RESONATORS, MODES, BEAMS

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Efficiency of the unstable resonator of a high-power laser with stochastic phase inhomogeneities in the active medium

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Abstract**. The specific features of operation of the unstable optical resonator of a large gas laser with an active medium containing stochastic phase inhomogeneities are considered. The output power of the laser, the Strehl number, the angular divergence and average far-field radiation intensity are studied as functions of the spatial scale and structure of random inhomogeneities of the refractive index of the active medium. Physical effects related to the deformation of the radiation pattern caused by a change in the spatial frequency of stochastic perturbations are analysed.**

Keywords: *unstable resonator, random phase inhomogeneities of an active medium, gas-dynamic laser, laser radiation divergence, radiation pattern.*

The problems of propagation of coherent radiation through a statistically inhomogeneous medium have been widely discussed in the scientific literature. Of special applied interest is the elucidation of the properties of propagation of directed radiation in a turbulent atmosphere [1] for determining the efficiency of energy transport to a required region located far away from a laser. This allows one to formulate the requirements to a system of dynamic correction of phase perturbations in atmosphere.

In this connection the problem of formation of a mode in the unstable resonator in a high-power laser with an active medium (AM) containing stochastic perturbations of the refractive index is undoubtedly of interest. Such perturbations are typical for gas flow lasers because in the case of ultrasonic flow velocities, not only instabilities can appear in the flow but also turbulence can develop.

The development of turbulence in the AM flow in gas flow lasers is mainly related to small-scale regions of viscous interaction, in particular, to regions of mixing of different components and also to the flow and near-wall layers. In reality, the situation is also complicated by the fact that both stochastic and regular inhomogeneities are present in flows. The active medium of gas-dynamic lasers is most typical in this respect. Regular phase inhomogeneities appear due to shock-wave perturbations in an ultrasonic flow and inhomogeneous distributions of gas-dynamic parameters in an accompanying turbulence layer.

The formation of the mode structure of radiation in an optical resonator with regular phase inhomogeneities in the AM was investigated in many papers. For example, the specific structure of a mode of an unstable resonator containing regular small-scale phase inhomogeneities was studied in detail in [2]. It was found that the laser radiation divergence increased resonantly when the period of a spatial phase perturbation in the AM was close to Fresnel zone size in the unstable resonator.

However, one can reasonably assume that the average parameters of the flow during lasing are virtually constant, so that regular perturbations will result in the appearance of the inhomogeneous but stationary wave front of radiation in the output aperture of the laser. Moreover, due to their regular and reproducible character, such wave-front perturbations can be efficiently separated for a subsequent correction by one of the known methods.

It is reasonable to assume that phase perturbations of different types present in the AM flow are linearly independent, i.e., the influence of regular and stochastic perturbations on a mode being formed and a subsequent propagation of radiation to the far-field zone will be of the superposition type. Because the role of regular perturbations was comprehensively considered in [3], it remains to elucidate the most important features of the influence of only stochastic perturbations of different spatial scales with correlation function of different types.

The algorithm for the adequate separation of regular and stochastic perturbations by means of a special digital filtration of the photometric field of the experimental Toepler diagram of the AM region in a gas-dynamic laser was proposed in [4]. It was found that at distances exceeding 10 diameters of the output cross section of the supersonic nozzle of a gas-dynamic laser in the far-wave region the turbulence had the anisotropic structure with the absolute integral scale along and perpendicular to the flow equal to ~ 9 and 7 mm, respectively. For the maximum diameter of the resonator aperture of 100 mm, the characteristic perturbation frequency f (the ratio of the aperture and inhomogeneity sizes) exceeds 10.

The effect of stochastic inhomogeneities on the laser radiation parameters in the single-pass amplifier regime was perhaps first investigated in detail by Sutton [5, 6]. In particular, the excess of the root-mean-square angle $\bar{\theta}$ of scattering from isotropic pulsations of the refractive index of a medium over the diffraction angular limit θ_0 was analytically estimated in [5] as

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$$\frac{\bar{\theta}}{\theta_0} = \left(\frac{\alpha L}{2\pi}\right)^{1/2} \frac{D}{\Lambda}.$$

Here, L is the length of the medium; D is the transverse size of the aperture; Λ is the integral scale of inhomogeneities; and α is the radiation power scattering coefficient. The estimate shows that for $D \gg \Lambda$ and $\alpha L \sim 1$, the turbulent scattering of radiation greatly exceeds the diffraction divergence of a beam. However, along with a considerable decrease in losses, the structure of the diffraction pattern weakly differs from the limiting structure, although the maximum far-field radiation brightness decreases proportionally to $\exp \alpha L$, resulting in the formation of a background halo in the radiation pattern.

It is obvious that the propagation of radiation in a resonator substantially differs from its propagation in a linear laser amplifier. In the resonator, diffraction effects appearing upon multiple reflections of radiation from mirrors play an important role. In the unstable resonator, the inclined propagation of beams should be also taken into account because in each cross section of the cone of a diverging beam between the resonator mirrors the relations between a fixed spatial scale of phase perturbations in the AM and the cross-section size are different.

The specific features of the formation of a radiation mode were studied by the digital simulation of the unstable resonator in the diffraction approximation. The calculation was performed by the spectral method [7], as in the study of the efficiency of the unstable resonator with the AM containing regular small-scale phase inhomogeneities [2]. As in [2], the model was reasonably simplified by assuming that the small-signal gain had a uniform distribution in the resonator-mode volume [8].

The stochastic distribution of the refractive index in a particular AM cross section orthogonal to the optical axis was described and the parameters of phase screens were determined by using a special simulation algorithm, which allowed the generation of phase screens for the stochastic distribution of the refractive index at the fixed spatial scale and level (modulation depth) of the inhomogeneity. Each of the screens along the optical axis z was generated independently of others, which provided the simulation of the stochastic distribution of the refractive index not only in the resonator cross section but also along the z axis.

The phase dispersion introduced by each of the screens into the wave front was the same. The phase dispersion S_φ^2 of the wave front after a single transit of radiation through the AM was a sum of dispersions on each screen.

The two-dimensional stochastic fields were generated by using the correlation functions of two types – exponential and linear, and the random-quantity distribution was described by the normal law. The algorithm for generating the two-dimensional stochastic field was based on the convolution and Wiener – Khinchin theorems [9]. A random sequence distributed according to the normal law was subjected to the Fourier transform. The obtained result was modulated by the spectral power density of the chosen two-dimensional correlation function, and then the inverse Fourier transform was performed. As a result, the two-dimensional stochastic field with the normal distribution law and correlation function of the specified form was obtained.

As the integrated characteristics of generated stochastic fields for the transmission coefficient of phase screens, we used

(i) the radiation phase dispersion S_φ^2 acquired by the initially plane wave front after a single transit through the AM and

(ii) the spatial scale Δ of a random inhomogeneity determined from the correlation function of a random field: for the exponential correlation function, the scale Δ was set equal to the doubled radius at which the value of the correlation function decreases by a factor of e compared to its maximum, while for the linear correlation function, the scale was set equal to the doubled radius at which the correlation function vanishes. Note that the characteristic size of the inhomogeneities of a stochastic field is determined somewhat conditionally unlike the case of regular periodic perturbations.

Similarly [4], the frequency of stochastic spatial perturbations was defined as

$$f = \frac{D}{\Delta},$$

where the transverse size D of the mode is equal to the size of a large mirror of the unstable resonator or the AM cross section.

Figure 1 shows stochastic phase screens for different scales of spatial inhomogeneities and different types of the correlation function. It is obvious that the asymptotic character of the exponential correlation function (i.e., its asymptotic tending to zero with increasing the spatial coordinate) gives rise to 'spots' with smooth edges in the structure, which describes turbulent perturbations from the physical point of view.

As in [2], an unstable telescopic resonator of the positive branch was simulated using the following parameters: the small-signal gain $g_0 = 1$ m^{-1}, the saturation intensity $I_s = 1$ kW cm^{-2}, the radiation wavelength $\lambda = 10.6$ μm, the magnification factor of the resonator $M = 2$, the distance between mirrors 4.1 m, the AM length 1 m, and the square aperture of the resonator with the side $D = 0.1$ m.

To determine the minimal sufficient amount of phase screens replacing the AM between the resonator mirrors, we performed a series of test calculations for different numbers of screens equal to 3, 5, 7, 10, and 20. The calculations

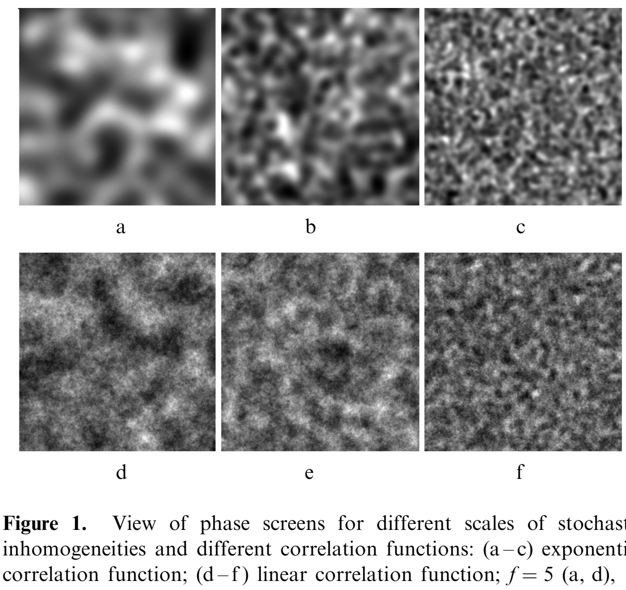

Figure 1. View of phase screens for different scales of stochastic inhomogeneities and different correlation functions: (a – c) exponential correlation function; (d – f) linear correlation function; $f = 5$ (a, d), 10 (b, e), and 20 (c, f).

showed that, when the number of screens was greater than five, variations in the distribution of the field amplitude at the output mirror of the resonator did not exceed 1 %. Because of this, we used five phase screens in further calculations and, therefore, four segments determining the propagation of light in the AM. Empty regions between each of the mirrors and the corresponding AM boundary were the other two segments of a free space.

We performed 50 calculations of the resonator and the radiation pattern for each value of S_φ^2 and the frequency f. Phase screens were generated independently in each calculation. As a result, we obtained the averaged far-field intensity distribution, which took into account the current wave-front inhomogeneity related not only to the AM inhomogeneity and diffraction effects of the mode establishment in the resonator but also to the stochastic character of intracavity perturbations.

The provision of the minimal sufficient amount of calculations to average adequately the radiation pattern is very important both for obtaining the required accuracy of calculations and reasonable time of simulations. This problem was studied in numerical experiments in which the number of realisations for the radiation-pattern averaging was varied from 20 to 500. We found that there was little reason for using the number of realisations exceeding 50 because variations in the averaged radiation parameters such as power, divergence angle, and the Strehl number did not exceed 1 % – 2 %. At the same time, for small values of f and 50 realisations used in simulations, a weak asymmetry of the averaged radiation pattern was nevertheless observed, which, however, did not affect the calculation accuracy of radiation parameters.

Figure 2 shows the one-dimensional cross sections of the angular distributions of the normalised (to the diffraction limit) far-field intensity for different frequencies f of the inhomogeneity. The distributions were obtained for $D_\varphi^2 = 0.11$ rad^2 per AM meter and the exponential correlation function.

One can see that for small f, the radiation pattern substantially differs from the ideal one, in particular, the distinct intensity minima at the angles multiple of the angle $\lambda M/[D(M-1)]$ are absent. Also, the above-mentioned asymmetry of the radiation pattern with respect to the optical axis of the resonator is observed. As the value of f increases, the radiation pattern tends qualitatively to the classical distribution with distinct minima and becomes more symmetric. In other words, the type of radiation-pattern transformation completely corresponds to the single-pass amplifier regime, as shown in [5].

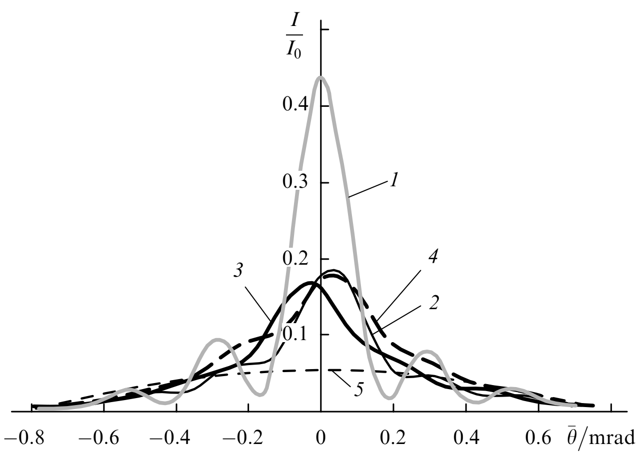

Figure 2. One-dimensional sections of the angular distributions of the far-field normalised intensity for $f = 85$ (*1*), 20 (*2*), 10 (*3*), and 5 (*4*); (*5*) is the spherical component of the radiation pattern for $f = 10$ separated by the method of least squares.

In addition, when radiation is formed in an unstable resonator, the Strehl number increases with increasing f, while the behaviour of the radiation divergence angle $\bar\theta$ substantially depends on the energy level at which this angle is determined (see Figs 3 and 4).

One can see from Fig. 3 that the angle $\bar\theta$ for the radiation power in the radiation-pattern cone equal to $\eta = 20$ % and 50 % increases up to a certain frequency and then either slightly decreases or slightly increases. The local maximum in the divergence angle is observed at $f = 10 - 20$. The transverse size of a stochastic inhomogeneity with this frequency approximately corresponds to the diameter of the first Fresnel zone of the unstable resonator, which in turn can be estimated as

$$d_{\rm F} = 2\left(\frac{2\lambda L}{M-1}\right)^{1/2}.$$

For the resonator under study, this diameter is $d_{\rm F} \sim 2$ cm.

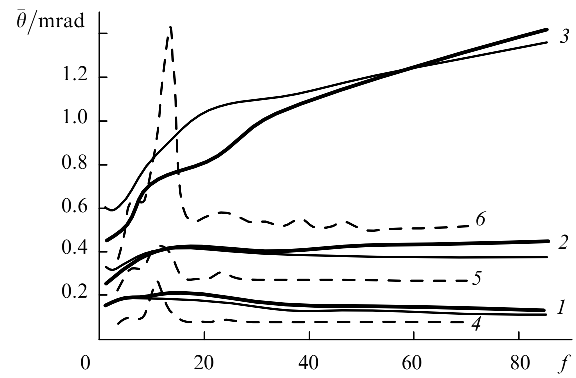

Figure 3. Dependences of the laser radiation divergence $\bar\theta$ on f for the values of η in the radiation-pattern cone equal to 20 % (*1*, *4*), 50 % (*2*, *5*), and 80 % (*3*, *6*). The thick curves and thin curves correspond to the exponential and linear correlation functions, respectively. The dashed curves correspond to the regular periodic perturbations in the AM under similar conditions in [2].

The divergence angle $\bar\theta$ for $\eta = 80$ % increases rather rapidly up to the indicated frequency and then its increase slows down somewhat. For comparison, we present in this figure the similar distributions for a regular small-scale inhomogeneity.

One can also see from Fig. 3 that the type of variations in the angle $\bar\theta$ for $\eta = 80$ % for regular and stochastic small-scale inhomogeneities is different: in the case of stochastic inhomogeneities, the angle $\bar\theta$ continues to grow above the critical value of f, whereas in the case of regular inhomogeneities, $\bar\theta$ virtually returns at high frequencies to the initial level.

This circumstance is explained by the fact that upon stochastic perturbations, aberrations are accumulated per transit of light through the AM irrespective of the direction of light propagation; in any case, a certain, although randomly varying structure appears at the output, which has approximately the same level and scale of spatial phase distortions independent of the angle of light propagation.

In the case of a regular structure, as shown in [2], even for a small angle between the vector of the wave normal and the optical axis of the resonator, the mechanism of self-compensation (integral averaging) of inhomogeneities begins to act, resulting in a considerable decrease in the dispersion of the wave front of propagating radiation. In the unstable resonator, especially of a large laser, self-compensation always occurs for oblique beams propagating from the output mirror to the highly reflecting mirror and also, but less distinctly, for beams propagating parallel to the resonator axis due to diffraction of light.

As the scale of regular periodic inhomogeneities in a resonator decreases, the influence of self-compensation effects increases, which explains a low divergence of radiation at high frequencies f. In the case of stochastic inhomogeneities, no self-compensation occurs, and for this reason the radiation divergence for $\eta = 80\,\%$ increases with increasing frequency, in accordance with the data presented in [5].

The radiation power P also depends on f (Fig. 4a), the type of this dependence being determined in some way by the form of the correlation function. In the case of the exponential correlation function, a minimum is observed at the critical frequency of a spatial inhomogeneity, while in the case of the linear correlation function, the power monotonically decreases with f.

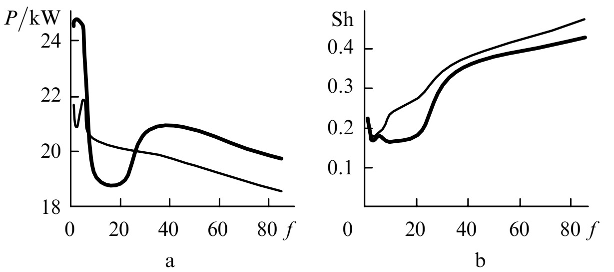

Figure 4. Dependences of the output power P (a) and the Strehl number Sh (b) on f. The thick and thin curves correspond to the exponential and linear correlation functions, respectively.

Both dependences of the Strehl Sh number on f (Fig. 4b) have a distinct minimum at the critical frequency, and the Strehl number then increases with f, in accordance with the radiation patterns shown in Fig. 2.

Thus, by analysing the dependences of $\bar{\theta}$, Sh, and P on the frequency f of a stochastic inhomogeneity, we can conclude that several interrelated effects take place in the unstable resonator.

First, the output power P and radiation-pattern parameters $\bar{\theta}$ and Sh depend on the relation between the sizes d_{F} of Fresnel zones of the resonator and characteristic scale Δ of the inhomogeneity. If these sizes approximately coincide, an unfavourable situation appears, as in the case of regular periodic inhomogeneities, when $\bar{\theta}$ (at least for $\eta = 20\,\%$ ad $50\,\%$) has a maximum, while Sh and P for the exponential correlation function have a minimum. The reason for this, which was explained in detail in [2] for regular inhomogeneities, can be completely explained in the case of stochastic perturbations as well.

Second, as f increases, the radiation pattern approaches the classical diffraction distribution. The intensity maximum described by the Strehl number also increases. The value of $\bar{\theta}$ for $\eta = 20\,\%$ slightly decreases with increasing f, which is obviously explained by the fact that $\bar{\theta}$, as Sh, characterises mainly the maximum axial intensity. The radiation divergence for $\eta = 80\,\%$, on the contrary, increases with increasing f.

For small f, these tendencies are explained by an important role of the beam yaw (random deviations of the intensity maximum from the resonator axis) in the radiation-pattern formation. The time-averaged radiation pattern strongly 'spreads' (Fig. 2), which explains a small value of the Strehl number and a large value of $\bar{\theta}$ for $\eta = 20\,\%$. As the inhomogeneity scale decreases (with increasing f), the beam yaw decreases at each instant of time, i.e., the radiation-pattern maximum not too strongly deviates from the optical axis direction, which explains both the increase in Sh and decrease in $\bar{\theta}$ for $\eta = 20\,\%$. Due to the scattering of radiation by a small-scale structure, the beam energy dissipates to the side diffraction orders of the radiation pattern, whose shape is preserved. As a result, $\bar{\theta}$ increases for $\eta = 80\,\%$.

The dependence of the radiation-pattern shape on the frequency of a spatial stochastic perturbation was further analysed by separating two components from the radiation pattern by the method of least squares. One of the components determined the spherical approximation of the deviation of minimal intensity values from the zero value, while another described the periodic intensity distribution corresponding to the ideal diffraction structure. Note that the first component is responsible for reducing the diffraction-pattern visibility. The integral of each component was normalised to the total power to determine the fractions W_1 and W_2 of the radiation power concentrated in the spherical component of the beam ('incoherent' component) and in the classical diffraction component ('coherent' component). It is important that the condition $W_1 + W_2 = 1$ will be fulfilled. As an example the spherical component [curve (*5*)] of the radiation pattern is shown in Fig. 2 for $f = 10$ and the exponential correlation function of a stochastic field. In other words, curve (*5*) is the spherical component separated from curve (*3*). Figure 5 shows the dependences of the fractions W_1 and W_2 of the radiation power on the frequency f. Because the fraction W_1 of the

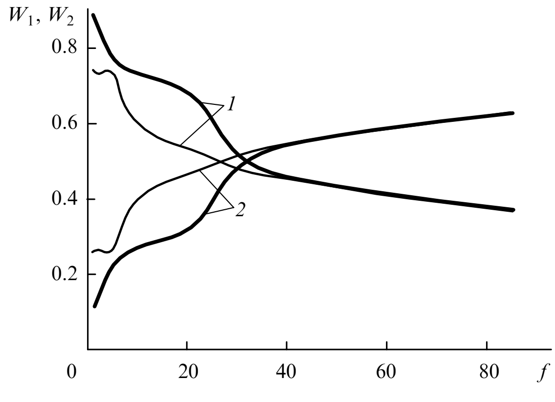

Figure 5. Dependences of the power fractions W_1 (*1*) and W_2 (*2*) on the frequency f. The thick and thin curves correspond to the exponential and linear correlation functions, respectively.

'incoherent' component noticeably decreases and W_2 correspondingly increases with increasing f, this confirms the above-mentioned effect of the recovery of the radiation-pattern structure, i.e., its approach to the classical shape with distinct minima and maxima. One can see that the fractions of the coherent and incoherent power components in the interval $f = 25 - 35$ (the spatial scale of the inhomogeneity is ~ 3 mm) become equal.

From the practical point of view, of special interest is the dependence of the far-field radiation brightness or average intensity on the frequency of stochastic inhomogeneities. The average far-field radiation intensity is proportional to the quantity

$$I_\chi = \chi \frac{P}{\theta_\chi^2},$$

where χ is the power fraction by which the divergence of the laser beam formed in the resonator is determined; in our case, $\chi = 0.2$, 0.5, and 0.8. The quantity I_χ can be also used as a criterion of the efficiency of a resonator with an AM taking into account the possible transport of radiation to the far-field zone.

Figure 6 presents the dependences of I_χ on the frequency f. The parameter $I_{0.2}$ has a distinct minimum at the critical frequency caused by the coincidence of the stochastic period of the inhomogeneity with the size of the Fresnel zone of the resonator. The values of $I_{0.5}$ and $I_{0.8}$ drastically decrease in the frequency range from the minimal frequency to the critical one and then decrease somewhat slower.

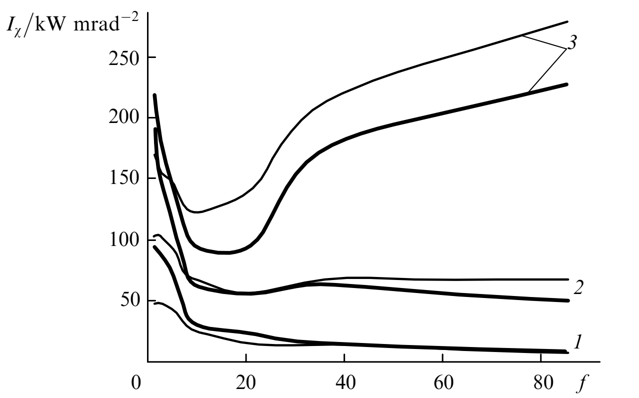

Figure 6. Dependences of the average intensity I_χ on the frequency f for $\eta = 80\,\%$ (*1*, $\chi = 0.8$), 50 % (*2*, 0.5), and 20 % (*3*, 0.2). The thick and thin curves correspond to the exponential and linear correlation functions, respectively.

We can make an obvious conclusion that small-scale inhomogeneities in the far-field zone in the unstable resonator with spatial scales considerably smaller than the size of the Fresnel zone of the resonator do not reduce the radiation intensity in the region of the central maximum of the radiation pattern. For high frequencies f, this intensity becomes equal to that for the ideal AM. However, the average intensity in a broader spatial region decreases.

In conclusion, we will formulate the main results of the paper:

(i) The presence of small-scale stochastic inhomogeneities in an unstable resonator results in a change in laser radiation parameters such as the output power, divergence angle, Strehl number, and far-field radiation intensity;

(ii) as the frequency of stochastic inhomogeneities increases (their spatial scale decreases), the radiation energy dissipates into the side diffraction orders of the radiation pattern; however, the maximum intensity of the central lobe of the radiation pattern and, hence, the Strehl number increase in this case, tending to the values inherent in the ideal AM;

(iii) the dependence of the divergence angle on the frequency of the spatial inhomogeneity is determined by the power at which this angle is measured: the divergence angle for $\eta = 20\,\%$ first decreases and then increases, tending to the value corresponding to the case of an unperturbed AM, while the radiation divergence for $\eta = 80\,\%$ increases due to the above-mentioned energy dissipation into the side diffraction orders of the radiation pattern; the average far-field radiation intensity behaves similarly;

(iv) there exists a critical spatial frequency of a stochastic inhomogeneity corresponding to the diameter of the Fresnel zone of an unstable resonator; in this case, the situation most unfavourable for the radiation parameters is observed, when the output power and the Strehl number of the radiation pattern decrease maximally and the radiation divergence increases for $\eta = 20\,\%$;

(v) the quantitative influence of stochastic inhomogeneities on radiation parameters not too strongly depends on the correlation function shape.

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