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Polarisation dynamics of single-longitudinal-mode Nd : YAG lasers with a weakly anisotropic cavity

I.V. Ievlev, P.A. Khandokhin, E.Yu. Shirokov

***Abstract*. The polarisation dynamics of a single-mode bipolarisation chip Nd : YAG laser with a weak phase anisotropy of the cavity is studied experimentally and theoretically. It is shown that the three types of relaxation oscillations exist in the bipolarisation lasing regime, the two of which are responsible for the antiphase dynamics of orthogonally polarised modes. It is demonstrated that pumping by linearly polarised radiation induces the gain anisotropy. It is found that the main properties of the low-frequency polarisation dynamics and the induced gain anisotropy are almost independent of the orientation of crystallographic axes of the active element. The model of a single-mode bipolarisation solid-state laser, which was developed earlier and takes into account the phase-sensitive interaction between orthogonally polarised modes, well describes the main properties of the low-frequency polarisation dynamics observed in experiments.**

Keywords: *Nd : YAG lasers, polarisation mode, relaxation oscillations, spatial inverse-population grating, angular burning of population inversion.*

1. Introduction

Unlike 'scalar' lasers, in which all radiation modes have the same fixed polarisation state, 'vector' (or bipolarisation) lasers are characterised by a complicated nonlinear dynamics, which is absent in scalar analogues [1]. The polarisation (vector) degree of freedom gives rise to new properties, which can be used for solving various fundamental and applied problems. In particular, vector lasers are promising for applications in telecommunications, in the development of optical computers and data encoding, in spectroscopy, Doppler velocity metres, vibrometres, etc. [2].

It is known that the polarisation characteristics of laser radiation are determined both by the cavity anisotropy and the gain anisotropy of the active medium. The cavity anisotropy is the phase anisotropy caused by the birefringence of intracavity optical elements (including the active element) and the loss anisotropy. The gain anisotropy is inherent in crystals with the preferred direction of the dipole moments of active centres (for example, yttrium aluminate doped with rare-earth ions). Neodymium-doped yttrium aluminium garnet (Nd : YAG) belongs to isotropic active media. To optimise the excitation conditions for the modes of a required polarisation, it is necessary to match all the factors determining the polarisation properties of the laser.

The absence of the loss anisotropy for modes with the orthogonal polarisation leads to the bipolarisation operating regime in lasers with isotropic active media. The studies of fibre lasers with an isotropic cavity [3 – 6] and a Nd : YAG laser [7] completely confirmed this statement. It was shown that the radiation of multimode fibre and Nd:YAG lasers with an isotropic cavity has two polarisation states, which are independent of the orientation of pump polarisation. They differ in the optical spectra and intensity fluctuation spectra. The total radiation of all the modes of the same polarisation is called a polarisation mode. The interaction of orthogonally polarised modes with the active medium causes the polarisation burning of the population inversion, resulting in the appearance in multimode lasers, along with well-known relaxation oscillations [8, 9], of low-frequency relaxation oscillations responsible for the antiphase dynamics of orthogonally polarised modes. As a rule, the fluctuation spectrum of the total intensity of all longitudinal modes of the same polarisation exhibits three resonance peaks at frequencies f_1, f_2, and f_3.

The high-frequency oscillation at the frequency f_1 is caused by in-phase intensity oscillations of all the modes without exception, while the two low-frequency peaks at frequencies f_2 and f_3 correspond, on the one hand, to in-phase oscillations of all the longitudinal modes with the same polarisation and, on the other hand, to antiphase intensity oscillations of polarisation modes.

The presence of several longitudinal modes in each polarisation mode casts some doubt that these low-frequency relaxation oscillations are purely polarisation oscillations. Only when each polarisation mode contains one longitudinal mode, so that orthogonally polarised modes have the same spatial structure, we can speak about a purely polarisation interaction.

Recently, experiments were performed with a chip Nd : YAG laser [10] whose cavity was sufficiently short and provided lasing only at a single longitudinal mode. This excluded completely the generation of low-frequency oscillations upon the interaction of modes with different spatial structures [8, 9]. Due to a large residual birefringence of the crystal, polarisation modes were generated at different

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optical frequencies, as demonstrated by the presence of a beat signal in the signal intensity fluctuation spectra. In this case, along with in-phase relaxation oscillations at the frequency f_1, only one additional peak was observed at the frequency f_2 of polarisation relaxation oscillations. The study of the model of a single-longitudinal-mode bipolarisation laser developed in [10] has shown that the dynamics of such lasers is mainly determined by the phase anisotropy δ of the cavity, which is equal to the difference of frequencies of the orthogonally polarised eigenmodes of the cavity. The phase anisotropy of the cavity can be estimated in experiments from the beat frequency [10]. The analysis of results obtained by using this model showed that small phase anisotropy of the cavity ($\delta \ll f_1$) should give rise to polarisation relaxation oscillations at the frequencies f_2 and f_3, whereas in the case of large phase anisotropy ($\delta \gg f_1$), oscillations at the frequency f_3 pass to the beat signal.

The aim of this paper is to study the properties of the polarisation dynamics of single-mode chip Nd : YAG lasers with small phase anisotropy of the cavity ($\delta \ll f_1$) and to prove the polarisation nature of low-frequency polarisation oscillations.

2. Experimental results

2.1 Experimental setup

The scheme of the experimental setup is shown in Fig. 1. Linearly polarised radiation from a laser diode LD was focused with lenses L1 and L2 on the active element in the form of a plane–parallel plate of diameter ~ 15 mm and thickness 0.5 mm. The input face had an AR coating for the 810-nm pump wavelength and a highly reflecting mirror (99.7 %) for the 1064-nm lasing wavelength. The output face had AR coatings ~ 95 % at the pump wavelength and 99 % at the laser wavelength. A phase half-wave plate for the pump wavelength placed in the pump beam was used to control the orientation of pump polarisation (angle Ψ_p). A filter F at the output of a Nd : YAG laser suppressed pump radiation. Laser radiation was incident on a phase half-wave plate which was used to match the orientations of laser radiation polarisations with the axes of a polariser P.

Lasing at the fundamental transverse TEM_{00} mode was achieved by focusing the laser beam on the input face of the crystal. The angular divergence of laser radiation was approximately 8.6×10^{-3} rad. The laser radiation contained two orthogonally polarised modes with the same spatial structure. The intensity ratio of these modes could be varied either by displacing the optical axis with respect to the active element or changing the orientation of the polarisation vector of pump radiation. The low-frequency polarisation dynamics and the induced gain anisotropy in Nd : YAG lasers were studied for several active elements with different orientations of crystallographic axes ([100] and [111]). It was found that the main properties of the polarisation dynamics of chip lasers were almost independent of the orientation of crystallographic axes.

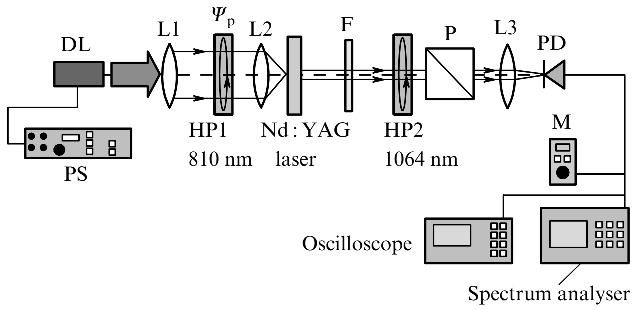


Figure 1. Scheme of the experimental setup: PS: power supply of the pump laser; LD: 810-nm pump diode laser; L1, L2, L3: focusing lenses; HP1, HP2: half-wave plates for the 810-nm pump wavelength and 1064-nm lasing wavelength; F: attenuating filter at 810 nm; P: polarisation cube; PD: photodiode; M: microamperemeter.

2.2 Relaxation oscillation spectrum

Because the chip lasers studied in the paper had weak phase anisotropy of the cavity caused by a small residual birefringence, the orthogonally polarised modes were degenerate in the optical frequency. Unlike the case investigated in [10], the intensity fluctuation spectrum for each polarisation mode exhibits three types of relaxation oscillations (Fig. 2). High-frequency relaxation oscillations at the frequency f_1 are caused by the in-phase intensity oscillations, while polarisation relaxation oscillations at frequencies f_2 and f_3 are caused by antiphase oscillations related to the competitive interaction between orthogonally polarised modes. These oscillations are absent in the spectrum of total intensity fluctuations, and therefore the intensity oscillations of polarisation modes occur out of phase. Note that the orthogonally polarised modes of the laser could be exactly identified only due to this feature of low-frequency relaxation oscillations because the optical frequencies of the modes coincided.

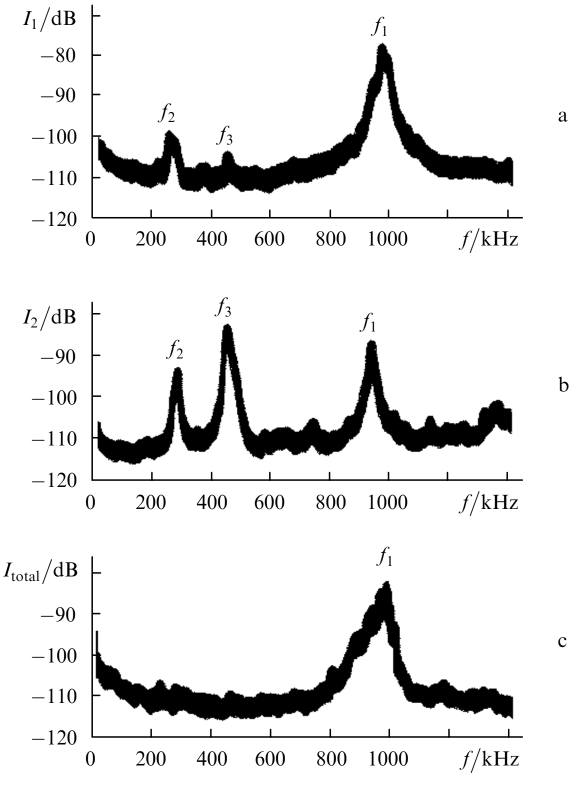


Figure 2. Intensity fluctuation spectra of the orthogonally polarised modes (a, b) and of the total intensity (c) of a laser with a weak phase anisotropy.

The polarisation eigenstates were determined by the following method. By observing variations in the intensity fluctuation spectrum of radiation transmitted through a polariser P and rotating a half-wave plate HP2, we obtained the maximum amplitude of the resonance peaks at frequencies f_2 and f_3. In this case, the polariser orientation corresponds to the transmission of one of the polarisation modes, i.e., as pointed out in [5–7, 10], to the maximum or minimum of the radiation intensity transmitted through the polariser (except the case of polarisation modes with the same intensity). The rotation of HP2 through 45° allows one to record the orthogonal polarisation mode. Figure 2 shows that the amplitudes of low-frequency relaxation peaks f_2 and f_3 for a strong mode I_1 are lower than the peak of in-phase relaxation oscillations at the frequency f_1. The opposite situation is observed for a weak mode I_2, the amplitudes of low-frequency relaxation peaks being higher than that of the high-frequency relaxation peak. For some intermediate position of the half-wave plate, a signal transmitted through the polariser proves to be proportional to the total intensity I_{total}, which is confirmed by the absence of the resonance peaks at frequencies f_2 and f_3 in the noise.

2.3 Gain anisotropy

We studied the dependences of relaxation oscillations at frequencies f_1, f_2, and f_3 (Fig. 3) and the intensities I_1 and I_2 of polarisation modes (Fig. 4) on the azimuth Ψ_{p} of the pump polarisation. It was found that the relaxation oscillation frequencies f_1, f_2, and f_3 were almost independent of Ψ_{p}, whereas the intensity of polarisation modes of the laser strongly depended on Ψ_{p}, which demonstrates the gain anisotropy induced by linearly polarised pump radiation [11, 12]. The linearly polarised radiation of the pump laser produced a nonuniform angular distribution of the population inversion, i.e., the gain anisotropy, which was initially absent in the isotropic active medium. This effect is observed most distinctly upon the rotation of the pump polarisation plane, when an increase in the gain for one polarisation mode was accompanied by its decrease for the orthogonal mode, resulting in a change in their intensities.

Figure 5 shows the dependences of the intensities I_1 and I_2 of polarisation modes on the pump parameter A, i.e., on the pump power normalised to the threshold for a strong mode (the I_1 mode in this case) for $\Psi_{\text{p}} = 45°$ (Fig. 4). For such an orientation of the pump polarisation, the single-mode regime is preserved up to the threshold pump level for the second polarisation mode $A_{\text{th2}} = 1.3 - 2$ and then passes to the bipolarisation regime. Above the threshold of the bipolarisation regime, polarisation relaxation oscillations appear at frequencies f_2 and f_3 in the relaxation oscillations spectrum (Fig. 6).

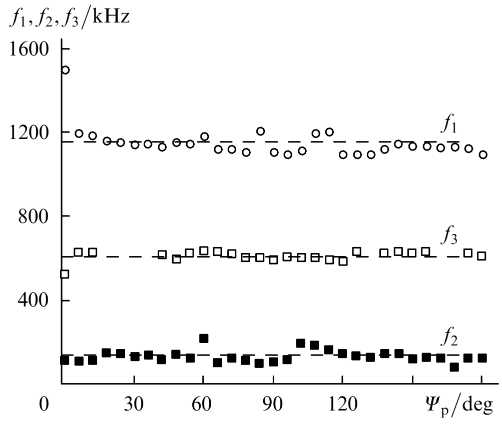


Figure 3. Dependences of the relaxation frequencies of a bipolarisation laser on the orientation Ψ_{p} of the polarisation plane of the pump radiation for the pump parameter $A = 3.5$.

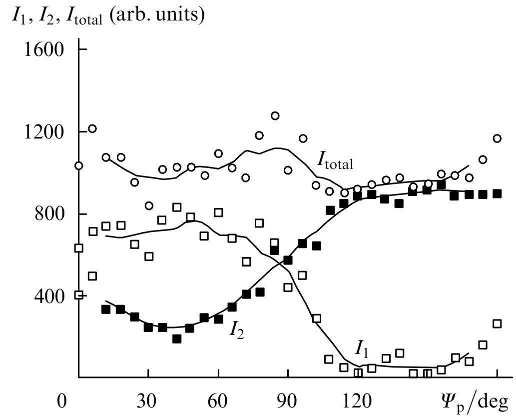


Figure 4. Dependences of the intensities of orthogonally polarised modes on the orientation Ψ_{p} of the polarisation plane of the pump radiation for the pump parameter $A = 3.5$.

The threshold A_{th2} of the bipolarisation regime depended on the selected active element. It was minimal

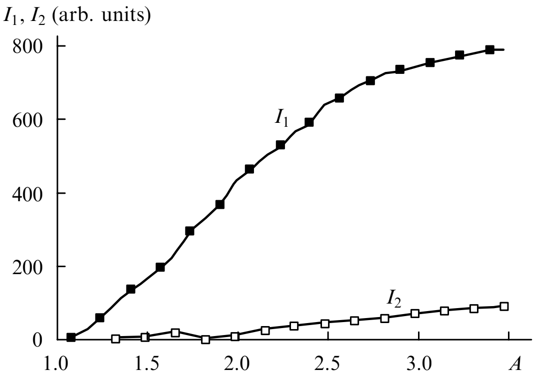


Figure 5. Dependences of the intensity of orthogonally polarised modes on the pump parameter A for $\Psi_{\text{p}} = 45°$.

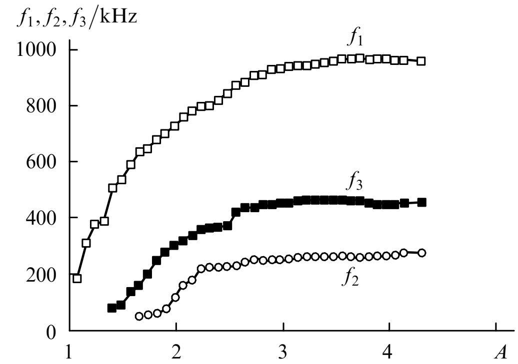


Figure 6. Dependences of the relaxation frequencies of a bipolarisation laser on the pump parameter A for $\Psi_{\text{p}} = 45°$.

for crystals with the [100] orientation of crystallographic axes and maximal for crystals with the [111] orientation of crystallographic axes. Note that the lasing threshold for a weak polarisation mode in [10] was $A_{\rm th2} = 4$. This discrepancy between the threshold values cannot be explained within the framework of the model developed in [10] because this model neglects the orientation of crystallographic axes in the active medium. We assume that this discrepancy can be explained by taking into account the exact orientation of active dipoles with respect to the laser cavity axis.

Note that, as the optical axes was displaced with respect to the active element, the dynamic behaviour of the laser changed from the stationary regime with the characteristic intensity fluctuation spectrum for polarisation modes described above to the nonstationary behaviour, including chaotic. The theoretical study of the influence of the phase anisotropy of the cavity suggests that this can be explained by small variations in the residual birefringence in the cross section of the active element caused by its inhomogeneity.

3. Results of the theoretical study

The model [10] of a single-longitudinal-mode solid-state laser, taking into account the phase-sensitive interaction between orthogonally polarised modes, well describes the properties of the low-frequency polarisation dynamics, which were observed in experiments considered above and in the study of a single-longitudinal-mode Nd : YAG laser with a large phase anisotropy of the cavity [10].

We used this model to analyse the influence of the weak phase anisotropy of the cavity on the behaviour of polarisation modes. Figure 7 presents the dependences of the intensity of orthogonally polarised modes on the phase anisotropy parameter δ (the difference of frequencies of the cavity eigenmodes) obtained by the numerical integration of the above model.

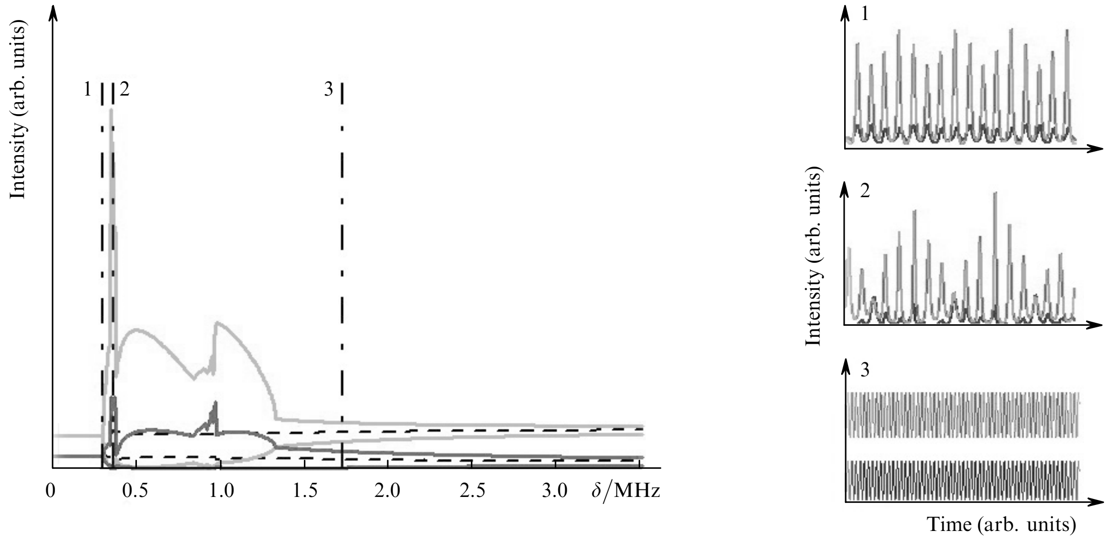
Figure 7a presents the minimal, maximal, and average intensities of the modes, and Fig. 7b shows their oscillograms for the values of δ corresponding to vertical dot-and-dash straight lines 1, 2, and 3 in Fig. 7a.

One can see that, as the phase anisotropy parameter is increased, the system passes from the stationary lasing regime ($\delta < \delta_{\rm cr}$) to the nonstationary regime ($\delta \geqslant \delta_{\rm cr}$, region 1–2 in Fig. 7a) and then to the chaotic polarisation dynamics regime. As the parameter δ is further increased, the system passes to the quasi-sinusoidal regime, which is typical for the beating regime (region 3 in Fig. 7b). These features of the influence of phase anisotropy on the laser behaviour explain the influence of the position of the pump focusing point of a sharp passage from the stationary regime to chaotic one observed in experiments. The presence of even small inhomogeneity in the distribution of residual birefringence in the crystal cross section can lead to such drastic variations in the laser behaviour if the phase anisotropy of the cavity is close to the critical value $\delta_{\rm cr}$. The numerical studies show that the value of $\delta_{\rm cr}$ depends on the frequency f_1 of in-phase relaxation oscillations as $\delta_{\rm cr} \approx \frac{1}{2} f_1$. The value of $\delta_{\rm cr}$ under our experimental conditions was approximately 300 kHz. By using the parameters of the system corresponding to this value, we can estimate the value of birefringence $n_{\rm o} - n_{\rm e}$. We used the following parameters of the laser: $L = 0.5$ mm, $n = 1.8$, $\delta = \nu_1 - \nu_2 \approx \delta_{\rm cr} \approx 300$ kHz, and the intermode interval $\Delta\nu_0 = c/2nL \approx 170$ GHz. The phase shift

$$\varphi = \frac{2\pi(n_{\rm o} - n_{\rm e})L}{\lambda}$$

caused by birefringence leads to the frequency difference δ between the orthogonally polarised cavity modes. The phase shift equal to π provides the frequency splitting of orthogonally polarised modes by $\Delta\nu_0/2$. Then, for an arbitrary phase shift

$$\delta = \frac{\Delta\nu_0}{2\pi}\varphi \quad \text{or} \quad \varphi = \frac{2\pi\delta}{\Delta\nu_0} \approx 10^{-5} \text{ rad},$$



a b

Figure 7. Theoretical dependences of the intensity of orthogonally polarised modes on the phase anisotropy parameter δ (solid curves are the minimal and maximal mode intensities; dashed curves are the average mode intensities in the nonstationary regime) (a) and the time dependences of the modes for different δ, $A = 3$, and $\Psi_{\rm p} = 20°$ (b).

we have

$$n_{\mathrm{o}} - n_{\mathrm{e}} = \frac{\lambda\varphi}{2\pi L} = \frac{\lambda}{2\pi L}\,\frac{2\pi\delta}{\Delta\nu_0} = \frac{\lambda}{L}\,\frac{\delta}{c}\,2nL = \frac{2n\lambda\delta}{c} \approx 4\times10^{-9}.$$

Let us estimate for comparison the birefringence of the crystal studied in [10]. This crystal had comparatively large phase anisotropy, so that the beat frequency was between 15 and 70 MHz. By assuming that $\delta = 60$ MHz, we obtain from the above calculations that the birefringence $n_{\mathrm{o}} - n_{\mathrm{e}}$ is $\sim 10^{-6}$.

4. Conclusions

The experimental study of the low-frequency dynamics of a bipolarisation single-longitudinal-mode Nd : YAG laser with a weak phase anisotropy of the Fabry – Perot resonator has shown that three relaxation peaks at frequencies f_1, f_2, and f_3 are observed in the intensity fluctuation spectra of each polarisation mode. The polarisation relaxation oscillations at frequencies f_2 and f_3 correspond to weak antiphase intensity oscillations of orthogonally polarised modes. These oscillations are compensated in the total radiation intensity of a bipolarisation laser and only the resonance peak at the frequency f_1 of in-phase relaxation oscillations is observed in the total intensity fluctuation spectrum.

We have found that the main properties of the low-frequency polarisation dynamics of Nd : YAG lasers and the induced gain anisotropy are almost independent of the orientation of crystallographic axes. Small differences were observed between the lasing thresholds for a weak polarisation mode in lasers with active crystals with the orientations of axis [100] ($A_{\mathrm{th2}} = 1.3 - 1.5$) and [111] ($A_{\mathrm{th2}} = 2$).

The model of a single-mode bipolarisation solid-state laser developed in [10], which takes into account the phase-sensitive interaction of orthogonally polarised modes, well describes as a whole the peculiarities of the low-frequency polarisation dynamics observed in experiments. This model allows one to estimate the phase anisotropy of the cavity (birefringence) from the dynamic behaviour of the laser when the frequency splitting $\delta = \nu_1 - \nu_2$ of the orthogonally polarised eigenmodes of the cavity cannot be directly measured due to its smallness.

The discrepancy between the lasing thresholds A_{th2} for a weak polarisation mode for active elements with different orientations of crystallographic axes cannot be explained within the framework of the model developed in [10]. It seems that this discrepancy can be explained by simulating the dynamics of bipolarisation lasers taking into account the orientation of active centres with respect to the laser cavity axis.

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