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INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA

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Quasi-stationary magnetic and electric waves produced by a pulsed shock-wave source

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***Abstract*. A merging mechanism of shock waves in a plasma with a magnetic field is considered. The merging criterion is found at which a point source produces low-frequency waves of magnetic and vortex electric fields in the surroundings.**

Keywords: *pulsed wave source, wave merging mechanism, quasi-stationary magnetic and electric fields, shock wave.*

The shock-wave merging mechanism (WMM) in an optical pulsed discharge (OPD) in a gas was considered in papers [1, 2], where criteria for its manifestation were found and the hypothesis about its universal nature was proposed. It was pointed out in [1, 2] that periodic perturbations are successively produced in a continuous medium, for example, shock waves with the initial velocity exceeding the sound velocity C_0. The propagation velocity V_0 of the pulsation region is smaller than C_0. Shock waves produce a quasi-stationary wave if the parameters of pulsations and a medium satisfy the wave-merging criteria. Depending on the pulsation structure, the WMM is manifested in effects which are characterised by a large length of a quasi-stationary wave – the region of elevated pressure. In this case, no restriction is imposed on the medium type and the energy of pulsations – artificial or natural. Quasi-stationary waves are also formed in the case of random pulsations if a shock-wave source satisfies the wave-merging criteria. Effects produced by the WMM in an OPD are of interest for a number of applications, while high-power repetitively pulsed lasers [3, 4] open up prospects for applications of the WMM in various media.

In this paper, we consider the WMM as applied to the outer space consisting of a plasma with a magnetic field. Longitudinal vibrations propagate in this medium at the magnetosonic velocity, similarly to waves in gases. The aim of this paper is to find the criterion for WMM manifestation in a space pulsed source generating quasi-stationary waves of the magnetic and vortex electric fields [for brevity, magnetic quasi-stationary waves (MQWs)]. In this case, the hypothesis about the universal character of WMM should be specified. A train of pulses from a source produces an MQW – a region of elevated (compared to the background) pressure and a magnetic field. A magnetic quasi-stationary wave forms a vortex electric field. The MQW length is much greater than the length of a shock wave produced by a single pulse with energy equal to that of the pulse-train energy. The conversion efficiency of the energy source in the MQW can be as high as $\sim 30\,\%$.

Shock waves in a plasma were studied in detail [5 – 10]. Sporadic pulsed energy release generating supersonic waves is typical for a number of processes proceeding in space objects, for example, reconnection processes of a magnetic field in the Earth magnetosphere or solar corona [11]. The merging of supersonic quasi-periodic waves can be also important for heating the upper solar corona by the Alfven waves [12].

Space plasmas are characterised by the presence of a magnetic field. In this case, the parameter $\beta = 8\pi nT/B^2$ (the ratio of plasma and magnetic-field pressures) describing its relative role changes in a broad range, from $\beta \gg 1$ in the Earth magnetosphere up to $\beta \sim 1$ in the solar wind and magnetosphere tail and $\beta \ll 1$ in the solar corona. Here, n, and T are the plasma concentration and temperature, respectively; and B is the magnetic field strength. The mean free path of particles is macroscopic compared to other characteristics. Due to the absence of collisions, the magnetic field proves to be 'frozen' into the plasma. MQWs are a source of low-frequency oscillations, in particular, electromagnetic oscillations. Unlike a gas-dynamic quasi-stationary wave, an MQW contains the vortex electromagnetic field $E = VB/c \approx C_0\delta B/c$, where V is the plasma velocity in the MQW propagation direction and C_0 and c are the speed of sound and light, respectively. The electric field involved in low-frequency processes can cause various resonance effects. For example, the quasi-stationary and large-scale vortex electric field can accelerate particles up to high energies.

To demonstrate clearly the MQW formation process, we solved here the problem in the cylindrical one-dimensional geometry. We considered a homogeneous, collisionless, completely ionised ideal plasma of infinite volume with the concentration n_0, temperature T_0, and a 'frozen' homogeneous magnetic field with the strength B_0. A perturbation source, which is laser or resonance cyclotron radiation in the case of the laboratory plasma, caused the local heating of the plasma in a cylindrical volume. The cylinder axis was fixed and directed along the magnetic field, the cylinder length was assumed infinite and the cross section radius R_q was much smaller than the dynamic

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radius (see below). Hereafter, each instant and local heating of the plasma will be called a pulse. The pulse duration is small compared to the expansion time of the heated region. The problem was solved by using magnetohydrodynamic equations, which can be written in the form [13]

$$\frac{\partial}{\partial t}n_{\rm p} + \frac{\partial}{r\partial r}(rVn_{\rm p}) = 0,$$

$$\frac{\partial}{\partial t}P + V\frac{\partial}{\partial r}P + \gamma\frac{\partial}{r\partial r}(rVP) = 0,$$

$$\frac{\partial}{\partial t}B + \frac{\partial}{r\partial r}(rVB) = 0,$$

$$n_{\rm p}M\left(\frac{\partial}{\partial t}V + V\frac{\partial}{\partial r}V\right) = -\frac{\partial}{\partial r}\left(\frac{B^2}{8\pi} + P\right),$$

where M is the ion mass; $\gamma = 5/3$ is the adiabatic index of the ideal plasma. As characteristic physical quantities, we used the unperturbed values n_0, P_0, B_0, and R_0. Similarly to quasi-stationary waves in a gas [1], we assume that the magnetosonic velocity corresponding to the propagation velocity of weak perturbations in a plasma with a magnetic field is

$$C_0 = \left(\frac{\gamma k T_0}{M} + \frac{B_0^2}{4\pi n_0 M}\right)^{1/2}$$

(in cm s^{-1}) and the dynamic radius (in cm) is

$$R_0 = (Q/\tilde{P}_0)^{1/2},$$

where Q is the pulse energy (erg cm^{-1}); k is the Boltzmann constant; and temperature T_0 is measured in kelvin. The total pressure in the plasma is

$$\tilde{P}_0 = n_0 k T_0 + B_0^2/8\pi$$

(in dyn cm^{-2}).

For $\tilde{P}_0$, R_0 and C_0 defined in this way, the wave merging criterion is the same as that for quasi-stationary waves in a gas (see below). The characteristic time is $t_0 = R_0/C_0$. The dimensionless equations for calculating $\rho = r/R_0$, $\tau = tC_0/R_0$, $v = V/C_0$, $p = P/P_0$, $n = n_{\rm p}/n_0$, and $b = B/B_0$ have the form

$$\frac{\partial}{\partial \tau}n + \frac{\partial}{\rho\partial\rho}(\rho v n) = 0,$$

$$\frac{\partial}{\partial \tau}p + v\frac{\partial}{\partial\rho}p + \gamma p\frac{\partial}{\rho\partial\rho}(\rho v) = 0, \tag{1}$$

$$\frac{\partial}{\partial \tau}b + \frac{\partial}{\rho\partial\rho}(\rho v b) = 0,$$

$$n\left(\frac{\partial}{\partial\tau}v + v\frac{\partial}{\partial\rho}v\right) = -\frac{\partial}{\partial\rho}\left(\frac{A_b b^2}{2} + A_p p\right),$$

where the dimensionless parameters are expressed in terms of $\beta = 8\pi n_0 T_0/B_0^2$: $A_b = 2/(2+\gamma\beta)$, $A_p = \beta/(2+\gamma\beta)$. The plasma heating pulse was specified as an instant increase in the thermal pressure P in the region $\rho_{\rm q} < 0.1$ for a constant density and strength of the magnetic field. The corresponding pressure jump in the dimensionless form is $\Delta p = (1+\beta^{-1})/\rho_{\rm q}^2 \gg 1$. Upon periodic energy release (a large number of pulses), the plasma concentration on the axis decreases. To eliminate this feature, the radiation beam radius $\rho_{\rm q}$ was taken so that to cover plasma with concentration of no more than $0.01n_0$, the absorption region radius for each next pulse was greater than that for the previous pulse. In this case, the axial concentration did not decrease below $10^{-4}n_0$. Therefore, the problem has two parameters: the dimensionless pulse repetition rate

$$\omega = fR_0/C_0 \tag{2}$$

and the parameter β related to the magnetic field strength (f is the pulse repetition rate in hertz). The dimensionless equations were solved by using the conservative scheme of calculations in Eulerian variables.

The WMM can be considered universal if merging criteria for shock waves have the same form in different media and for different sources of pulsation energy. For a source moving in a gas, these criteria contain two dimensionless parameters – the source velocity with respect to a gas and the frequency of its pulsations [1, 2]. If a source is at rest, the wave merging and similarity are described only by frequency ω, whereas two parameters ω and β are used in a plasma with a magnetic field. However, in the latter case, as shown below, only ω can be used because the influence of β is small. In this case, the boundary frequency ω, expressed in dimensionless variables normalised to the sound speed and dynamical radius, at which the WMM begins to act, also weakly depends on β.

The formation of an MQW in a strong magnetic field is shown in Fig. 1, where the calculated 'oscillograms of a probe' measuring the drop of a total pressure at the distance $R_{\rm p} = 100$ from the source of pulsations are presented. As follows from our calculations, the waves are combined into MQWs within a region of size $\sim R_0$. The nonlinear interaction of shock waves in a train and the formation of a stationary shape and the spectrum of MQW trains are completed at the distance $(10-20)R_0$. A point with the coordinate $R_{\rm p} = 100$ can be treated as located at infinity. The wave intensity in a strong magnetic field is determined by the magnetic field pressure, so that it is reasonable to characterise the MQW by the total pressure drop, which is written in the dimensionless form as

$$\delta p = \frac{P}{P_0} - 1 = \frac{b^2 - 1 + \beta(p-1)}{1+\beta}. \tag{3}$$

For small β, the parameter δp is determined exclusively by the magnetic field: $\delta p \approx b^2 - 1$. Two boundary frequencies, ω_1 and ω_2, characterising the wave merging can be distinguished. One can see from Fig. 1 that for $\omega < \omega_1 \sim 0.2$, the signal is a train of shock waves. The compression and low-pressure phases of neighbouring shock waves are separated in time. For $\omega > \omega_2 \sim 1$, the shock waves of a train combine to form an extended high-pressure region with strong magnetic and electric fields (MQW). In the case of high-frequency pulsations ($\omega \gg \omega_2$), a signal produced by a pulse train is the same as for a single pulse with the total train energy (see Fig. 1, $\omega = 8$). The intermediate region is located in the frequency range $\omega_1 <$

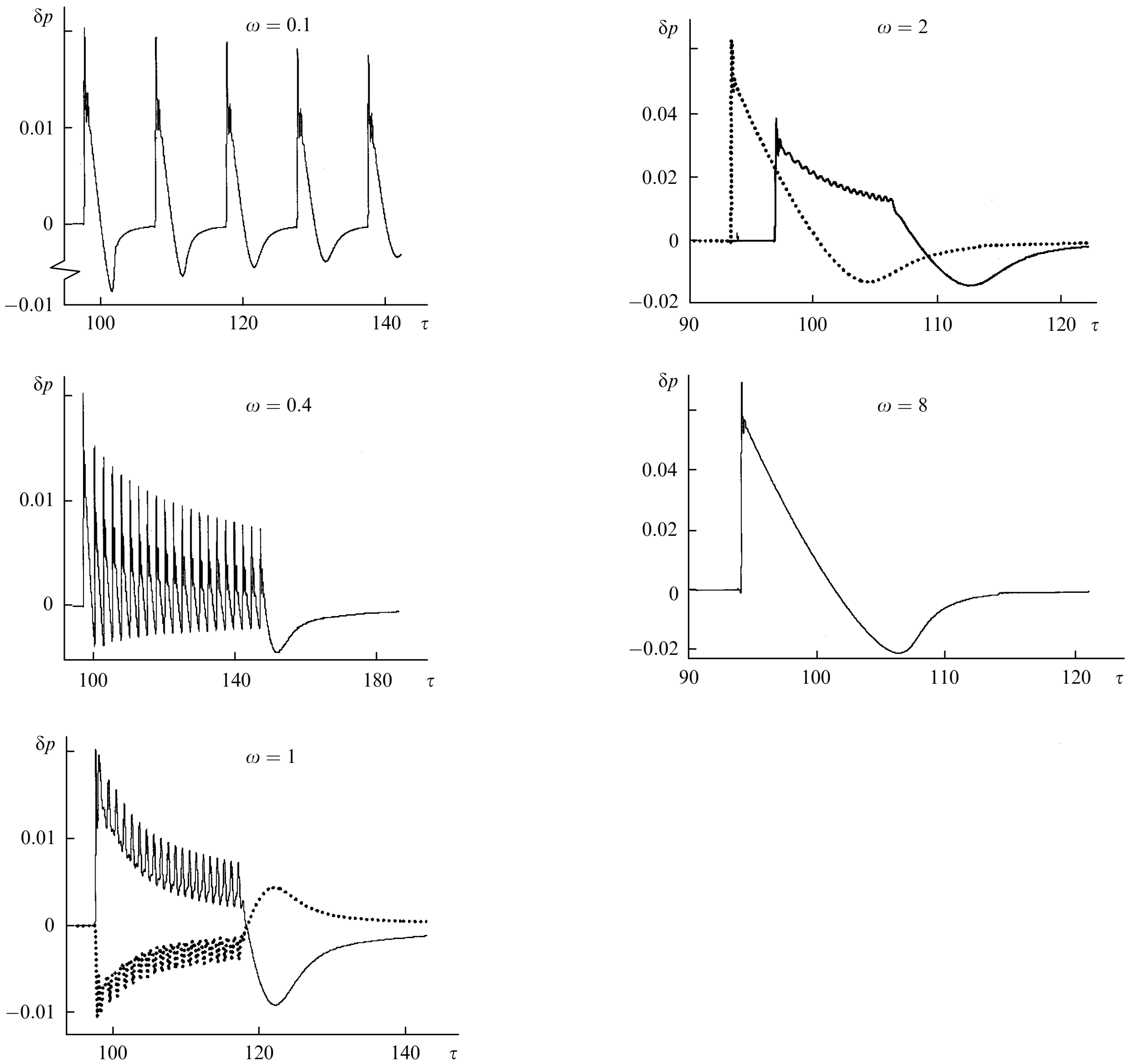

Figure 1. Total pressure drop δp at the distance $R_{\mathrm{p}} = 100$ from the source of pulsations for different pulse repetition rates ω for the number of pulses in a train equal to 20 and $\beta = 0.1$. The dotted curves for frequency $\omega = 1$ show the quasi-stationary vortex electric field $E = vb$, and for $\omega = 2$ – the signal from a pulse with energy equal to the total energy of a train of 20 pulses.

$\omega < \omega_2$. The values of ω_1 and ω_2 weakly depend on β. This follows from a series of calculations in which to each value of β the frequency ω^* corresponded at which the minimum concentration of plasma in the MQW did not exceed the initial concentration ($n_{\mathrm{p}}/n_0 \geqslant 1$) along the entire length of the MQW compression phase produced by a train containing many pulses. The value of ω^* changed from 0.82 to 0.765 in the range $\beta = 0.01 - 100$.

The most important difference of an MQW from a shock wave of the same energy is that when a train contain many pulses, the MQW is much longer than a shock wave because the MQW length increases linearly with the number N of pulses in the train, whereas the length of the shock-wave compression phase produced by a single pulse with the energy equal to the train energy depends on N weaker. The ratio of the MQW and shock-wave lengths is $\sim N^{0.68}$.

The MQW efficiency for generation of low-frequency oscillations was studied by the Fourier method. We considered a sequence of many trains of duration $T = 50$ (much more than unity) and following with the period $2T$, which corresponds to the train repetition rate $\Omega = 1/2T = 0.01$ (much smaller than unity). Each of the trains is filled with pulses with frequency ω, the number of pulses being $N = T\omega$. The shape of signals at a detector placed at the point $R_{\mathrm{p}} = 100$ is close to that shown in Fig. 1, where $N = 20$. Figure 2 shows the power spectra L_{s} of signals for different pulse frequencies. The conversion efficiency of high-frequency pulsations to low-frequency MQWs is characterised by the fraction of power in the spectrum

$$\delta_{\mathrm{c}}(\omega_{\mathrm{s}}) = \int_0^{\omega_{\mathrm{s}}} L_{\mathrm{s}} \mathrm{d}\omega_{\mathrm{s}}.$$

If $\omega < \omega_1$, the waves are not combined (Fig. 2a) and the value of δ_{c} in the low-frequency region Ω is small. As ω approaches ω_2, the value of $\delta_{\mathrm{c}}(\Omega)$ increases. Due to the formation of an MQW at $\omega > \omega_2$, the main part of energy ($\sim 70\,\%$) is concentrated at the frequency Ω. Spectral analysis confirms the nonlinear wave merging mechanism. Figure 2d presents the spectrum of a signal obtained by adding single pulses with the period $1/\omega = 1$ whose shape is shown in Fig. 1 ($\omega = 0.1$). It follows from a comparison of

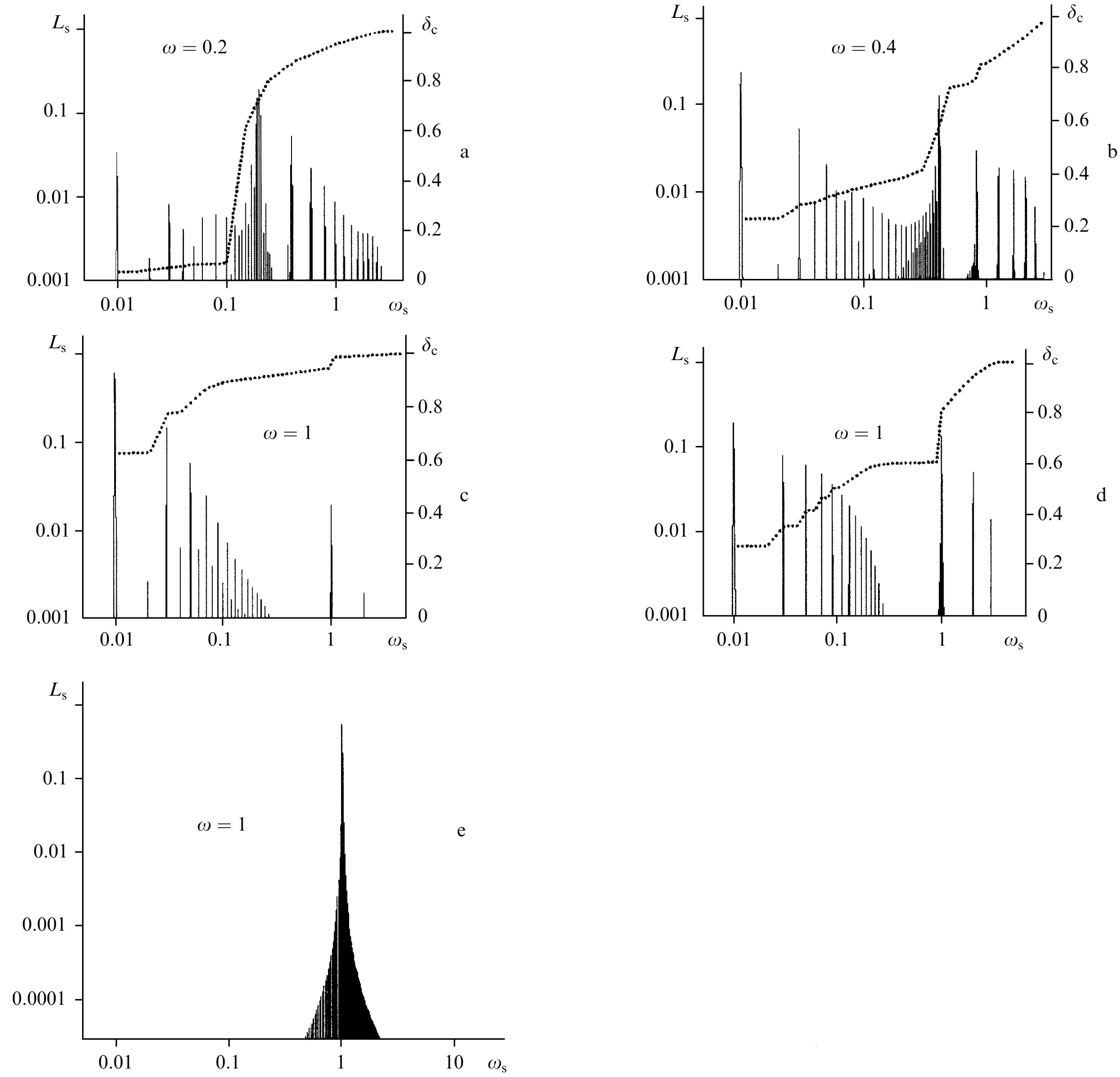

Figure 2. Power spectra L_s of trains with the repetition rate $\Omega = 0.01$ and different pulse repetition rates in trains. The dotted curves (δ_c) are the fraction of the spectrum power in the frequency interval $0 - \omega_s$. For $\omega \ll 1$, the carrier frequency ω and its overtones dominate in the spectrum. When the MQW is formed ($\omega > 1$), the train repetition rate and its overtones dominate. Spectrum d corresponds to a linear summation of the signal; e is the spectrum of trains filled with a sinusoidal signal.

the values of δ_c in Figs 2c, d that the MQW generates low-frequency harmonics much more efficiently. In addition, it was interesting to compare the spectra of trains during the MQW action with the spectrum of trains filled with a harmonic signal of frequency $\omega = 1$. The spectrum of sinusoid trains is shown in Fig. 2e. One can see that the main part of power is concentrated near $\omega_s \sim \omega$ (with the linewidth $\sim \Omega$), whereas the line intensity is negligible at the train repetition rate $\omega_s = \Omega = 0.01$.

The problem of MQW formation has two dimensionless parameters ω and β. However, we managed to find the representation of R_0, C_0 and ω, at which the wave merging and their similarity are determined, as in a gas, only by the frequency ω. A weak influence of β as the independent wave similarity and merging parameter is confirmed by calculations performed in the range $\beta = 0.01 - 100$. The similarity of the mechanisms of shock-wave formation in plasmas ($\beta \gg 1$) and MQWs in a magnetic field ($\beta \ll 1$) is demonstrated in Fig. 3, where signals of a remote probe are presented for different magnetic fields. The dimensionless pulse repetition rates and distances from the probe are the same in all the cases. One can see that these signals are very close to each other despite the fact that the value of δp in a strong magnetic field is proportional to a change in the magnetic field, while in a weak field it is proportional to a change in the thermal pressure. A slight difference is caused by the influence of β. Therefore, a pulsed source of shock waves produces MQWs in a plasma with a magnetic field if the wave-merging criterion

$$\omega \equiv fR_0/C_0 > \omega_2 \sim 1 \qquad (4)$$

is fulfilled. The frequency ω is at the same time the similarity parameter, which is achieved by using expression (3) for the total pressure drop in the MQW. The choice of the right-hand side of (4) admits some uncertainty. Shock waves begin interacting with each other already for $\omega > \omega_1$. This is manifested in the change in the spectrum and

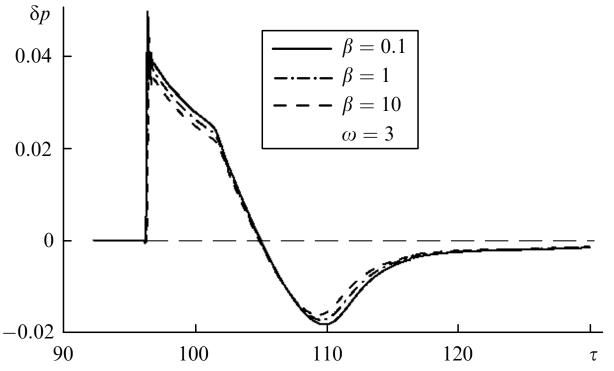

Figure 3. Total pressure drop δp at the distance $R_p = 100$ for a fixed pulse repetition rate in a strong ($\beta = 0.1$), moderate ($\beta = 1$), and weak ($\beta = 10$) magnetic fields. The number of pulses in the train is 20.

fraction of power at the train repetition rate. Adopted criterion (4) corresponds to the case when more than 60 % of the power of the spectrum of periodic trains is contained in the main line in the spectrum of trains following at the repetition rate Ω. Another approach is also possible, when expression (4) is written in the form

$$\omega \equiv fR_0/C_0 > \omega^* \sim 0.8 .$$

In this case, $\delta_c(\Omega)$ is smaller but the condition is fulfilled according to which the concentration of particles in the MQW compression phase is higher than the background concentration (see above). A comparatively narrow range $\omega_1 - \omega_2$ corresponds to a very broad region of the dimensional values of energy and pulsation frequency of the source.

Note that the boundary frequencies ω_1 and ω_2 depend on the symmetry type. Their values prove to be close to those obtained in gases for the cylindrical symmetry of a pulsation source [14]. For the spherical symmetry, the values of ω_1 and ω_2 are several times larger [1, 2, 15].

Let us estimate the spatiotemporal scales of events in the solar corona, which can result in the MQW formation. For the average magnetic field ~ 1 G and the concentration of particles $\sim 10^9$ cm^{-3}, the average value of the magnetosonic velocity C_0 is $\sim 10^7$ cm s^{-1} and $\beta = 0.1$. The explosive energy release (flares) occurs in local regions where magnetic fields can be ~ 1 kG. The merging of shock waves from separate flares will occur only in the case of a certain relation between the energy and frequency of the events. Because flares are distributed randomly in space, the estimates for the spherical geometry should be used [1, 2], in which the boundary frequencies of the shock-wave merging are approximately three times higher than those for the cylindrical geometry. For the observation of WMM effects at the solar corona scale $\sim 10^{10}$ cm, the events with the dynamical scale of separate flares $R_0 < 10^9$ cm, corresponding to $Q < 4 \times 10^{25}$ erg, should be investigated. Events with such an energy are classified in solar physics as micro- and nanoflares [16]. The required repetition rate of events depends on the energy as $f > 10^7/Q^{1/3} > 0.03$, i.e., the microflare repetition period in the solar corona is no less than 30 s. By using criteria [1], we can find both the maximum distance and energy scatter of a train of flares at which MQWs are formed.

Therefore, a fixed source generating a train of shock waves produces a quasi-stationary wave of the magnetic and vortex electric fields in a plasma with a strong magnetic field. The merging of the waves and formation of an MQW, as in a gas, is determined only by the dimensionless pulsation frequency ω. This additionally confirms the hypothesis about the universal mechanism of wave merging. When the number of pulses is large, the MQW length is much greater than the length of a shock wave produced by a single pulse with the energy equal to the train energy. The main power in MQW trains is concentrated at their repetition frequency. In the outer space, processes with parameters corresponding to the MQW formation conditions can occur.

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