the initial part of the fibre.

Our experimental results have shown that scattering by stationary inhomogeneities is not a key factor of the destruction of spatial coherence in a fibre (the number of radiation modes at the fibre output in this case should not depend on the radiation power at the fibre input). In addition, radial multimode fibres have a rather high spatial homogeneity of the refractive-index distribution in the fibre core, and hence to obtain a noticeable destruction of the spatial coherence of radiation, long fibres are required. At the same time, the spatial modulation of the radiation intensity appears in the multimode fibre already at a small distance from its input because of the coherent summation of waves with random phases, which appear due to scattering both from stationary inhomogeneities of the refractive index of the fibre and the inhomogeneities caused by geometrical irregularities of the fibre (bendings along the fibre, variations in the fibre cross section and diameter). In the case of nonlinear interaction, these intensity fluctuations induce spatial inhomogeneities of the refractive index of the fibre core. The magnitude of nonlinear variation in the refractive index is proportional to the input radiation power. Scattering of radiation by inhomogeneities of this type leads, as follows from our experimental results, to a considerable transformation of the spatial coherence of radiation in the fibre.

Although the efficiency of coherence destruction in the waveguide medium is lower than in an extended scattering medium, it is preferable to use fibres for producing light beams with the specified coherent properties. This is explained by the fact that along with the destruction of radiation coherence, a fibre allows the radiation transport to any place, including intracavity volumes. In this case, the required degree of radiation coherence can be obtained by varying the fibre length and input radiation power.

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