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Double-exposure holographic interferometry with recording a series of overlapped holograms in one recording medium

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Abstract. **A method is proposed for recording a series of double-exposure holograms in one recording medium by rotating the medium between the recording cycles and reconstructing the interference patterns characterising a change in the object state in time. The conditions for obtaining interference patterns in fringes of infinite and finite widths are considered. The results of experiments on recording a series of nine double-exposure holograms in an FG-690 holographic film are presented.**

Keywords: *holographic interferometry, interference pattern, compensation for aberrations, recording-medium turn.*

1. Introduction

Holographic interferometry offers a number of obvious advantages over classical double-beam interferometry, especially in the study of rapid processes in the case of strong aberrations of an optical system forming interferograms. First of all this is the elimination of aberrations introduced by optical elements of poor quality. Aberrations in holographic interferometry are usually eliminated during the reconstruction of an interference pattern either by means of a reference hologram recorded without an object or upon the double-exposure hologram recording [1 – 3]. The double-exposure holographic interferometry does not require the use of high-precision holders unlike the method of combining the reference and object holograms. In the latter case, to eliminate aberrations completely, it is necessary to combine the reference and object holograms with an accuracy of 3 – 5 μm [1].

Holographic interferometric studies of rapid processes are usually based on recording a series of reconstructed aberration-free interference patterns visualising the parameters of an object at different instants of time. Such a variant of holographic interferometry can be realised either by recording each hologram in different recording media [3] or by recording a series of overlapped holograms in one recording medium [4 – 8]. In the second case, each hologram is coded by the carrier frequency by turning the reference beam.

The reconstruction of a series of interference patterns from overlapped holograms in one recording medium visualising the states of an object at certain time intervals is performed in a separate device by two coherent beams by separating the waves diffracted from the corresponding holographic structures [7, 8]. The use of such a device for reconstructing interference patterns considerably complicates the experimental method. To reconstruct the interference pattern from a doubly exposed hologram, neither two coherent beams nor coherent radiation are required [1, 2], which makes this method the most attractive for studying various objects and processes.

The aim of this paper is the development of a more efficient method for recording a series of double-exposure holograms in the same recording medium by rotating it upon passing from one recording cycle to another, which allows the interference patterns to be reconstructed by means of a simple optical scheme with an incoherent radiation source. Unlike the recording methods considered in earlier papers, this method does not require the use of two coherent light beams for reconstructing interference patterns.

2. Hologram recording

Figure 1 presents the optical scheme for hologram recording. Radiation from helium – neon laser (*1*) was directed with fold mirror (*2*) through telescopic system (*3*), (*5*) to a Mach – Zehnder interferometer formed by beamsplitters (*6*), (*10*) and mirrors (*7*), (*9*). To improve the laser-beam quality, ~ 10-μm aperture (*4*) was placed in the rear focal plane of microobjective (*3*). The image of object (*8*) was focused by objective (*11*) on recording medium (photoemulsion) (*12*). The axes x and y of the coordinate system xyz lie in the plane of recording medium (*12*) and the z axis coincides with the propagation direction of the object beam.

In the usual method of double-exposure holographic interferometry, recording medium (*12*) is successively exposed to light first without an object and then with object (*8*). Upon both exposures, either one reference beam is used without changing the illumination direction to reconstruct the interference pattern in fringes of infinite width or a beam with a changed illumination direction to obtain the interference pattern in fringes of finite width. In this case, the intensity distributions in holographic structures obtained after the first and second exposures (structures of the first and second exposures) can be represented in the form

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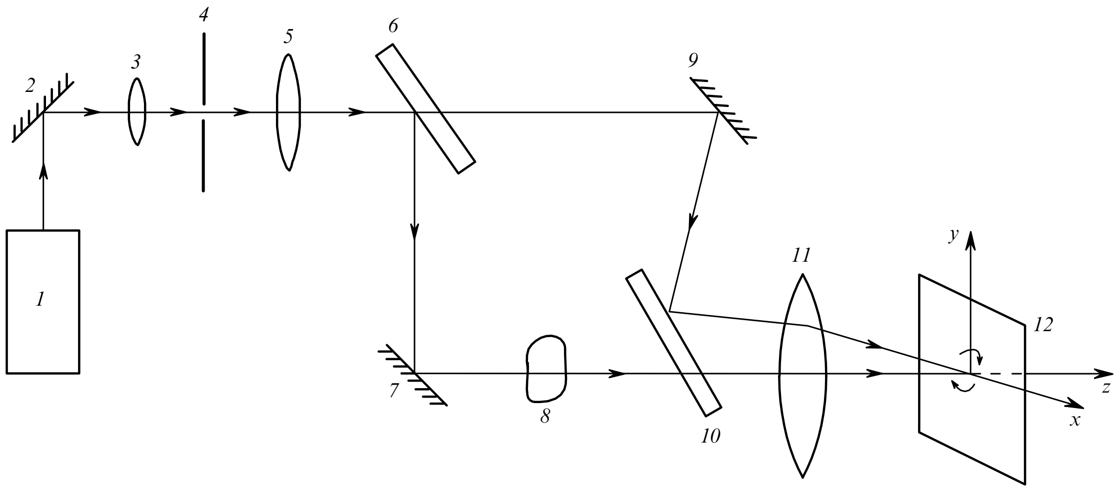

Figure 1. Optical scheme for hologram recording: (*1*) helium – neon laser; (*2*) fold mirror; (*3, 5*) objectives of a telescopic system; (*4*) diaphragm; (*6, 10*) beamsplitters; (*7, 9*) mirrors; (*11*) objective; (*12*) recording medium (photographic plate).

$$I'(x,y) = a_1^2 + a_2^2 + 2a_1^2a_2^2$$
$$\times \cos[2\pi(\xi_0 x + \eta_0 y) + \varepsilon(x,y)], \qquad (1)$$

and

$$I''(x,y) = a_1^2 + a_2^2 + 2a_1^2a_2^2$$
$$\times \cos[2\pi(\xi x + \eta y) + \varepsilon(x,y) + \varphi(x,y)], \qquad (2)$$

respectively, where a_1 and a_2 are the real amplitudes of the object and reference beams; ξ_0, η_0 and ξ, η are the components of the spatial frequencies of the fringes of holographic structures of the first and second exposures, which characterise the period and are determined by the orientation of the reference beam with respect to the object beam [5]; $\varepsilon(x,y)$ are phase distortions caused by aberrations of the optical scheme used for hologram recording; and $\varphi(x,y)$ are phase variations caused by the object during recording. The amplitude transmission of such a double-exposure hologram is proportional to $[I'(x,y) + I''(x,y)]^{-\gamma/2}$, where γ is the contrast coefficient of the photoemulsion.

Let us assume that it is necessary to record N double-exposure holograms in one recording medium, which characterise phase variations caused by the object at certain instants of time. To code each of these holograms by the carrier frequency, we will turn the recording medium through some angle $\Delta\alpha$ with respect to the z axis. At the first stage of recording without an object, N reference holographic structures are exposed to light. In this case, beginning from the recording of the second holographic structure, the recording medium is turned through the angle $\Delta\alpha$ before exposing. After the recording of the last (N-th) holographic structure, each of the structures will be turned through the angle $\alpha_l = \Delta\alpha(N - l)$ with respect to its initial position corresponding to the recording of the first structure, and the illumination distribution accumulated in the recording medium by overlapping N reference holographic structures can be represented in the form

$$I'_\Sigma(x,y) = N(a_1^2 + a_2^2) + 2Na_1a_2 \times$$
$$\times \sum_{l=1}^{N} \cos[2\pi(\xi_{0l}x + \eta_{0l}y) + \varepsilon_l(x,y)], \qquad (3)$$

where ξ_{0l}, and η_{0l} are the components of the spatial frequency of fringes of the l-th reference holographic structure characterising its orientation by the instant of recording the last (N-th) structure after the rotation of the recording medium through the angle α_l. The view of the function $\varepsilon_l(x,y)$ will depend on the angle α_l, and the function $\varepsilon(x,y)$ can be transformed to $\varepsilon_l(x,y)$ by using the transformation formulas for the rotation of the coordinate system through the angle α_l [9].

It is obvious that to prevent the overlap of holographic structures of the first and last exposures, which gives rise to a 'parasitic' moire pattern, the maximum values of $\Delta\alpha$ and N are determined from the condition $\alpha_l < 180°$.

After the last exposure of the N-th holographic structure, the recording medium can either remain in its position or return to the initial position. Here, it is not important for double-exposure holographic interferometry which of the object holographic structures is overlapped on the recorded reference structure.

At the second stage, N object holographic structures are exposed to light together with the object under study. In this case, as upon recording reference structures, beginning from the second-structure writing, the recording medium is rotated before its exposure through some angle $\Delta\alpha$. Each object holographic structure characterising the object state during such recording will overlap on its own reference structure recorded at the first stage. In this case, the illumination distribution accumulated in the recording medium upon the overlap of N object holographic structures can be represented in the form

$$I''_\Sigma(x,y) = N(a_1^2 + a_2^2) + 2Na_1a_2$$
$$\times \sum_{l=1}^{N} \cos[2\pi(\xi_l x + \eta_l y) + \varepsilon_l(x,y) + \varphi_{ll}(x,y)], \qquad (4)$$

where ξ_l, η_l are the components of the spatial frequency of fringes of the l-th object holographic structure characterising its orientation by the instant of recording the last (N-th) structure. The function $\varphi_{ll}(x,y)$ describes the phase shift caused by the object at the instant of recording the l-th holographic structure. Unlike $\varepsilon_l(x,y)$, this function depends not only on the rotation angle α_l of the recording medium but also on the object state at the instant of recording the l-th object holographic structure. In this case, information on N states of the object separated in time and represented by the functions $\varphi_1(x,y)$, $\varphi_2(x,y), \ldots, \varphi_l(x,y), \ldots, \varphi_N(x,y)$ is written in the recording medium.

Thus, after two stages of recording a series of overlapped reference and object holographic structures in one recording medium, the expression for the amplitude transmission of the hologram in the case of linear recording ($\gamma = -2$) can be written in the form

$$\tau_\Sigma(x,y) = \tau_0 + \sum_{l=1}^{N} \tau_{l0}\cos[2\pi(\xi_{0l}x + \eta_{0l}y) + \varepsilon_l(x,y)] + \sum_{l=1}^{N} \tau_l \cos[2\pi(\xi_l x + \eta_l y) + \varepsilon_l(x,y) + \varphi_{ll}(x,y)], \tag{5}$$

where τ_0 is the constant component of the amplitude transmission; τ_{l0}, τ_l are the coefficient characterising the amplitude modulation of the l-th reference and object holographic structures.

Figure 2 shows a photograph of the diffraction spectrum of a hologram with the amplitude transmission of type (5) for the case of recording nine double-exposure holograms by rotating each holographic structure by the angle $\Delta\alpha = 20°$. The diffraction spectrum was photographed in the rear focal plane of the objective by illuminating the hologram with a collimated beam from a helium–neon laser.

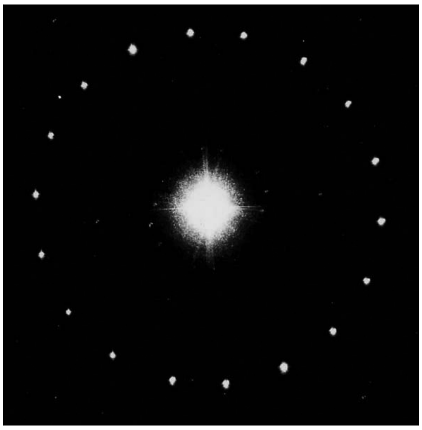

Figure 2. Diffraction spectrum of a hologram for nine overlapped reference and object holographic structures.

3. Production of interference patterns

To obtain an interference pattern characterising the state of an object at a certain instant of recording the object holographic structure, it is sufficient to filter the diffraction orders corresponding to this structure.

Figure 3 presents the scheme for producing interference patterns. Radiation from light source (*1*) is transformed with telescopic system (*2*), (*3*) to a collimated beam illuminating hologram (*4*). The corresponding diffraction order was filtered in the rear focal plane of objective (*5*) by a hole in diaphragm (*6*),

$$A_{0l}(x,y) = a_0 \exp\{\mathrm{i}[2\pi(\xi_0 x + \eta_0 y) + \varepsilon(x,y)]\}, \tag{6}$$

$$A_l(x,y) = a_l \exp\{\mathrm{i}[2\pi(\xi x + \eta y) + \varepsilon(x,y) + \varphi_l(x,y)]\}, \tag{7}$$

where a_0 and a_l are the real amplitudes of the waves. It is obvious that, if the exposures for recording of the l-th reference and object holographic structures were equal, then $\tau_l \approx \tau_{l0}$ and, hence, $a_0 \approx a_l$.

In plane (*7*), optically conjugated by means of objective (*5*) with hologram (*4*), an interference pattern is formed upon the overlap of waves (6) and (7) separated by diaphragm (*6*). The intensity distribution in this interference pattern is

$$I_l(x,y) = 2a_0^2\{1 + \cos[2\pi(\xi - \xi_0)x + (\eta - \eta_0)y + \varphi_l(x,y)]\}. \tag{8}$$

The form of expression (8) corresponds to the interference pattern obtained from a usual double-exposure hologram. Aberrations $\varepsilon(x,y)$ of the system for hologram recording are excluded in interference pattern (8).

One can see from (8) that the quantities $|\xi - \xi_0|$ and $|\eta - \eta_0|$ determine the orientation and width of interference fringes [1, 2]. For $|\xi - \xi_0| \to 0$ and $|\eta - \eta_0| \to 0$, the interference pattern in fringes of infinite width is observed. The interference pattern in fringes of finite width parallel to the x axis is observed for $|\xi - \xi_0| \to 0$ and in fringes parallel to the y axis for $|\eta - \eta_0| \to 0$. The width of interference fringes in the former case is $1/|\eta - \eta_0|$ and $1/|\xi - \xi_0|$ in the latter.

4. Experimental approbation

Double-exposure holographic interferometry by recording a series of overlapped reference and object holographic structures in one recording medium was experimentally tested by studying the distribution of thermal air-flow fields

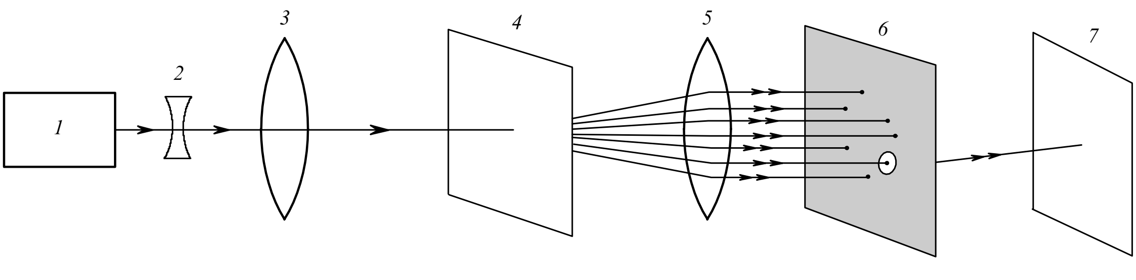

Figure 3. Optical scheme for obtaining interference patterns: (*1*) light source; (*2*, *3*) objectives of a telescopic system; (*4*) hologram; (*5*) objective; (*6*) diaphragm; (*7*) observation plane of the interference pattern.

near a heated copper rod. The latter was mounted vertically and heated by an electric heater placed in the lower part of the rod. The rod with the heater was placed in the object arm of the interferometer (Fig. 1). Interference patterns were recorded in an FG-690 holographic film mounted in a special optical holder allowing the film to be rotated around the z axis through fixed angles multiple of 20°. Before rod heating, nine overlapped reference holographic structures were recorded in the film. Interference patterns in fringes of infinite width were obtained by recording a series of object holographic structures at the second stage with the reference beam illuminating the recording medium having the same direction as during the recording of reference holographic structures at the first stage. To obtain interference patterns in fringes of finite width, the angle between the reference and object beams was slightly changed during recording a series of object holographic structures at the second stage.

During copper rod heating, a series of object holographic structures was written in the recording medium every 10 s.

Figure 4 shows interference patterns in infinite-width fringes obtained from holograms in the reconstruction device (Fig. 3). The interference patterns correspond to the object holographic structures recorded within 20, 50, and 70 s after the beginning of rod heating.

Figure 5 presents interference patterns in finite-width fringes. Before recording a series of object holographic structures, the angle between the reference and object beams was slightly changed in the yz plane compared to the holography angle. The interference patterns correspond to the object holographic structures recorded within 20, 40, and 80 s after the beginning of rod heating.

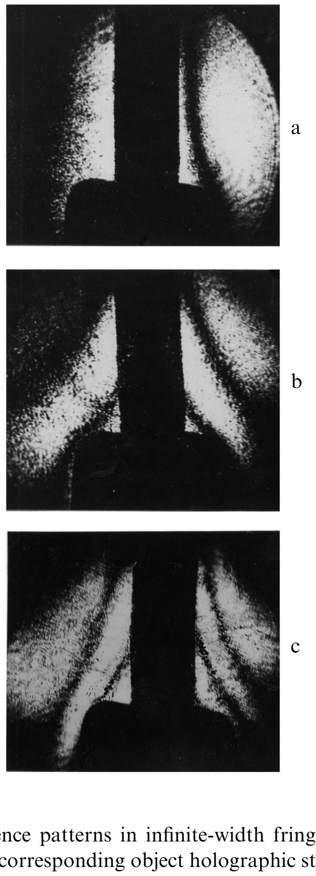

Figure 4. Interference patterns in infinite-width fringes obtained from holograms and the corresponding object holographic structures recorded within 20 (a), 50 (b), and 70 s (c) after the beginning of heating.

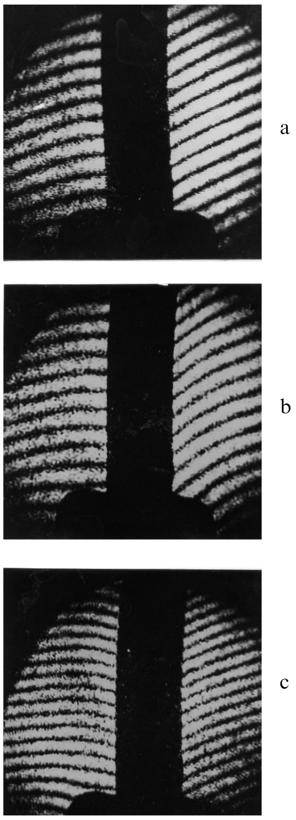

Figure 5. Interference patterns in finite-width fringes obtained from holograms and the corresponding object holographic structures recorded within 20 (a), 40 (b), and 80 s (c) after the beginning of heating.

Note that aberrations in the system for hologram recording are completely compensated in the interference patterns presented in Figs 4 and 5.

5. Conclusions

The main advantage of the proposed method of holographic interferometry over other known methods is that the recording of a series of overlapped holograms in one recording medium based on the rotation of the recording medium on passing from one recording cycle to another allows one to obtain interference patterns both in infinite- and finite-width fringes by using a simple optical scheme for interference-pattern reconstruction without employing two coherent light beams. The main requirement to such a reconstruction scheme is the possibility of spatial filtration of diffraction orders.

The experimental approbation of the method of double-exposure holographic interferometry proposed in the paper showed that this method can be used for studying nonstationary processes. It is obvious that the minimal interval between two successive cycles of holographic-structure recording cannot be shorter than the time required for rotating the recording medium through the angle $\Delta\alpha$ and it mainly depends on the kinematic scheme of the motion controller of an optical holder.

References

1. Beketova A.K., Belozerov A.F., Berezkin A.N., et al. *Golograficheskaya interfrometriya fazovykh ob'ektov* (Holographic Interferometry of Phase Objects) (Leningrad: Nauka, 1979).
2. Vest C.M. *Holographic Interferometry* (New York: Wiley, 1979; Moscow: Mir, 1982).
3. Zeilikovich I.S., Spornik N.M. *Golograficheskaya diagnostika prozrachnykh sred* (Holographic Diagnostics of Transparent Media) (Minsk: Izd. Universitetskoe, 1988).
4. Hildebrand B.R., Haines K.A. *J. Opt. Soc. Am.*, **57**, 155 (1967).
5. Collier R.J., Burckhardt C.B., Lin L.H. *Optical Holography* (New York: Academic Press, 1971; Moscow: Mir, 1973).
6. Zakharov Yu.N., Mensov S.N., in *Fizicheskie osnovy i prikladnye voprosy golografii* (Physical Foundations and Applied Problems of Holography) (Leningrad: Leningrad Institute of Nuclear Physics, 1984) p. 138.
7. Lyalikov A.M. *Zh. Tekh. Fiz.*, **6**, 56 (2002).
8. Lyalikov A.M. *Opt. Spektr.*, **93**, 146 (2002).
9. Korn G.A., Korn T.M. *Mathematical Handbook for Scientists and Engineers* (New York: McGraw-Hill, 1961; Moscow: Nauka, 1971).