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On stimulated VUV emission of the atomic helium in a Bose – Einstein condensate: 1.

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***Abstract*. A scenario of the experiment on the observation of stimulated VUV emission at a wavelength of 62 nm from metastable states of helium atoms in a Bose – Einstein condensate moving along an extended quantum well (trap) is considered. The quantitative estimates of parameters are presented.**

Keywords: *Bose – Einstein condensate, stimulated emission, metastable states.*

1. Introduction

Increasing the emission frequency of lasers beyond the near-UV spectral range involves considerable difficulties caused by a drastic decrease in the excited-state lifetime and a catastrophic increase in the spontaneous emission rate, which requires the same drastic increase in the pump intensity. This circumstance was pointed out already in the pioneering work of Schawlow and Townes in 1958 [1]. This obstacle can be overcome in a natural way by using metastable states, from which stimulated emission can be observed without any pumping at all if the lifetime of a metastable state is long enough [2, 3]. It is generally believed, however, that another impediment appears in this case due to a very low cross section for stimulated emission at a strongly forbidden transition from the metastable state. As a matter of fact, as follows directly from the Einstein thermodynamic derivation of emission laws, this impediment is absent in principle because after the elimination of the excess broadening of a emission line over its natural radiative width, the stimulated emission cross section σ is always equal to $\lambda^2/2\pi$ irrespective of the multiplicity and the degree of forbiddeness of the transition [4, 5]. Therefore, the problem of observation of stimulated emission from long-lived metastable states is reduced to the suppression or even complete elimination of the excess line broadenings of all types. The excess broadening is mainly caused, as a rule, by the thermal motion of emitters. To suppress efficiently this broadening, it was proposed to use the zero-phonon Mössbauer line with the natural width [2, 3], the deep cooling of free emitters by the methods of laser manipulation by neutral atoms [6], the inclusion of atoms into a Bose – Einstein condensate (BEC) [4, 5, 7], etc.

The aim of this study based on the above considerations is to analyse the possibility of observing stimulated VUV emission from the $2\,^3S_1$ metastable state of atomic helium [8] with the energy $E_\omega = 19.820$ eV and spontaneous lifetime τ equal to a few milliseconds [9] (below, the value $\tau \approx 0.003$ s is used for numerical estimates). This problem is important as a demonstration experiment with one of the few atomic metastable states, which confirms the concepts adopted above and opens up approaches to the creation of VUV lasers, and especially as a model study of the possibility of extending the considered approach to nuclear metastable states.

If all possible sources of the helium line broadening are suppressed, the cross section for stimulated emission from the $2\,^3S_1$ state is not low at all, being equal to $\sigma \approx 6 \times 10^{-12}$ cm^2, which, for example, for the concentration of metastable atoms equal to 5×10^{10} cm^{-3} gives the total gain $G \approx 1.35$ at the medium length $L = 1$ cm. However, in this case, to decrease the Doppler broadening down to the natural radiative width by cooling an atomic helium gas, an extremely low temperature below 10^{-12} K would be required, which hardly can be achieved at present. Therefore, based on the results of successful experiments on the observation of a helium BEC [10, 11] in which the condensate concentration equal to $(3.8 \pm 0.7) \times 10^{13}$ cm^{-3} was achieved in the middle of a trap [10] for a total number of condensed metastable atoms equal to 5×10^5, the most promising seems the approach involving the introduction of helium atoms into a BEC [4, 5, 7, 8].

A general scenario of this approach includes a sequence of operations with a directed flow of atomic helium propagating through a number of zones in an extended quantum trap channel. This sequence performs the following functions: excitation of atoms by an electron impact accompanied by the formation of metastable states, deep laser cooling of atoms down to temperature T, increasing the atomic concentration in the channel by kinematic methods with increasing the critical temperature T_0 of the BEC formation in a flow up to $T_0 > T$, the condensation of metastable atoms and, finally, stimulated VUV emission from metastable states of the condensate atoms.

First we recall some physical concepts forming the basis of this approach.

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2. Stimulated emission cross section according to Einstein

According to the thermodynamic (i.e., the most general) derivation of the emission laws by Einstein, the stimulated emission cross section has the form

$$\sigma = \frac{4\hbar\omega B_{21}}{c\Delta\omega_{\rm tot}} = \frac{A_{21}\lambda^2}{2\pi\Delta\omega_{\rm tot}} = \frac{\lambda^2}{2\pi\tau\Delta\omega_{\rm tot}} = \frac{\lambda^2}{2\pi}\beta, \tag{1}$$

where ω is the emission frequency; τ is the lifetime with respect to the radiative spontaneous decay; B_{21} and A_{21} are the Einstein coefficients; and

$$\beta = \frac{\Delta\omega_{\rm rad}}{\Delta\omega_{\rm tot}} \leqslant 1 \tag{2}$$

is the ratio of the natural radiative linewidth $\Delta\omega_{\rm rad}$ to the total linewidth $\Delta\omega_{\rm tot}$ including all the types of excess broadenings, both the homogeneous (for example, caused by a limited observation time of an emitter flying through a region to which a field is applied) and inhomogeneous (for example, Doppler broadening caused by a scatter in the velocities of emitters) broadenings.

Due to the generality of the thermodynamic approach, in which any details of a quantum-mechanical transition such as its multipolarity, the matrix element value, the forbiddeness degree, etc., are absent, expression (1) has an absolute sense. The above details are reflected only in the lifetime τ of a state with respect to spontaneous decay, which, of course, depends on them and can vary in a broad range [4, 5].

The excess line broadening is predominantly determined by thermal motion and, in a condensed medium, by the interaction of an emitter with the environment.

3. Bose – Einstein condensation as a method for eliminating the excess broadening of an emission line

It is known that the BEC atoms occupy the same quantum state, have a common wave function and can be treated as a 'megaatom'. Therefore, their individual motions and mutual displacements are strongly restricted, resulting in a drastic suppression of the inhomogeneous broadening of the emission line related to these motions. Thus, emitters of the BEC represent a medium that is probably most convenient for observing stimulated emission directly from long-lived metastable states [4, 5, 7]. All the estimates made below assume that any type of the excess line broadening is absent for the radiative transition from the metastable state of atomic helium ($\beta \to 1$).

The phase transition to the BEC occurs at the gas temperature T below the degeneracy temperature T_0 [12] ($T < T_0$)

$$T_0 = 3.3\frac{\hbar^2 n^{2/3}}{(2J+1)^{2/3}k_{\rm B}M} \approx 4\times10^{-15}n^{2/3}, \tag{3}$$

where n is the gas concentration; M is the atom mass; J is its angular momentum; and $k_{\rm B}$ is the Boltzmann constant; the numerical value is related to helium.

This process can proceed in two ways: by decreasing temperature down to $T < T_0$ at $n = {\rm const}$ or by increasing the degeneracy temperature up to $T_0 > T$ due to an increase in the gas concentration n at $T = {\rm const}$ [13]. In both cases, the concentration of BEC atoms is

$$n_{\rm BEC} = n\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right]. \tag{4}$$

One can see that due to a small mass of a helium atom, the degeneracy temperature T_0 proves to be moderately low and, hence, the gas temperature T required to obtain an appreciable ratio $n_{\rm BEC}/n$ is also not too low (thus, $T_0 \approx 10^{-7}$ K for $n = 10^{12}$ cm^{-3} and $n_{\rm BEC}/n \approx 0.91$ for $T/T_0 = 0.2$; hereafter, the numerical estimates give only the orders of magnitude of quantities and are not intended for any optimisation).

Note here that, strictly speaking, expressions (3) and (4) are valid only for a continuous spectrum of free atoms. However, the estimates obtained from them will be used for atoms in potential channels with a discrete spectrum of states, which leads to a small quantitative but not qualitative difference appearing in passing to a discrete spectrum.

4. Limitations of the interatomic collision and flight time

If we even leave aside possibly too optimistic estimates [7] based on the assumption that the excess line broadening of the transition from the metastable state of atomic helium is caused only by an incomplete quantum coherence of condensate atoms, the condition $\beta \to 1$ can be fulfilled when at least two more criteria are satisfied: the average lifetime $\Delta t_{\rm BEC}$ of condensate atoms and their flight time $\Delta t_{\rm L}$ in the region of interaction with the field should noticeably exceed the lifetime τ of the metastable state.

The lifetime $\Delta t_{\rm BEC}$ of condensate atoms in any case cannot exceed the average time between their collisions with atoms outside the condensate with the collision cross section $\sigma_{\rm col}$. Therefore, the estimate

$$\Delta t_{\rm BEC} = [\sigma_{\rm col}v(n - n_{\rm BEC})]^{-1} = \left[\sigma_{\rm col}vn\left(\frac{T}{T_0}\right)^{3/2}\right]^{-1} > \tau \tag{5}$$

is valid, where $v = v(T)$ is the temperature-dependent thermal velocity of atoms outside the condensate. It follows from (5) that

$$n < 0.43\left(\frac{M}{\hbar\sigma_{\rm col}\tau}\right)^{3/4}(2J+1)^{1/4}\left(\frac{T_0}{T}\right)^{3/2}$$
$$\text{or}\quad T < \frac{1.9\hbar^{3/2}}{k_{\rm B}[(2J+1)M\sigma_{\rm col}\tau]^{1/2}}. \tag{6}$$

The flight time is equal to the ratio of the length L of the interaction region to the transport velocity V of the atomic flow, so that the second criterion can be represented in the form

$$\Delta t_{\rm L} = \frac{L}{V} > \tau. \tag{7}$$

Assuming, for example, that $\sigma_{\rm col} = 10^{-16}$ cm^2, we obtain from (6) that $T < 10^{-4}$ K and $n < 10^{18}$ cm^{-3} for $T/T_0 = 0.2$, and from (7) that $V < 330$ cm s^{-1} for $L = 1$ cm. One can see that the estimated limitations prove to be comparatively mild.

5. Kinematics of neutral atoms in extended potential channels

The preparation of an amplifying medium of the BEC of metastable helium atoms in the proposed method is based on kinematic phenomena in a flow of neutral atoms propagating in a channel with a potential well over the two transverse coordinates x and y, which have discrete energy eigenvalues E_{mk} and experience a continual motion along the third longitudinal coordinate z [13]. Such potential channels, which usually have a parabolic dependence of the potential on the transverse coordinates and solutions of the harmonic oscillator type [14], are known in experimental practice [15].

The lowest energy eigenvalue ($m = k = 0$) is described by the expression

$$E_{00} = \hbar\left(\frac{2\alpha}{M}\right)^{1/2}, \tag{8}$$

and the longitudinal component of the wave vector (the longitudinal momentum of an atom) is

$$p_{00} = \pm[2M(E - E_{00})]^{1/2} = \pm\left\{2M\left[E - \hbar\left(\frac{2\alpha}{M}\right)^{1/2}\right]\right\}^{1/2}, \tag{9}$$

where α is the proportionality coefficient in the expression for the parabolic transverse potential of the channel

$$U(x,y) = \alpha(x^2 + y^2), \tag{10}$$

and E is the total energy of the atom. For a channel with the circular cylinder symmetry of type (10), it is convenient to introduce the efficient diameter in the lowest state

$$D = 2\left(\frac{2\hbar^2}{\alpha M}\right)^{1/4} = 2\hbar\left(\frac{2}{ME_{00}}\right)^{1/2}. \tag{11}$$

A flow of atoms with energy E and a full intensity Q propagates along the cannel in the positive direction of the z axis, the value of Q being constant for all z due to the flow continuity. The concentration of atoms in the lowest state is

$$n = \frac{4Q}{\pi D^2 V} = \frac{Q}{\pi\hbar V}\left(\frac{\alpha M}{2}\right)^{1/2}, \tag{12}$$

where

$$V = \frac{p_{00}}{M} = \left[\frac{2}{M}(E - E_{00})\right]^{1/2} \tag{13}$$

is the transport flow velocity.

Let us assume that the coefficient α in (10) is no longer a constant but slowly increases with z:

$$\frac{d\alpha(z)}{dz} > 0. \tag{14}$$

Then, the quantities determined by expressions (11), (10), (8), (9), and (13) also slowly change [13]:

$$\frac{dD}{dz} < 0,\ \frac{dU}{dz} > 0,\ \frac{dE_{00}}{dz} > 0,\ \frac{d|p_{00}|}{dz} < 0,\ \frac{dV}{dz} < 0. \tag{15}$$

As a result, as the atomic flow with $Q = \text{const}$ is propagating along the z axis, its concentration increases (12)

$$\frac{dn}{dz} > 0, \tag{16}$$

and, hence, the degeneracy temperature (3) of the gas increases:

$$\frac{dT_0}{dz} > 0. \tag{17}$$

Densification of the flow is caused both by its slowing down and the decrease in diameter (11) during the transformation of the kinetic energy of the longitudinal motion of the flow to the potential energy of the quantum state in the transverse well [13]. When the increasing degeneracy temperature T_0 exceeds the gas temperature at $z = z_{\text{BEC}}$ ($T_0 > T$), a part of atoms forms a condensate, as pointed out in Section 3 [13].

The compression coefficient, i.e., the ratio of gas concentrations for two successive longitudinal coordinates of the flow entry into the compression region (z_{in}) and the condensate formation (z_{BEC}) is

$$\Xi_{\text{comp}} = \frac{n(z_{\text{BEC}})}{n(z_{\text{in}})} = \left[\frac{D(z_{\text{in}})}{D(z_{\text{BEC}})}\right]^2\left[\frac{1 - 8\hbar^2/MED^2(z_{\text{in}})}{1 - 8\hbar^2/MED^2(z_{\text{BEC}})}\right]^{1/2}$$

$$\approx \left[\frac{D(z_{\text{in}})}{D(z_{\text{BEC}})}\right]^2\left\{1 + \frac{4\hbar^2}{ME}\left[D^{-2}(z_{\text{BEC}}) - D^{-2}(z_{\text{in}})\right]^{-2}\right\}, \tag{18}$$

where the approximate equality is valid in the case $E \gg E_{00}(z_{\text{BEC}})$.

If the channel acquires new properties at $z > z_{\text{BEC}}$, which are characterised by new but constant values of $\alpha(z \geqslant z_{\text{BEC}}) = \text{const}$ and $D(z \geqslant z_{\text{BEC}}) = \text{const}$, then a mixture of the condensate and non-condensed gas in a proportion determined by (4) propagates through the channel with the longitudinal velocity $V(z_{\text{BEC}})$ determined by expression (13) for the new value $E_{00} = E_{00}(z_{\text{BEC}})$.

6. Recoil of an atom, the shift of emission and absorption lines and hidden inversion. The gain anisotropy

The recoil upon emission or absorption of a VUV photon by a light helium atom is comparatively large, and the corresponding shifts of the lines

$$\frac{\Delta\omega_{\text{rec}}}{2\pi} = \pm\frac{E_\omega^2}{4\pi\hbar Mc^2} \tag{19}$$

exceed in modulus the radiative width by several orders of magnitude: $|\Delta\omega_{\text{rec}}/2\pi| \approx 10^7$ Hz $\gg \tau^{-1} \approx 330$ Hz. Therefore, the so-called hidden inversion [6] is always present in a medium of metastable helium atoms, at which for any ratio of the populations of the upper and lower transition levels, even smaller than unity, the photon flux can be amplified due to the absence of absorption of photons by ground-state atoms, which are not resonant with emitted photons.

A finite longitudinal transport velocity $V(z_{\rm BEC})$ of the atomic flow causes the Doppler frequency shifts of opposite signs for the transition from the metastable state

$$\frac{\Delta\omega_{\rm D}}{2\pi} = \pm\frac{V(z_{\rm BEC})}{c}\frac{E_\omega}{2\pi\hbar}, \tag{20}$$

whose modulus can substantially exceed the radiative linewidth. For this reason, photon fluxes propagating in the opposite directions along the z axis experience the frequency anisotropy of the gain, which is determined by the double modulus of the Doppler shift (20).

7. Losses of VUV photons

The nonresonance losses of photons are determined by photoionisation of helium atoms with the cross section $\sigma_{\rm ph}$ and by other types of scattering and absorption by atomic electrons with the averaged cross section χ, as well as by the transmission and absorption of mirrors if the latter are used.

Resonance photons with the energy $E_\omega \approx 20$ eV cannot be absorbed due to photoionisation of a helium atom from the ground state with the ionisation potential of about 24.59 eV, but they can be absorbed only upon photoionisation of atoms in the excited metastable state with the concentration n^*. Therefore, the coefficient of photoionisation losses is proportional to n^* and is equal to $\sigma_{\rm ph}n^*$, where $\sigma_{\rm ph} < 10^{-17}$ cm^2. The averaged coefficient of scattering losses of all types is proportional to the total gas concentration n and is equal to χn.

Photon losses on mirrors are determined by their reflection coefficients, which achieve the acceptable values of the order of tens of percent in the VUV region when multilayer structures are used [16].

8. Scheme of the scenario

As mentioned above, the main element of the scheme is an atomic helium flow transported through regions I–IV, each of them fulfilling certain functions. Consider the sequence of these regions.

In region I, helium atoms are excited by the electron impact with the formation of the metastable $2\,^3\mathrm{S}_1$ states. In the stationary case, the amount of atomic helium in the region is continuously supplemented by an external flow of new atoms.

In region II, helium atoms are cooled by laser manipulation methods down to microkelvin temperature T and an atomic beam is formed with $Q = $ const and the longitudinal transport velocity $V = (2E/M)^{1/2}$. Due to a small mass of helium atoms, their cooling time is much shorter than the lifetime τ, so that the concentration of metastable states weakly decreases due to spontaneous decay during cooling. The flow of cooled atoms with the energy E, the total concentration $n(z_{\rm in})$, and the concentration of metastable atoms $n^*(z_{\rm in})$ enters into region III at the point $z = z_{\rm in}$.

Region III is a potential channel with the effective dimeter D decreasing from $D(z_{\rm in})$ at $z = z_{\rm in}$ down to $D(z_{\rm BEC}) < D(z_{\rm in})$ at $z = z_{\rm BEC}$. Upon entering this channel with a transverse quantum well, the transport velocity of the flow decreases from V to

$$V_{\rm in} = \left(\frac{2E}{M}\right)^{1/2}\left[1 - \frac{8\hbar^2}{MED^2(z_{\rm in})}\right]^{1/2}. \tag{21}$$

As the atomic flow further propagates in the channel with the decreasing efficient diameter D, its velocity decreases from the longitudinal transport velocity $V_{\rm in}$ down to

$$V(z_{\rm BEC}) = \left(\frac{2E}{M}\right)^{1/2}\left[1 - \frac{8\hbar^2}{MED^2(z_{\rm BEC})}\right]^{1/2} \tag{22}$$

with the slowing down coefficient

$$\Xi_V = \frac{V(z_{\rm BEC})}{V_{\rm in}} = \left[\frac{1 - 8\hbar^2/MED^2(z_{\rm BEC})}{1 - 8\hbar^2/MED^2(z_{\rm in})}\right]^{1/2}. \tag{23}$$

The flow cross section simultaneously decreases with the coefficient

$$\Xi_D = \left[\frac{D(z_{\rm BEC})}{D(z_{\rm in})}\right]^2. \tag{24}$$

As a result, the flow is compressed with the coefficient

$$\Xi_{\rm comp} = (\Xi_V\Xi_D)^{-1}, \tag{25}$$

which increases the concentration of helium atoms at the coordinate $z_{\rm BEC}$ up to $n(z_{\rm BEC})$ and, according to (3), rises the degeneration temperature up to $T_0 > T$ with the formation of a Bose condensate at concentration (4).

After entering region IV with the longitudinal coordinate $z_{\rm BEC}$, the narrowing potential channel of region III passes to a channel with a constant effective diameter $D = D(z_{\rm BEC})$ and length L, in which the flow of a mixture of the Bose condensate of metastable and unexcited atoms and non-condensed atoms propagates, which forms the amplifying medium. In this medium, the VUV photon flux is amplified at stimulated transitions for the metastable state with the total gain G.

The total small-signal gain G of the VUV photon flux, when a decrease in the population of metastable levels can be neglected, in the stationary case, taking into account the discussion presented in sections 6 and 7, is described by the expression

$$\begin{aligned}\ln G &= \sigma V(z_{\rm BEC})\tau n^*_{\rm BEC}(z_{\rm BEC})\left\{1 - \exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\} \\ &\quad\times\left[1 - \frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}\right] - \chi n(z_{\rm BEC})L \\ &\approx \sigma V(z_{\rm BEC})\tau n^*_{\rm BEC}(z_{\rm BEC})\left\{1 - \exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\} \\ &\quad - \chi n(z_{\rm BEC})L,\end{aligned} \tag{26}$$

where $n(z_{\rm BEC})$ is the total concentration of atoms; $n^*(z_{\rm BEC})$ is the total concentration of metastable atoms; $n^*_{\rm BEC}(z_{\rm BEC})$ is their concentration in the condensate at the input to region IV at $z = z_{\rm BEC}$. The condition for the threshold one-pass amplification with $G \geqslant 1$ is described by the expression

$$\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm th} \geqslant \frac{\chi L}{\sigma V(z_{\rm BEC})\tau} \times\left\{1-\exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\}^{-1}\left[1-\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}\right]^{-1} \approx \frac{\chi L}{\sigma V(z_{\rm BEC})\tau}\left\{1-\exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\}^{-1}. \quad (27)$$

The threshold condition in the resonator case should, as usual, include additional losses of the photon flux on mirrors taking into account the unidirectional gain due to its frequency anisotropy. The maximum value of the gain $G_{\max}$ determined by the expression

$$\ln G_{\max} = \sigma V(z_{\rm BEC})\tau n^*_{\rm BEC}(z_{\rm BEC})\left\{1-\frac{\chi n(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})} \times\left[1+\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\chi n(z_{\rm BEC})} + \ln\left(\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})-\sigma_{\rm ph}n^*(z_{\rm BEC})}{\chi n(z_{\rm BEC})}\right)\right]\right\} \approx \sigma V(z_{\rm BEC})\tau n^*_{\rm BEC}(z_{\rm BEC})\left\{1-\frac{\chi n(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})} \times\left[1+\ln\left(\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})}{\chi n(z_{\rm BEC})}\right)\right]\right\} \quad (28)$$

is achieved for the amplification length

$$L_{\max} = V(z_{\rm BEC})\tau\ln\left[\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})-\sigma_{\rm ph}n^*(z_{\rm BEC})}{\chi n(z_{\rm BEC})}\right] \approx V(z_{\rm BEC})\tau\ln\left[\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})}{\chi n(z_{\rm BEC})}\right]. \quad (29)$$

Approximate expressions in (26)–(29) are valid when the inequality

$$\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n^*(z_{\rm BEC})} \gg \frac{\sigma_{\rm ph}}{\sigma} \approx 10^{-6} \quad (30)$$

is fulfilled, as a rule.

In region V, a collector collects the atoms coming from region IV, completing the sequence of operations.

9. Quantitative estimates

It is convenient to perform the simplest quantitative estimates of processes proceeding in the regions of the scenario considered above beginning from region IV by using inequality (30) and approximate expressions (26)–(29).

(1) By assuming that $n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC}) = 10^{-2}$ and $\chi/\sigma = 10^{-3}$, we obtain $L_{\max}/[V(z_{\rm BEC})\tau] = 2.3$, in accordance with condition (7).

(2) For $n(z_{\rm BEC}) = 10^{13}$ cm^{-3} and $L_{\max} = 1$ cm, we obtain $V(z_{\rm BEC}) = 145$ cm s^{-1}, $G_{\max} = 1.3$, $[n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC})]_{\rm th} = 0.255\times10^{-2}$, and the degeneracy temperature $T_0 \approx 0.5 \times10^{-6}$ K (3) corresponding to the total concentration of atoms $n(z_{\rm BEC}) = 10^{13}$ cm^{-3} at the point $z = z_{\rm BEC}$.

(3) If the temperature T achieved upon gas cooling in region II is 10^{-7} K, then, according to (4), the relative concentration $n_{\rm BEC}/n$ of condensed atoms at the point $z = z_{\rm BEC}$ is 0.91, i.e., $n_{\rm BEC}(z_{\rm BEC}) = 0.91\times10^{13}$ cm^{-3} and $n^*_{\rm BEC}(z_{\rm BEC}) = 10^{11}$ cm^{-3}.

(4) If the potential well of the channel is assumed relatively shallow, for example, with $E_{00}(z_{\rm BEC}) \approx 10^{-11}$ eV, then the effective diameter is $D(z_{\rm BEC}) \approx 1.4\times10^{-4}$ cm at the point $z = z_{\rm BEC}$.

(5) The atomic flow with the above parameters should be prepared in region III by compressing the initial flow with the concentration $n(z_{\rm in}) = n(z_{\rm BEC})\Xi^{-1}_{\rm comp}$, where $\Xi_{\rm comp}$ is the compression coefficient (18), (25). In this case, the value of $n(z_{\rm in})$ should not exceed the concentration which produces, according to (3), the critical temperature $T_0 > T = 10^{-7}$ K and causes the formation of a Bose condensate of atoms of the initial flow, i.e., $n(z_{\rm in}) < 10^{11}$ cm^{-3} and, hence, the flow compression coefficient should be $\Xi_{\rm comp} \approx 100$.

(6) Because for the values of parameters obtained above (which are not optimised), the total and invariable energy of atoms in the flow is almost equal to the kinetic energy of their transport motion ($E_{00} \ll MV^2/2$), the total compression coefficient is mainly determined by its component ($\Xi_{\rm comp} \approx \Xi_D^{-1}$). Therefore, according to (24), $D(z_{\rm in}) \approx 10D(z_{\rm BEC}) \approx 14\times10^{-4}$ cm, and the transport velocity $V_{\rm in}$ at the input to region III virtually coincides with the velocity $V(z_{\rm BEC}) = 145$ cm s^{-1}.

(7) If the required tenfold decrease in the effective diameter from $D(z_{\rm in})$ to $D(z_{\rm BEC})$ occurs over the length of region III of about 0.1 cm, the spontaneous decay of metastable states reducing their concentration during the flight time through this region, which does not exceed 0.7 ms, is negligible.

10. On the process of transition to stationary stimulated emission

We considered above the established stationary process of stimulated VUV emission of metastable helium atoms forming the BEC. At the same time, the transition to the stationary state from the initial state, in which emission is absent, has peculiarities requiring a separate analysis, which will be performed in the second part of this work.

Among these peculiarities is, first, a comparatively low level of the spontaneous photon background caused by a long lifetime of metastable states. For this reason, the external injection of resonance seed photons into the modes of the laser resonator may be required to initiate the self-excitation process. In addition, analysis of the stability of stationary lasing will be needed.

The second peculiarity, concerning both the one-pass amplification of an external input signal and the lasing regime, is related to the asymptotic behaviour of the stimulated emission cross section, which is called sometimes the 'laser lethargy' [17–20] and lies in the fact that upon the incidence of resonance emission on a quantum oscillator, the current value of the transition cross section increases gradually from zero at the initial moment to the stationary value (1), with the characteristic rise time of the order of the natural radiative lifetime of the state. This circumstance, which is especially important in the case of long-lived metastable states, may considerably affect the character of the start transient process.

11. Conclusions

The numerical estimates obtained in the paper show that the stationary stimulated VUV emission from the metastable states of atomic helium can be observed in a transport BEC flow according to the scenario considered.

The important features of the start transient process, pointed out in section 10, will be considered in the second part of the paper.

The study of the entire scope of problems mentioned in the paper is important, in particular, for modelling stimulated gamma emission of metastable isomeric nuclei.

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