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Transmission and self-imaging of submillimetre laser beams in rectangular metal waveguides

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***Abstract*. The mode technique for calculating radiation propagation in rectangular metal waveguides excited by linearly polarised Gaussian beams is developed on the basis of the input field representation in the form of a spectrum of propagating modes. The conditions for optimal excitation are studied theoretically and experimentally, the degree of polarisation is calculated, and conditions for reconstruction of submillimetre beams transmitted through a waveguide are considered.**

Keywords: *submillimetre laser, rectangular metal waveguide, mode approach, excitation, polarisation, reconstruction.*

1. Introduction

The problem of developing channelling systems is one of the basic scientific and technical problems arising in mastering the submillimetre (SMM) range. To solve this problem, it was proposed to use quasi-optical transmission channels such as lens and mirror lines, hollow superdimensional waveguides and microstrip lines [1]. The parameters required for one or another case by using a hollow waveguide can be obtained by choosing appropriately the reflecting boundary surface and the shape of the cross section. Theoretical and experimental studies of the transmission of SMM radiation from gas-discharge lasers and optically pumped lasers in hollow circular waveguides are available in the literature [2 – 10]. From the point of view of polarisation stability, however, the circular cross section is not optimal. In fact, no information is available on the transmission of SMM laser radiation through rectangular waveguides. In order to develop transmission lines for such radiation using rectangular waveguides, information is required on energy losses in inhomogeneous beams in such systems, the conditions of their optimal excitation, the nature and magnitude of distortions introduced into the signal being transmitted, as well as on the ways of minimising these distortions [11].

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In this work, we studied theoretically and experimentally the transmission of SMM laser beams with a Gaussian intensity profile through hollow rectangular metal waveguides in order to determine the conditions for their optimal excitation, minimal depolarisation, undistorted transmission of the initial beam and to develop recommendations for using these waveguides in SMM transmission lines.

2. Theoretical relations

2.1 Radiation transmission

Let a linearly polarised axially symmetric Gaussian radiation beam be incident on the input face of a waveguide directed along the z axis and having dimensions $2a \times 2b$ ($a > b$) in the transverse plane x, y so that its polarisation vector is directed along the broad or narrow wall of the waveguide (Fig. 1). The electric field $\boldsymbol{E}_0 = \boldsymbol{x}_0 E_0(x, y, 0)$ or $\boldsymbol{E}_0 = \boldsymbol{y}_0 E_0(x, y, 0)$ in the plane $z = 0$ of the source has the form

$$E_0(x, y, 0) = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{w_0} \exp\left(-\frac{x^2 + y^2}{w_0^2}\right), \qquad (1)$$

where $\boldsymbol{x}_0$ and $\boldsymbol{y}_0$ are the unit vectors of Cartesian coordinates in x and y directions, and w_0 is the beam radius at the 1/e level of the maximum amplitude.

It is known [12] that the transverse components of the electric field in a metal waveguide may be presented as a series expansion in orthogonal TE and TM waveguide modes. In our case, the normalised transverse components of the electric field for waves in a rectangular waveguide have the form

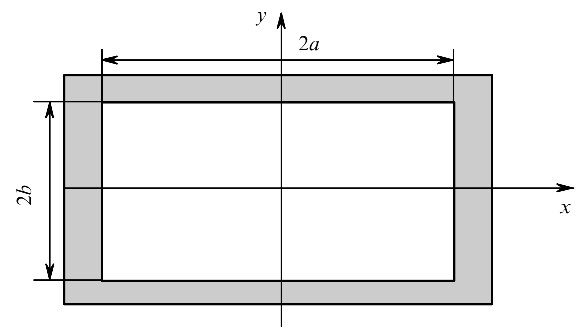

Figure 1. Rectangular waveguide cross section.

$$\boldsymbol{V}_{mn}^{\mathrm{TE}}(x,y)=\boldsymbol{x}_0\frac{1}{2b}\frac{n(\varepsilon_m\varepsilon_n)^{1/2}}{(m^2b/a+n^2a/b)^{1/2}}\cos\left[\frac{m\pi}{2a}(x+a)\right]$$
$$\times\sin\left[\frac{n\pi}{2b}(y+b)\right]-\boldsymbol{y}_0\frac{1}{2a}\frac{m(\varepsilon_m\varepsilon_n)^{1/2}}{(m^2b/a+n^2a/b)^{1/2}}$$
$$\times\sin\left[\frac{m\pi}{2a}(x+a)\right]\cos\left[\frac{n\pi}{2b}(y+b)\right],$$

where $m,n=0,\ 1,\ 2,...$ (the $m=n=0$ mode does not exist);

$$\varepsilon_{m,n}=\begin{cases}1 & \text{for } m,\ n=0,\\ 2 & \text{for } m,\ n\neq 0;\end{cases}$$

$$\boldsymbol{V}_{mn}^{\mathrm{TM}}(x,y)=-\boldsymbol{x}_0\frac{1}{a}\frac{m}{(m^2b/a+n^2a/b)^{1/2}}\cos\left[\frac{m\pi}{2a}(x+a)\right]$$
$$\times\sin\left[\frac{n\pi}{2b}(y+b)\right]-\boldsymbol{y}_0\frac{1}{b}\frac{n}{(m^2b/a+n^2a/b)^{1/2}}$$
$$\times\sin\left[\frac{m\pi}{2a}(x+a)\right]\cos\left[\frac{n\pi}{2b}(y+b)\right];$$

$m,n=1,\ 2,\ 3,\ldots$.

In this case, the field distribution over the waveguide cross section at a distance L from the input face is

$$\boldsymbol{E}(x,y,L)=\sum_{m,n}C_{mn}\boldsymbol{V}_{mn}^{\mathrm{TE}}(x,y)\exp(\mathrm{i}\gamma_{mn}^{\mathrm{TE}}L)$$
$$+\sum_{m,n}D_{mn}\boldsymbol{V}_{mn}^{\mathrm{TM}}(x,y)\exp(\mathrm{i}\gamma_{mn}^{\mathrm{TM}}L),\qquad(2)$$

where the amplitudes C_{mn} and D_{mn} of the modes excited at the waveguide input are defined by the relations

$$C_{mn}=\iint\boldsymbol{E}_0\,\boldsymbol{V}_{mn}^{\mathrm{TE}}\,\mathrm{d}S,\quad D_{mn}=\iint\boldsymbol{E}_0\,\boldsymbol{V}_{mn}^{\mathrm{TM}}\,\mathrm{d}S.$$

Here, $\gamma_{mn}=\beta_{mn}+\mathrm{i}\alpha_{mn}$ are the propagation constants for TE and TM modes [12].

The radiation power passing through the cross section is

$$P_{\mathrm{out}}(L)=\sum_{mn}|C_{mn}|^2\exp(-2\alpha_{mn}^{\mathrm{TE}}L)$$
$$+\sum_{mn}|D_{mn}|^2\exp(-2\alpha_{mn}^{\mathrm{TM}}L).$$

The above relations allow us to determine the coefficient $T(L)$ of radiation transmission in the waveguide and the degree of polarisation $\Pi(L)$ of the output radiation:

$$T(L)=\frac{P_{\mathrm{out}}(L)}{P_{\mathrm{in}}},\quad \Pi(L)=\frac{I_y(L)-I_x(L)}{I_y(L)+I_x(L)},$$

where

$$P_{\mathrm{in}}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\boldsymbol{E}_0(x,y,0)|^2\mathrm{d}x\,\mathrm{d}y$$

is the input beam radiation power, and

$$I_{x,y}(L)=\int_{-a}^{a}\mathrm{d}x\int_{-b}^{b}|E_{x,y}(x,y,L)|^2\mathrm{d}y.$$

2.2 Self-imaging (reconstruction) of radiation

The possibility of transmitting images over a multimode waveguide was mentioned earlier in [13, 14]. Since waves of different modes propagate in the waveguide with different phase velocities and attenuations, their superposition in an arbitrary cross section $z>0$ of the waveguide does not repeat the input field distribution of type (1) in the cross section $z=0$. However, in the paraxial case, when $(\lambda/\lambda_{mn})^2\ll 1$, where

$$\lambda_{mn}=\frac{2}{[(m/2a)^2+(n/2b)^2]^{1/2}}$$

is the critical wavelength for TE and TM waveguide modes [12], there may exist in-phase cross sections $z=z_s$ ($s=1,\ 2,\ 3,...$) in which the phase relations between any modes with arbitrary indices mn and kl are multiples of 2π:

$$\exp[\mathrm{i}z_s(\beta_{mn}-\beta_{kl})]=1.\qquad(3)$$

In these cross sections, the superposition of modes (2) reproduces the form of the input field distribution (1) with an accuracy determined by the number of allowed modes. The conditions of reconstruction of the form of the input signal are determined by the specific shape of the characteristic spectrum of phase constants β_{mn}.

Following the analysis performed in [13, 14], we consider the fulfilment of conditions (3) for a rectangular metal waveguide whose characteristic phase constant spectrum for TE and TM modes is

$$\beta_{mn}=2\pi\left[\left(\frac{1}{\lambda}\right)^2-\left(\frac{1}{\lambda_{mn}}\right)^2\right]^{1/2}.$$

In this case, the phase difference φ between the waves of any two modes with arbitrary indices mn and kl in the paraxial case is

$$\varphi_{mn,kl}\approx 2\pi\,\frac{z\lambda}{32a^2}\left[k^2-m^2+\left(\frac{a}{b}\right)^2\left(l^2-n^2\right)\right].\qquad(4)$$

In a rectangular waveguide with an integer ratio of transverse dimensions (a/b), the sum in the brackets in the above expression is also an integer. The condition that the phase difference between modes is a multiple of 2π means that the equality

$$z_s=\frac{32a^2}{\lambda}\,s$$

describes a sequence of in-phase cross sections z for a rectangular metal waveguide, in which the superposition of paraxial modes reproduces the form of the input field.

Apart from the basic in-phase cross sections z_s, additional cross sections may also exist for special types of excitation. Consider the special types of excitation that are important for our analysis. Suppose that the excitation spectrum of a rectangular waveguide with an integer ratio of transverse dimensions contains only TE_{mn} and TM_{mn} waveguide modes with an even number of field half-waves between the centre and the waveguide wall along the x axis (see Fig. 1) which are denoted by the index m, an odd number of half-waves between the centre and the waveguide wall along the y axis, which are denoted by the index n. In this case, it can be shown easily from (4) that the in-phase cross sections are located at distances $z_s = (8a^2/\lambda)s$. If, however, the excitation spectrum contains modes with an odd index m and an even index n, the in-phase cross sections are located at distances $z_s = (4a^2/\lambda)s$.

However, the above expressions can be used only to predict the approximate position of in-phase cross sections. The reasons for this are explained in [13, 14]. The spectral technique described above must be used for a more precise evaluation of the position $z_{s'}$ of cross sections for reconstruction of the Gaussian SMM radiation and estimation of the error in such a reconstruction.

3. Comparison of experimental and theoretical results

Experiments were performed using the setup described in [2], the only difference being in the radiation source, which in our case was an optically pumped 432.6-μm SMM formic acid (HCOOH) laser. For radii of curvature of mirror (*5*) equal to 100 and 50 cm, the laser beam diameters at the ~ 1/10 level of the maximum intensity were 6.2 and 3.8 mm, respectively. To produce beams of a smaller diameter, spherical mirror (*5*) was replaced by a plane mirror and Teflon lenses (*6*) of focal lengths 24, 14 and 19 cm were mounted at a distance of 24 cm from this mirror. This resulted in the formation of beams of diameters 4.4, 3.2 and 2.7 mm, respectively, at the waist. Measurements of beam diameter, transmission coefficient, and degree of polarisation of radiation were made in the same way as in [2].

The proposed technique was used for computer calculations and experimental measurement of the transmission coefficient and degree of polarisation of radiation in rectangular copper waveguides excited by linearly polarised Gaussian beams of SMM laser radiation with a field of type (1). The relation $R_s = 2.625 \times 10^{-7}(c/\lambda)^{1/2}$ was used in calculations to take into account the surface resistance of copper [8]. The relative beam radius $w = w_0/a$ was varied in the interval 0.1 – 0.9 (in the region of its 'weak' diffraction [15]).

Figure 2 shows the results of theoretical and experimental studies of the dependence of radiation transmission coefficient T on the relative radius w of a beam entering a copper waveguide of dimensions 11 × 5.5 mm and $L =$ 500 mm. The polarisation vector of the beam is directed along the broad and narrow walls of the waveguide. Table 1 shows the calculated relative energy fractions in the emission spectrum of the main TE and TM waveguide modes

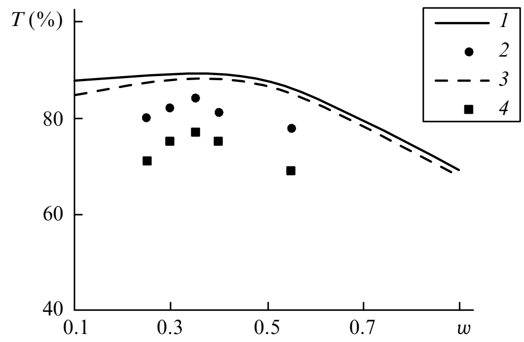

Figure 2. Theoretical [curves (*1, 3*)] and experimental (*2, 4*) dependences of the radiation transmission coefficient T on the relative radius w of the exciting beam polarised linearly along the broad (*1, 2*) and narrow (*3, 4*) walls in a rectangular copper waveguide of size $2a = 11$ mm, $2b = 5.5$ mm, and $L = 500$ mm.

emerging at the waveguide input for such excitations. The energy fractions of the excited waveguide modes were defined as

$$U_{mn}^{\mathrm{TE}} = \frac{C_{mn}^2}{\sum_{m,n} C_{mn}^2 + D_{mn}^2},$$

$$U_{mn}^{\mathrm{TM}} = \frac{D_{mn}^2}{\sum_{m,n} C_{mn}^2 + D_{mn}^2}.$$

The increase in the transmission coefficient with w is explained by the fact that the relative fraction of energy of the fundamental waveguide modes having a lower damping than the other modes increases in the input radiation spectrum. This is confirmed by calculations presented in Table 2 which contains the transmission coefficient of radiation for the fundamental eigenmodes of the waveguide under study. The decrease in T for $w > 0.5$ is observed because a part of the beam energy does not enter the waveguide. The dependence of T on the direction of the

Table 2. Transmission coefficient T for fundamental TE and TM eigenmodes of the waveguide for different values of mode indices.

T (%)	n	m = 0	1	2	3
T_{mn}^{TE}	0	–	89.98	89.97	89.94
	1	94.83	88.07	85.30	83.55
	2	94.72	89.28	87.94	86.43
	3	94.55	89.39	88.68	87.72
T_{mn}^{TM}	1	–	82.69	85.34	87.08
	2	–	81.43	82.64	84.04
	3	–	81.09	81.71	82.57

Table 1. Relative fractions of energy U in the emission spectrum of fundamental TE and TM waveguide modes excited at the waveguide input by a Gaussian laser beam of radius w with the polarisation vector directed along the broad and narrow walls of the waveguide.

w	Polarisation vector direction: along the broad wall of the waveguide			along the narrow wall of the waveguide		
	U_{01}^{TE} (%)	U_{21}^{TE} (%)	U_{21}^{TM} (%)	U_{10}^{TE} (%)	U_{30}^{TE} (%)	U_{12}^{TM} (%)
0.2	17.79	14.60	14.60	23.90	16.12	20.47
0.4	49.86	22.68	22.68	71.19	14.66	10.40
0.6	72.40	13.68	13.68	94.02	2.33	3.20
0.8	77.20	5.99	5.99	98.13	0.03	1.10

Table 3. Relative fraction of energy U in the emission spectrum of fundamental TE and TM waveguide modes for different values of mode indices for a beam of radius $w = 0.37$ polarised linearly along the narrow wall of the waveguide.

U (%)	n	m						
		0	1	2	3	4	5	6
U_{mn}^{TE}	0	–	71.19	0	14.66	0	0.63	0
	1	0	0	0	0	0	0	0
	2	0	0.65	0	0.82	0	0.06	0
	3	0	0	0	0	0	0	0
U_{mn}^{TM}	1	–	0	0	0	0	0	0
	2	–	10.40	0	1.46	0	0.04	0
	3	–	0	0	0	0	0	0

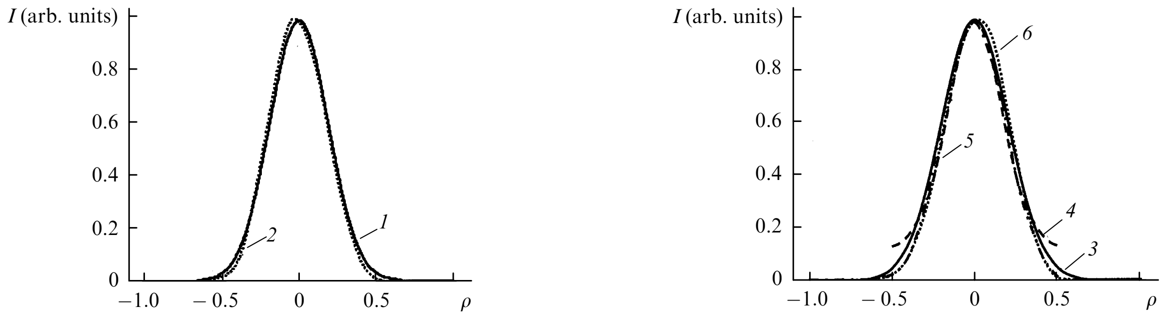

Figure 3. Theoretical [curves (*1*, *3*, *4*)] and experimental [curves (*2*, *5*, *6*)] relative transverse distributions of field intensity at the input [curves (*1*, *2*)] and output [curves (*3*–*6*)] of the waveguide [curves (*1*, *2*, *3*, *6*) correspond to distributions along the broad wall and curves (*4*) and (*5*) to distributions along the narrow wall of the waveguide for $w = 0.37$, $\rho = x/a$].

polarisation vector of the exciting beam is caused by lower losses in the TE_{0n} modes than in the TE_{0m} modes, which provide transmission of a radiation beam with the polarisation vector directed along the broad and narrow walls of the waveguide, respectively (Tables 1–3).

The difference between theoretical and experimental results is caused by variations in the cross section, surface roughness, and possible difference in the theoretical values of the material constants for the waveguide used in our studies.

The results of theoretical and experimental studies show that the degree of polarisation of the transmitted radiation is close to 100 % for rectangular metal waveguides and is preserved for all the beams investigated by us.

Figure 3 shows the theoretical and experimental results of self-imaging of initial SMM beams with a field of type (1) polarised linearly along the narrow wall of a rectangular copper waveguide. One can see from Table 3 that in the case of excitation of the waveguide by such a radiation, the signal spectrum at the waveguide input contains only waveguide modes with odd indices m and even indices n. Unlike other types of excitation (see Section 2.2), additional in-phase cross sections of beam reconstruction are situated at the shortest distance from the input end of the waveguide.

Experiments were made on the basis of a rectangular copper waveguide of dimensions 11×55 mm with an integer ratio a/b of its sides. The length $L = 287$ mm ($s' \approx 1.03$) of the waveguide was chosen preliminarily from the condition of in-phase positioning of the first additional reconstruction cross section for the given radiation beam ($s = 1$) and was refined in calculations by using the spectral technique described above. Figure 3 shows the theoretical and experimental relative transverse distributions of the field intensity at the waveguide input and output for an incoming radiation beam of radius $w = 0.37$. The non-zero field at the broad wall of the waveguide in the reconstruction cross section is attributed in theoretical calculations to the presence of the field of corresponding waveguide modes at this wall [12], which synthesise the input radiation beam.

The reconstruction errors in the theoretical and experimental relative input distributions of the field intensity shown in Fig. 3 were calculated from the expressions [16]

$$\delta_x = \left\{ \int_{-a}^{a} [I_{\mathrm{e}}(x, 0, L) - I_{\mathrm{t}}(x, 0, L)]^2 \mathrm{d}x \right\}^{1/2},$$

$$\delta_y = \left\{ \int_{-b}^{b} [I_{\mathrm{e}}(0, y, L) - I_{\mathrm{t}}(0, y, L)]^2 \mathrm{d}y \right\}^{1/2},$$

where I_{e} and I_{t} are respectively the experimental and theoretical relative distributions of the field intensity in the reconstruction cross section. The reconstruction of the input field was achieved with rather small errors: $\delta_x = 0.35$ % and $\delta_y = 0.47$ %.

4. Conclusions

We have studied theoretically and experimentally the propagation of linearly polarised Gaussian SMM laser beams with a plane phase front in hollow rectangular metal waveguides. By presenting the input field as a spectrum of propagating modes, we have developed the mode technique for calculating the transmission characteristics of radiation in such waveguides.

The conditions of optimal excitation of rectangular metal waveguides have been studied. It is shown that a change in the excitation beam radius does not significantly affect the radiation transmission coefficient in such waveguides, and a distinct optimum is not observed as in the case of a dielectric waveguide. The degree of polarisation of the transmitted radiation is close to 100 % and is preserved for all investigated beams.

The conditions for undistorted transmission of Gaussian SMM laser beams in hollow rectangular metal waveguides have been studied. It has been shown that self-imaging can be realised in practice at the minimum distance from the input end of the waveguide in the case of an integer ratio of its transverse dimensions when the length of the waveguide is approximately equal to the quadrupled ratio of the squared half-width of the broad wall to the wavelength of the radiation being transmitted.

References

1. Meriakri V.V. *Zarubezhn. Radioelektron. Usp. Sovr. Radioelektron.*, **12**, 1 (2002).
2. Gurin O.V., Degtyarev A.V., Maslov V.A., et al. *Kvantovaya Elektron.*, **35**, 175 (2005) [*Quantum Electron.*, **35**, 175 (2005)].
3. Ensheng F. *J. Opt. Soc. Am. B*, **13**, 702 (1996).
4. Epishin V.A., Maslov V.A., Ryabykh V.N., et al. *Kvantovaya Elektron.*, **17**, 480 (1990) [*Sov. J. Quantum Electron.*, **20**, 415 (1990)].
5. Belland P., Crenn J.P. *Int. J. Infrared Millimeter Waves*, **10**, 1279 (1989).
6. Frost B.S., Gourlay P.M. *Appl. Opt.*, **26**, 5112 (1987).
7. Crenn J.P. *Appl. Opt.*, **24**, 3648 (1985).
8. Roser H.P., Yamanaka M., Wattenbach R., Schultz G.V. *Int. J. Infrared Millimeter Waves*, **3**, 839 (1982).
9. Crenn J.P. *IEEE Trans. Microwave Theory Techn.*, **27**, 573 (1979).
10. Kneubuhl F.K., Alfolter E., in *Infrared and Millimeter Waves* (New York: Acad. Press, 1979) Vol. 1, p. 235.
11. Fesenko V.S., Rogozhnikova O.A. *Opt. Spektr.*, **97**, 498 (2004).
12. Fel'd Ya.N. (Ed.) *Spravochnik po volnovodam* (Waveguides Handbook) (Moscow: Sov. Radio, 1952).
13. Rivlin L.A., Shil'dyaev V.S. *Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz.*, **11**, 572 (1968).
14. Grigor'ev E.E., Semenov A.T. *Kvantovaya Elektron.*, **5**, 1877 (1978) [*Sov. J. Quantum Electron.*, **8**, 1063 (1978)].
15. Belland P., Crenn J.P. *Appl. Opt.*, **21**, 522 (1982).
16. Vasilenko G.I., Taratorin A.M. *Vosstanovlenie izobrazhenii po proektsiyam* (Projection Reconstruction of Images) (Moscow: Radio i Svyaz', 1986).