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SCATTERED RADIATION

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Statistic and coherent properties of scattered light fields for different geometrical parameters of rough surfaces

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***Abstract*. The relation between the correlation and coherent properties of light fields scattered by rough surfaces and the geometrical parameters of these surfaces is analysed. It is shown that, if the coherence time of illuminating radiation is $\tau_c > 10/\omega_0$ (where ω_0 is the central frequency of the emission spectrum), then, by averaging the field intensity scattered by a rough surface over time $T > 10\tau_c$, we can determine the stationary regions and coherent parameters of the scattered field and the conditions imposed on the narrowband and chromatic (spectral) parameters of illuminating radiation. These regions, parameters, and conditions are specified by the following parameters: the coherence length $L_c = \tau_c c$ of illuminating radiation, the transverse size d of the backscattering region of the surface, the depth L_s of the backscattering region, the distance r_c from a receiving aperture to the surface, the size d_ρ of the receiving aperture, the central wavelength $\lambda_0 = c/\omega_0$ of illuminating radiation, and the root-mean-square deviation σ of the height of irregularities of the surface. The obtained results give the relations between L_c, L_s, and σ at which illuminating radiation behaves as monochromatic, quasi-monochromatic or polychromatic radiation and the scattered field behaves as coherent, partially coherent or incoherent field.**

Keywords: *scattering by rough surfaces, stationarity and coherence of scattered fields, speckles in the scattered field, narrowband and chromatic parameters of illuminating radiation.*

1. Introduction

The concept of coherence of wave processes has long been used in radiophysics and optics [1 – 7]. In this paper, we are dealing with the coherence of light fields scattered by rough surfaces illuminated by radiation with the spectral width $\Delta\omega \approx 2\pi/\tau_c \ll \omega_0$ (where τ_c, $\Delta\omega$ and ω_0 are the coherence time of radiation, the width and central frequency of the spectrum, respectively), which is assumed, as a rule, narrow [1]. However, the concept of coherence is still defined somewhat inaccurately, for example, in the case of detecting the radiation intensity scattered by surfaces for the time $T > 10\tau_c$ (usually, $T > 10^{-6}$ s). Also, the influence of geometrical parameters of scattering surfaces and spectral characteristics of illuminating radiation on the relation between the statistical, correlation (spatial), and coherent (temporal) properties of scattered fields has not been analysed. For example, it is not clear how these characteristics affect the contrast and correlation radius of the intensity distribution in scattered fields, which determine the speckle pattern in these fields. In particular, in the case of nonplanar scattering surfaces, the dependences of the contrast and correlation radius on $\Delta\omega$, macroscopic parameters of the surface shape, the root-mean-square deviation σ and the correlation radius l of the height of irregularities of the scattering surface are not analysed, as a rule. Note that the condition under which the illuminating radiation can be treated as narrowband radiation is also not specified distinctly enough.

In this paper, we solve the problem of determining the dependence of statistical and coherent properties of the radiation field scattered by rough surfaces and of the corresponding narrowband and chromatic (spectral) characteristics of illuminating radiation on the geometrical parameters of these surfaces. For this purpose, we introduce the normalised correlation function and the intensity contrast of the scattered field averaged over the time $T \geqslant 10\tau_c$ instead of the complex degree of the mutual coherence of scattered fields, which is commonly used to analyse the coherent properties of the scattered field [1, 2].

The features of the introduced functions found in our paper, in which the size d and depth L_s of the backward scattering region play the most important role, allow us to solve this problem. In particular, they permit us to determine distinctly the condition under which the illuminating radiation can be treated as narrowband radiation, which is closely related to the stationarity (homogeneity) of the averaged intensity of the scattered field. This condition was unknown earlier. The found features also allow us to formulate sufficiently distinct and convenient geometrical conditions determining the regions of existence within which the fields scattered by rough surfaces illuminated by narrowband radiation behave as coherent, partially coherent or incoherent fields.

We obtained the contrast of the averaged intensity of scattered fields within each of these regions and showed that it decreased almost to zero upon transformation of the coherent scattered field to the incoherent field. The illuminating radiation corresponding to these regions can be monochromatic, quasi-monochromatic, and polychromatic.

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Such a classification was recently performed by assuming that a surface is illuminated by radiation from a Q-switched pulsed laser [8]. In this paper, we performed the classification for rough surfaces illuminated by narrowband radiation from pulsed and cw lasers and by thermal sources. This cannot be done with the help of the complex mutual coherence function [1].

2. Correlation characteristics of scattered fields

The expression for a random field scattered by a surface can be written in the Kirchhoff approximation in the form [3, 4] (Fig. 1)

$$E(\boldsymbol{\rho}, t) = E_0 \frac{\mathrm{i}}{r_{\mathrm{c}}\lambda_0} \iint k(\boldsymbol{r}_\Sigma) \exp\left[\mathrm{i}\omega_0\left(t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}_{\mathrm{s}}| + |\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right)\right]$$

$$\times U\left(t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}_{\mathrm{s}}| + |\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right) \mathrm{d}\boldsymbol{r}_\Sigma,$$

where E_0 is the field amplitude of illuminating radiation on the surface; $\boldsymbol{r}_\Sigma$ is the radius vector of a rough region of the scattering surface; $k(\boldsymbol{r}_\Sigma)$ is the distribution of Fresnel reflection coefficients on the surface; $\boldsymbol{r}_{\mathrm{c}}$ is the radius vector of the surface point nearest to the receiving aperture; ω_0 and $\lambda_0 = c/\omega_0$ are the central frequency and wavelength of illuminating radiation; c is the speed of light; $\boldsymbol{\rho}$ is the radius vector on the receiving aperture; $\boldsymbol{\rho}_{\mathrm{s}}$ is the radius vector of the source; $U(t)$ is the slowly varying (compared to $\exp(\mathrm{i}\omega_0 t)$ dimensionless nonperiodic modulation function of illuminating radiation with the spectral width $\Delta\omega$.

Let us now introduce the random distribution $\xi(\boldsymbol{r})$ of the heights of irregularities of the scattering surface (where $\boldsymbol{r}$ is the radius vector of the mean surface) as a set of deviations of this surface from the mean surface. Then, $\boldsymbol{r}_\Sigma \approx \boldsymbol{r} + \boldsymbol{N}(\boldsymbol{r})\xi(\boldsymbol{r})$, where $\boldsymbol{N}(\boldsymbol{r})$ is the normal to the mean surface; $\boldsymbol{r} = \boldsymbol{r}(u, v)$; u, and v are the orthogonal coordinates on the mean surface. Let us also introduce the one-dimensional $[w_1(\xi)]$ and two-dimensional $[w_{12}(\xi_1, \xi_2)]$ probability densities of this distribution, so that, for example, the dispersion ξ is determined by the relation $\sigma^2 = \int \xi^2 \times w_1(\xi)\mathrm{d}\xi$. Let us place an illuminating radiation source at the centre of the receiving aperture ($\rho_{\mathrm{s}} = 0$). Because the heights of irregularities are small $[\xi(\boldsymbol{r}) \ll r]$, we have with an accuracy to unimportant factors

$$E(\boldsymbol{\rho}, t) \sim \exp(\mathrm{i}\omega_0 t) \int k(\boldsymbol{r}) \exp\left[-\frac{\mathrm{i}\omega_0(r + |\boldsymbol{r} - \boldsymbol{\rho}|)}{c}\right]$$

$$\times \exp\left[-\frac{\mathrm{i}q_N\xi(\boldsymbol{r})\omega_0}{c}\right] U\left[t - \frac{r + |\boldsymbol{r} - \boldsymbol{\rho}| + q_N\xi(\boldsymbol{r})}{c}\right] \mathrm{d}\boldsymbol{r},$$

where $q_N = \boldsymbol{qN}$; and $\boldsymbol{q} \approx \boldsymbol{2r}_{\mathrm{c}}/r_{\mathrm{c}}$ is the scattering vector (i.e., the sum of unit vectors directed from the source to the object and from the object to the receiving aperture) [3, 4].

The correlation function $B_{\mathrm{a}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ of the time-averaged intensity distribution of the scattered field has the form

$$\bar{I}(\boldsymbol{\rho}) = \frac{1}{T}\int_{t_0}^{t_0+T} I(\boldsymbol{\rho}, t)\mathrm{d}t,$$

where $I(\boldsymbol{\rho}, t) = |E(\boldsymbol{\rho}, t)|^2$ is the instant intensity of this field and t_0 is the initial instant of averaging. As a rule, the field $E(\boldsymbol{\rho}, t)$ has a Gaussian distribution [4]. It can be shown that in this case

$$B_{\mathrm{a}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{1}{T^2} \times$$

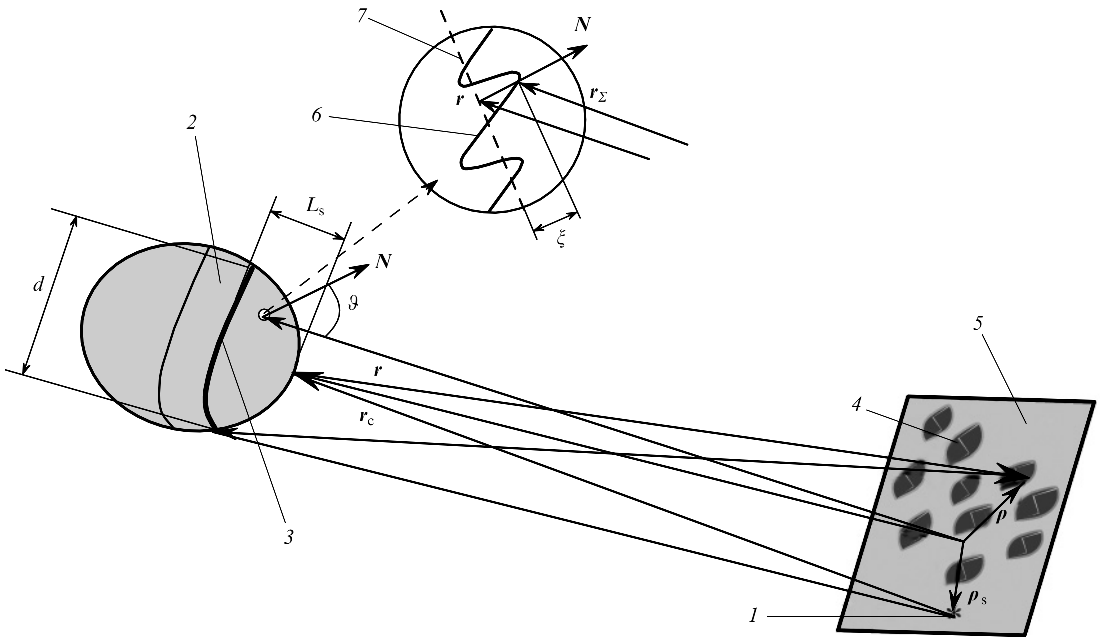

Figure 1. Scheme of the scattering of a light field by a rough surface (beams propagating from a source to the centre and periphery of the backward scattering region are shown): (*1*) illuminating radiation source; (*2*) surface; (*3*) backward scattering region boundary; (*4*) speckles; (*5*) receiving aperture [the observation region of the scattered field $E(\boldsymbol{\rho}, t)$]; (*6*) strongly magnified profile of a small site of the rough surface; (*7*) mean surface.

$$\times\int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T}|\langle E(\boldsymbol{\rho}_1,t_1)E^*(\boldsymbol{\rho}_2,t_2)\rangle_{\rm r}|^2{\rm d}t_1{\rm d}t_2, \quad (1)$$

where

$$\langle E(\boldsymbol{\rho}_1,t_1)E^*(\boldsymbol{\rho}_2,t_2)\rangle_{\rm r}=\iint E(\xi_1,\boldsymbol{\rho}_1,t_1)E^*(\xi_2,\boldsymbol{\rho}_2,t_2)$$

$$\times\,w_{12}(\xi_1,\xi_2){\rm d}\xi_1{\rm d}\xi_2$$

is the correlation function of the scattered field $E(\boldsymbol{\rho},t)$, which is determined from numerous realisations; and $\xi_k=\xi(\boldsymbol{r}_k)$. The normalised correlation function

$$B_{\rm n}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\frac{B_{\rm a}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)}{\langle I(\boldsymbol{\rho}_1)\rangle_{\rm r}\langle I(\boldsymbol{\rho}_2)\rangle_{\rm r}} \quad (2)$$

is also considered. Below, we assume that $\sigma\gg\lambda$. Then in the Fresnel approximation [4], we have

$$\langle E(\boldsymbol{\rho}_1,t_1)E^*(\boldsymbol{\rho}_2,t_2)\rangle_{\rm r}\sim\iint k_{\rm s}(\boldsymbol{r})\exp\left[{\rm i}\frac{2\pi\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]$$

$$\times U\left(t_1-\frac{r+\boldsymbol{r}\boldsymbol{\rho}_1/r_{\rm c}+q_N\xi}{c}\right)$$

$$\times U^*\left(t_2-\frac{r+\boldsymbol{r}\boldsymbol{\rho}_2/r_{\rm c}+q_N\xi}{c}\right)w_1(\xi){\rm d}\xi{\rm d}\boldsymbol{r}, \quad (3)$$

where $k_{\rm s}(\boldsymbol{r})\approx w_{\rm s}(\xi'_u=q_\perp l_u/(q_N\sigma))\,w_{\rm s}(\xi'_v=q_\perp l_v/(q_N\sigma))|k(\boldsymbol{r})|^2$ is the backward scattering coefficient of the surface; $q_\perp=(\boldsymbol{q}^2-q_N^2)^{1/2}$; l_u and l_v are the correlation radii of the heights of irregularities along the coordinates u and v; $w_{\rm s}(\xi'_u)$ and $w_{\rm s}(\xi'_v)$ are the probability densities of the distributions of slopes ξ'_u and ξ'_v of irregularity heights ξ. We will assume below that the irregularity heights are statistically isotropic, i.e., $l_u=l_v=l$. Then, $k_{\rm s}(\boldsymbol{r})\approx w_{\rm s}^2(\xi'_u=(l/\sigma)\tan\vartheta)|k(\boldsymbol{r})|^2$, where $\vartheta=q_\perp/q_N$ (Fig. 1). If, for example, the probability density is a Gaussian [3], i.e., $w_1(x)=[1/(\sqrt{2\pi}\sigma)]\exp(-\xi^2/\sigma^2)$, then $k_{\rm s}(\boldsymbol{r})\approx(l/\sigma)^2|k(\boldsymbol{r})|^2\times\exp\{-[(l/\sigma)\tan\vartheta]^2\}$. The equation $\tan\vartheta_{\rm b}(\boldsymbol{r})=\sigma/l$ determines the boundary of the backscattering region of the surface, and its distance from the surface point nearest to the receiving aperture gives the depth $L_{\rm s}$ of this region (Fig. 1). For a planar mean surface parallel to the receiving aperture, the scattered field intensity is $I(\boldsymbol{\rho},t)\sim w_1^2(\xi'_u=(l/\sigma)\tan\vartheta_\rho)$, where $\vartheta_\rho=\rho/r_{\rm c}$ is the angle between the normal to the mean surface and the direction to the observation point of the scattered field. This means [3] that in this case the scattering indicatrix is determined by the probability density of the distribution of irregularity slopes.

Taking relations (1) and (3) into account, we obtain the expressions

$$\langle I(\boldsymbol{\rho})\rangle\sim\int k_{\rm s}(\boldsymbol{r}){\rm d}\boldsymbol{r}, \quad (4)$$

$$B_{\rm a}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\iint k_{\rm s}(\boldsymbol{r}_1)k_{\rm s}(\boldsymbol{r}_2)\exp\left[{\rm i}\frac{2\pi(\boldsymbol{r}_1-\boldsymbol{r}_2)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]$$

$$\times\iint\frac{1}{T^2}\iint U\left(t_1-\frac{r_1+\boldsymbol{r}_1\boldsymbol{\rho}_1/r_{\rm c}+q_N\xi_1}{c}\right)\times$$

$$\times U^*\left(t_2-\frac{r_1+\boldsymbol{r}_1\boldsymbol{\rho}_2/r_{\rm c}+q_N\xi_1}{c}\right)$$

$$\times U^*\left(t_1-\frac{r_2+\boldsymbol{r}_2\boldsymbol{\rho}_1/r_{\rm c}+q_N\xi_2}{c}\right)$$

$$\times U\left(t_2-\frac{r_2+\boldsymbol{r}_2\boldsymbol{\rho}_2/r_{\rm c}+q_N\xi_2}{c}\right)\Bigg]$$

$$\times w_1(\xi_1)w_1(\xi_2){\rm d}\xi_1{\rm d}\xi_2{\rm d}\boldsymbol{r}_1{\rm d}\boldsymbol{r}_2{\rm d}t_1{\rm d}t_2=\iint k_{\rm s}(\boldsymbol{r}_1)k_{\rm s}(\boldsymbol{r}_2)$$

$$\times\exp\left[{\rm i}\frac{2\pi(\boldsymbol{r}_1-\boldsymbol{r}_2)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]\rho_{\rm t}(\boldsymbol{r}_1,\xi_1,\boldsymbol{\rho}_1,\boldsymbol{r}_2,\xi_2)$$

$$\times\rho_{\rm t}^*(\boldsymbol{r}_1,\xi_1,\boldsymbol{\rho}_2,\boldsymbol{r}_2,\xi_2)w_1(\xi_1)w_1(\xi_2){\rm d}\xi_1{\rm d}\xi_2{\rm d}\boldsymbol{r}_1{\rm d}\boldsymbol{r}_2, \quad (5)$$

where

$$\rho_{\rm t}(\boldsymbol{r}_1,\xi_1,\boldsymbol{\rho}_j,\boldsymbol{r}_2,\xi_2)=\frac{1}{T}\int_{t_0}^{t_0+T}U\left(t-\frac{r_1+\boldsymbol{r}_1\boldsymbol{\rho}_j/r_{\rm c}+q_N\xi_1}{c}\right)$$

$$\times U^*\left(t-\frac{r_2+\boldsymbol{r}_2\boldsymbol{\rho}_j/r_{\rm c}+q_N\xi_2}{c}\right){\rm d}t$$

are the self-convolutions of the function $U(t)$; $j=1$, 2. The dependence of the functions $\rho_{\rm t}$ on $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ indicate the deviation of the random process $\bar{I}(\boldsymbol{\rho})$ deviates from the stationary one, which is usually weak (see Appendix). The contrast of speckles of $\bar{I}(\boldsymbol{\rho})$ is described by the expression

$$C(\boldsymbol{\rho})=\frac{B_{\rm a}(\boldsymbol{\rho},\boldsymbol{\rho})}{\langle\bar{I}(\boldsymbol{\rho})\rangle^2}.$$

We assume below that, if the nonperiodic function $U(t)$ has few extrema during the time T of recording the scattered field intensity, it behaves as a determinate function with the temporal width $\tau_{\rm c}$, which belongs to the class of functions for which

$$\frac{1}{T}\int_{t_0}^{t_0+T}U\left(t-\frac{\alpha}{c}\right)U^*\left(t-\frac{\beta}{c}\right){\rm d}t=B\left[\frac{(\alpha-\beta)^2}{L_{\rm c}^2}\right]$$

for $\tau_{\rm c}\ll T$, where $L_{\rm c}=c\tau_{\rm c}$; α and β are the parameters having the dimension of length. This occurs, for example, when a Q-switched pulsed laser is used to illuminate a surface. Its coherence time $\tau_{\rm c}$ is the temporal width of the function $U(t)$. Then,

$$\rho_{\rm t}(\boldsymbol{r}_1,\xi_1,\boldsymbol{\rho}_j,\boldsymbol{r}_2,\xi_2)$$

$$\approx B\left\{-\frac{[r_1-r_2+(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{\rho}_j/r_{\rm c}+q_N\xi_1-q_N\xi_2]^2}{L_{\rm c}^2}\right\}.$$

In particular, if $U(t)$ is a Gaussian, i.e., $U(t)=\exp(-t^2/\tau^2)$, then

$$\rho_{\rm t}(\boldsymbol{r}_1,\xi_1,\boldsymbol{\rho}_j,\boldsymbol{r}_2,\xi_2)$$

$$=\exp\left\{-\frac{[r_1-r_2+(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{\rho}_j/r_{\rm c}+q_N\xi_1-q_N\xi_2]^2}{L_{\rm c}^2}\right\}. \quad (6)$$

If the nonperiodic function $U(t)$ has many extrema during the time T of recording the scattered field intensity, it is reasonable to assume that this function behaves as a random process with the correlation function $B_U(t_1, t_2) = \langle U(t_1)U^*(t_2)\rangle_U$. This takes place, for example, when illumination is performed by using a cw laser or a thermal radiation source. By assuming that the random process $U(t)$ is stationary, we obtain that $B_U(t_1, t_2) = B_U[(t_1 - t_2)^2/\tau_c^2]$. Here, τ_c is the correlation time of the process, which we also define as the coherence time. For $\tau_c = 2\pi/\Delta\omega \ll T$, the approximation

$$\frac{1}{T}\int_{t_0}^{t_0+T} U\left(t - \frac{\alpha}{c}\right)U^*\left(t - \frac{\beta}{c}\right)\mathrm{d}t \approx B_U\left[\frac{(\alpha-\beta)^2}{L_c^2}\right]$$

is valid; and

$$\rho_t(\boldsymbol{r}_1, \xi_1, \boldsymbol{\rho}_j, \boldsymbol{r}_2, \xi_2) \approx B_U\left\{-\frac{[r_1 - r_2 + (\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_j/r_c + q_N\xi_1 - q_N\xi_2]^2}{L_c^2}\right\}.$$

In particular, if the correlation function B_U is a Gaussian for which $B_U[(t_1 - t_2)^2/\tau_c^2] = \exp[-(t_1 - t_2)^2/\tau_c^2]$, the function ρ_t is determined by relation (6).

Below, we will use the term the coherence length of illuminating radiation $L_c = c\tau_c$, which is often encountered in the literature [1]. Because $U(t)$ is a slowly varying dimensionless function, we have $\tau_c > 2\pi/\omega_0$, or $L_c > \lambda_0$. Thus, both for determinate and random functions $U(t)$, the expression for ρ_t can be written in the form

$$\rho_t(\boldsymbol{r}_1, \xi_1, \boldsymbol{\rho}_j, \boldsymbol{r}_2, \xi_2) \approx G\left\{-\frac{[r_1 - r_2 + (\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_j/r_c + q_N\xi_1 - q_N\xi_2]^2}{L_c^2}\right\},$$

where the function G satisfies the conditions

$$G(0) = 1, \quad G(1) \approx 1/\mathrm{e}, \text{ and } G(\pm\infty) = 0. \tag{7}$$

3. Classification of the coherent properties of scattered fields and the chromatic properties of radiation illuminating a rough surface

We will call illuminating radiation narrowband if its spectral width is $\Delta\omega = 2\pi/\tau_c \leqslant 0.25\omega_0 M^{-1/2}$ (where $M = (d_\rho d)^2/(\lambda_0 r_c)^2$ is the number of speckles in the scattered field within the receiving aperture and d_ρ is the receiving aperture size [4]), i.e., the coherence length of illuminating radiation is

$$L_c = \frac{2\pi c}{\Delta\omega} \geqslant L_{cM}, \tag{8}$$

where $L_{cM} = 4\lambda_0 M^{1/2}$. It follows from Appendix that in this case

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \sim \iiiint k_s(\boldsymbol{r}_1)k_s(\boldsymbol{r}_2)\exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] \times\left|G\left[-\frac{(r_1 - r_2 + q_N\xi_1 - q_N\xi_2)^2}{L_c^2}\right]\right|^2 \times w_1(\xi_1)w_1(\xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \tag{9}$$

The dependence of $B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ on $\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$ demonstrates the stationarity of the random process $\bar{I}(\boldsymbol{\rho})$ upon the fulfilment of condition (8) determining the lower boundary L_{cM} of the coherence length of illuminating radiation at which this radiation is narrowband. Because $\langle\bar{I}(\boldsymbol{\rho})\rangle$ is independent of $\boldsymbol{\rho}$, the contrast $C(\boldsymbol{\rho}) = B_a(\boldsymbol{\rho}, \boldsymbol{\rho}/\langle\bar{I}(\boldsymbol{\rho})\rangle^2$ in the case of a stationary random process $\bar{I}(\boldsymbol{\rho})$ at any point of the receiving aperture is almost constant.

Thus, radiation illuminating a rough surface is narrowband if the time-averaged intensity distribution of the scattered radiation field is a random stationary process. This means that such a distribution represents a homogeneous interference pattern demonstrating the coherent properties of the scattered field. We will analyse this distribution for different relations between the coherence length L_c and geometrical parameters of the surface.

Let, for example, condition (8) be fulfilled and $L_c > q_N\sigma$. In this case,

$$B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \sim \iint k_s(\boldsymbol{r}_1)k_s(\boldsymbol{r}_2)\exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] \times\left|G\left[-\frac{(r_1 - r_2)^2}{L_c^2}\right]\right|^2\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \tag{10}$$

Here, the two most interesting cases are possible:

(i) $L_c > 2L_s$. Then, as follows from (7), for $L_c \gg L_s$, we have $|G[-(r_1 - r_2)^2/L_c^2]| \approx 1$. Therefore,

$$B_{a0}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \left|\int k_s(\boldsymbol{r})\exp\left[\mathrm{i}\frac{2\pi\boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right]\mathrm{d}\boldsymbol{r}\right|^2,$$

$$C = \frac{B_a(\boldsymbol{\rho}, \boldsymbol{\rho})}{\langle\bar{I}(\boldsymbol{\rho})\rangle^2} \sim 1.$$

Such a high contrast of speckles is explained by the fact that in this case the scattered field is formed at each point of the observation region by summation of the amplitudes of waves (interference) scattered by the entire region of backward scattering, which is typical for illuminating radiation with an infinite coherence length ($L_c = \infty$). For this reason, illuminating radiation behaves for $L_c > 2L_s$ as a monochromatic field, while the scattered field behaves as a coherent field.

(ii) $2L_s > L_c$. In this case, without loss of generality, we define the correlation function of the time-averaged intensity distribution of the scattered field $\bar{I}(\boldsymbol{\rho})$ for the mean surface, which represents a paraboloid of revolution with the axis perpendicular to the receiving aperture. Then, $r \approx r_c + R^2/(2\rho_0)$, where R is the distance from the paraboloid axis to the point with the radius vector $\boldsymbol{r}$; ρ_0 is the radius of curvature of the top of the surface under study. We can show in this case that the transverse size of the backward scattering region (Fig. 1) is $d \approx \rho_0\sigma/l$ and its depth, i.e. the longitudinal size is $L_s \approx (\rho_0/8)(\sigma/l)^2$. In this case,

$$\langle \bar{I}(\boldsymbol{\rho})\rangle \sim \int k_s(\boldsymbol{r})\mathrm{d}\boldsymbol{r} = \iint k_s(R)R\,\mathrm{d}R\,\mathrm{d}\varphi,$$

where R and φ are the polar coordinates of the mean surface. Taking conditions (7) into account, for $L_s \gg L_c > q_N\sigma$, the function $|G[-(r_1 - r_2)^2/L_c^2]|^2$ has a smaller width along the coordinate R than other functions in (10), so that the approximation $r_1 - r_2 \approx R_1(R_1 - R_2)/\rho_0$ is valid. Then, for a small value of $|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|$, we obtain

$$B_{a0}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \rho_0 L_c \int \left[1 - \frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2 d^2}{(\lambda_0 r_c)^2}\right] k_s^2(R)R\,\mathrm{d}R$$

and the contrast of speckles in the scattered field is

$$C = \frac{B_a(\boldsymbol{\rho}, \boldsymbol{\rho})}{\langle \bar{I}(\boldsymbol{\rho})\rangle} \approx \frac{L_c}{2L_s} < 1.$$

Such a relatively low contrast is explained by the fact that in this case the scattered radiation field is formed at each point of the observation region by summation of the amplitudes of waves scattered only by some parts of the backward scattering region. Therefore, by weakening somewhat the relation between the coherence length and the depth of the backward scattering region ($2L_s > L_c > q_N\sigma$), we obtain that illuminating radiation behaves as quasi-monochromatic and the scattered field – as partially coherent.

Let now $q_N\sigma \gg L_c > L_{cM} = 4\lambda_0 M^{1/2}$. Under this condition, we have

$$B_{a0}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim L_c \iiint k_s(\boldsymbol{r}_1)k_s(\boldsymbol{r}_2) \times \exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] \times w_1(\xi_1)w_1(r_1 - r_2 + \xi)\mathrm{d}\xi\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2.$$

We will assume below that the function $w_1(\xi_1)$ belongs to the class of functions for which

$$\int w_1(\xi)w_1(\gamma + \xi)\mathrm{d}\xi = \frac{1}{\sigma}\exp\left[-\frac{\gamma^2}{(q_N\sigma)^2}\right].$$

Then,

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \frac{L_c}{q_N\sigma}\iint k_s(\boldsymbol{r}_1)k_s(\boldsymbol{r}_2) \times \exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right]\exp\left[-\frac{(r_1 - r_2)^2}{(q_N\sigma)^2}\right]\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \quad (11)$$

Here, the two most interesting cases are also possible:

(i) $L_s > q_N\sigma$. Then, for $L_s \gg q_N\sigma$, the function $\exp[-(r_1 - r_2)^2/(q_N\sigma)^2]$ has a much smaller width along the coordinate r than other functions in (11). Taking this condition into account and using a paraboloid of revolution as the mean surface, we obtain

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \rho_0 L_c \int \left[1 - \frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2 d^2}{(\lambda_0 r_c)^2}\right] k_s^2(R)R\,\mathrm{d}R$$

for a small value of $|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|$, and, hence, $C \approx L_c/(2L_s) \ll 1$. Such a low contrast of speckles is explained by the fact that in this case the scattered field at each point of the observation region is formed by summation of the amplitudes of the waves scattered both by separate parts of the backward scattering region of the surface and separate parts of each irregularity on it. Because of this, under the condition $L_s \gg q_N\sigma$ and for the coherence length satisfying the inequalities $q_N\sigma \gg L_c > L_{cM} = 4\lambda_0 M^{1/2}$, the illuminating radiation behaves as polychromatic radiation and the scattered field behaves as an incoherent field.

(ii) $L_s \ll q_N\sigma$. In this case, $q_N\sigma \gg L_c > L_{cM} = 4\lambda_0 M^{1/2}$. Then, $|r_1 - r_2| \ll \xi$ and $\exp[-(r_1 - r_2)^2/(q_N\sigma)^2] \approx 1$, resulting in the relations

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \left|\int k_s(\boldsymbol{r})\exp\left[\mathrm{i}\frac{2\pi\boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right]\mathrm{d}\boldsymbol{r}\right|^2$$

and $C = L_c/(2\sigma) \ll 1$, $q_N = 2$. This case is realised rather rarely, for example, upon illuminating a flat on average surface, which is parallel to the receiving aperture, by a comparatively narrow polychromatic beam. The scattered field is formed at each observation point by summation of the amplitudes of waves scattered by separate parts of each irregularity on the surface. The scattered field is also incoherent. An interesting example can be a concave rough spherical surface (Fig. 2) with the depth of the backward scattering region $L_s = 0$.

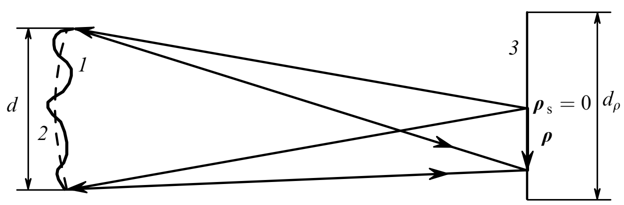

Figure 2. Scheme of the scattering of a light field by a shallow object in the form of a concave rough surface with parameters $q_N = 2$ and $L_s = 0$; (*1*) rough spherical surface; (*2*) mean spherical surface; (*3*) receiving aperture.

Note that for such a surface for $L_s \ll q_N\sigma$ and $L_c \gg 2\sigma$, the analytic expression can be obtained for the correlation function $B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$. Then, the backward scattering region coincides with the entire scattering surface and therefore $k_s(\boldsymbol{r}) = (l/\sigma)^2|k(\boldsymbol{r})|^2$. By using the approximation $|k(\boldsymbol{r})|^2 \approx \exp[-(x^2 + y^2)/d^2]$ (where x and y are the Cartesian coordinates) and assuming for simplicity that $\boldsymbol{\rho}_1 = (\rho_{1x}, 0)$, $\boldsymbol{\rho}_2 = (r_{2x}, 0)$, we obtain

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \int k_s(\boldsymbol{r}_1)k_s(\boldsymbol{r}_2)\exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c} - \frac{[(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_1/r_c]^2}{L_c^2} + \frac{[(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_2/r_c]^2}{L_c^2}\right]\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2 \sim B_n(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2),$$

where

$$B_n(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \left[1 + \frac{d^2(\rho_{1x}^2 + \rho_{2x}^2)}{(r_c L_c)^2}\right]^{-1/2} \times$$

$$\times \exp\left\{ - \pi^2(\rho_{1x}-\rho_{2x})^2 \frac{d^2}{(\lambda_0 r_c)^2}\left[1+\frac{d^2(\rho_{1x}^2+\rho_{2x}^2)}{(r_c L_c)^2}\right]^{-1}\right\},$$

and

$$C(\boldsymbol{\rho}) = \frac{B_a(\boldsymbol{\rho},\boldsymbol{\rho})}{\langle \bar{I}(\boldsymbol{\rho})\rangle^2} \approx \left[1+\frac{2d^2\rho_x^2}{(r_c L_c)^2}\right]^{-1/2}.$$

These expressions characterise variations in $B_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ and $C(\boldsymbol{\rho})$ upon removing the observation point of the scattered field from the receiving-aperture centre, in the vicinity of which a random distribution $\bar{I}(\boldsymbol{\rho})$ is stationary (homogeneous), to its periphery where deviations from the stationary distribution are maximal (Fig. 3). In this case, the contrast $C(\boldsymbol{\rho})$ of speckles of $\bar{I}(\boldsymbol{\rho})$ decreases, achieving its minimum $C_{\min} = (1+N_b^2/2)^{-1/2}$ at the edge of the receiving aperture ($\rho_x = d_\rho/2$), where $N_b = d_\rho d(r_c L_c)^{-1}$ is the number of regions of the surface making statistically independent contributions to the scattered field at the receiving-aperture edge. One can see that under the condition $L_c > 4\lambda_0 \times M^{1/2} = 4\rho_x d/r_c$, i.e., inside the region of the receiving aperture with $\rho_x < L_c r_c/(4d)$, the deviations of the random distribution $\bar{I}(\boldsymbol{\rho})$ from the stationary distribution are insignificant and the contrast in this region is approximately constant [$C(\boldsymbol{\rho}) \approx 1$]. Under the condition $L_c < 0.5\lambda_0 M^{1/2}$, i.e., for $\rho_x > 2L_c r_c/d$, the contrast is $C(\boldsymbol{\rho}) \approx 1/(\sqrt{2}n_x)$ near the receiving-aperture edge, where $n_x = \rho_x d/(r_c L_c)$ is the number of regions of the surface making statistically independent contributions to the scattered field, which rapidly increases with ρ_x. The contrast $C(\boldsymbol{\rho})$ rapidly decreases upon approaching to the receiving-aperture edge. This is caused by a noticeable smoothing of speckles of $\bar{I}(\boldsymbol{\rho})$ because the illuminating radiation begins to act as broadband near the receiving-aperture edge. Figure 3 shows the intensity distribution averaged over the time $T \gg \tau_c$ (for $T > 10\tau_c$) for the field $\bar{I}(\boldsymbol{\rho})$ scattered by a rough surface, which is shown in Fig. 2.

Note especially that the correlation radius ρ_c of speckles of the scattered field is the same ($\rho_c \approx \lambda_0 r_c/d$) for any relation between L_c and L_s. Note also that because usually $q_N \approx 2$ in practice, q_N can be replaced by 2.

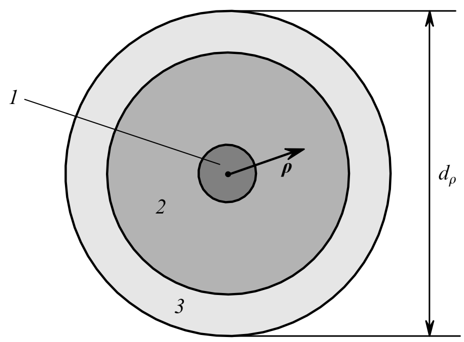

Figure 3. Intensity distribution $\bar{I}(\boldsymbol{\rho})$ averaged over the time $T > 10\tau_c$ for the field scattered by a rough surface: (*1*) region of a circular receiving aperture $\rho \leqslant L_c r_c/(4d)$, satisfying the condition $L_c > 4\lambda_0 M^{1/2}$ of the narrowband illuminating radiation [here, $M = (\pi\rho d)^2/(4\lambda_0 r_c)^2$ is the number of speckles in the scattered field], in which the distribution $\bar{I}(\boldsymbol{\rho})$ is virtually stationary; (*2*) region $L_c r_c/(4d) \geqslant \rho \geqslant L_c r_c/d$, where the distribution $\bar{I}(\boldsymbol{\rho})$ noticeably deviates from the stationary distribution; (*3*) region $\rho \geqslant L_c r_c/d$, where the distribution $\bar{I}(\boldsymbol{\rho})$ is virtually stationary.

In conclusion, we analyse the possibility of estimating the correlation function $B_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ of the time-averaged intensity distribution of the scattered field $\bar{I}(\boldsymbol{\rho})$ by approximating it by the spatial correlation function determined from one realisation of this distribution

$$B_s(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \frac{1}{S_\rho}\int \bar{I}(\boldsymbol{\rho}_1)\bar{I}(\boldsymbol{\rho}_1+\Delta\boldsymbol{\rho})A(\boldsymbol{\rho}_1)\mathrm{d}\boldsymbol{\rho}_1 - \left[\frac{1}{S_\rho}\int \bar{I}(\boldsymbol{\rho})A(\boldsymbol{\rho})\mathrm{d}\boldsymbol{\rho}\right]^2,$$

where S_ρ is the receiving-aperture area; $\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1$; and $A(\boldsymbol{\rho})$ is the function of the receiving-aperture pupil. If the receiving-aperture size d_ρ is such that the number of speckles of the scattered field within the aperture is $M = (d_\rho d)^2/(\lambda_0 r_c)^2 \geqslant 400$, then

$$\frac{\{\langle[B_s(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) - B_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)]^2\rangle_r\}^{1/2}}{\langle[B_s(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_r} \approx M^{-1/2} \leqslant 0.05.$$

Therefore, for $M \geqslant 400$, the approximation $B_s(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \approx B_a(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ is correct. Note that for $M = 400$, by using relation (8), the condition under which the illuminating radiation can be treated as narrowband can be written in the form

$$L_c = \frac{c}{\Delta\omega} \geqslant L_{cM} = 4\lambda_0 M^{1/2} = 80\lambda_0.$$

In this case, $\Delta\omega \leqslant 0.0125\omega_0 \lll \omega_0$. This means that for $M \geqslant 400$, the conventional condition $\Delta\omega \lll \omega_0$ that radiation is narrowband is fulfilled.

The results obtained above are of interest both from the methodological and practical points of view. They can be used to determine the geometrical parameters of a surface. In particular, the contrast of the intensity distribution $\bar{I}(\boldsymbol{\rho})$ averaged over the time $T > 10\tau_c$ for the field scattered by a surface illuminated by a sufficiently narrow polychromatic beam is $C = L_c/(2\sigma)$. By using this expression, we can determine the root-mean-square deviation σ of the heights of irregularities of the surface from the relation $\sigma = L_c/(2C_s)$, where

$$C_s = \frac{B_s(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)}{\left[(1/S_\rho)\int \bar{I}(\boldsymbol{\rho})A(\boldsymbol{\rho})\mathrm{d}\boldsymbol{\rho}\right]^2}$$

is the estimate of the contrast from one realisation of $\bar{I}(\boldsymbol{\rho})$.

4. Conclusions

We have obtained the following results:

(i) If rough surfaces are illuminated by a light signal described by the expression $U(t)\exp(\mathrm{i}\omega_0 t)$ (where ω_0 is the central frequency of the radiation spectrum and $U(t)$ is a slowly varying nonperiodic modulation function), the narrowband and chromatic characteristics of illuminating radiation can be determined from the intensity distribution $\bar{I}(\boldsymbol{\rho})$ averaged over the time $T > 10\tau_c$ for the light field scattered by these surfaces. The corresponding coherent properties of the scattered field can be also determined. These characteristics and properties depend on the coherence length L_c of illuminating radiation, the transverse size d

of the backward scattering region, the depth L_s of the backward scattering region, the distance r_c from the receiving aperture to the surface, the size d_ρ of the receiving aperture, the central wavelength $\lambda_0 = c/\omega_0$ of illuminating radiation, and the root-mean-square deviation σ of the heights of irregularities of the surface.

(ii) The illuminating radiation acts on the scattering surface as narrowband radiation if its spectral width satisfies the condition $\Delta\omega = 2\pi/\tau_c \leqslant 0.25\omega_0 M^{-1/2}$ or equivalent condition $L_c \geqslant 4\lambda_0 M^{-1/2}$, where $M = (d_\rho d)^2/(\lambda_0 r_c)^2$ is the number of speckles in the scattered field within the receiving aperture. Under this condition, the scattered-field intensity distribution averaged over the time $T > 10\tau_c$ is a random stationary process. Speckles in the radiation intensity distribution have the same contrast C at any site of the observation region for any relation between the length L_s and length L_c. If $L_c < \lambda_0 M^{-1/2}$, the contrast of speckles of the distribution $\bar{I}(\boldsymbol{\rho})$ noticeably decreases with removing the observation point of the scattered field to the periphery of the receiving aperture, i.e., the deviation of the random distribution $\bar{I}(\boldsymbol{\rho})$ from the stationary one increases. This means that illuminating radiation acts as broadband radiation closer to the periphery of the receiving aperture.

(iii) The illuminating narrowband radiation behaves as monochromatic and the scattered field – as coherent radiation if $2L_s < L_c$. In this case, the scattered field is formed at each point of the observation region as a sum of the amplitudes (interference) of the waves scattered by the entire backscattered region, and the contrast of speckles in this field is $C \approx 1$.

(iv) The illuminating narrowband radiation behaves as quasi-monochromatic and the scattered field – as partially coherent if $2L_s > L_c > 2\sigma$. In this case, the scattered field is formed at each point of the observation region as a sum of the amplitudes of several statistically independent waves scattered by individual sites of the backward scattering region, and the contrast of speckles in this field is $C \approx L_c/(2L_s) < 1$.

(v) The illuminating narrowband radiation behaves as polychromatic and the scattered field – as incoherent radiation if $2\sigma > L_c > L_{cM} = 4\lambda_0 M^{1/2}$ and $L_s > \sigma$. Then, the scattered field is formed at each point of the observation region as a sum of the amplitudes of many statistically independent waves scattered both by the individual sites of the backward scattering region and by the individual sites of each irregularity on the surface. The contrast of speckles in this field is $C \approx L_c/(2L_s) \ll 1$. The same situation takes place if $2\sigma \gg L_c > L_{cM}$ and $L_s \ll \sigma$ In this case, the scattered field is formed at each point of the observation region as a sum of the amplitudes of the waves scattered by the individual sites of each irregularity on the scattering surface. The contrast of speckles in this field is $C \approx L_c/(2\sigma) \ll 1$.

Appendix: The estimate of the condition under which the illuminating radiation can be treated as narrowband

The condition $L_c > 4\lambda_0 M^{1/2}$ under which the illuminating radiation can be treated as narrowband can be quite simply obtained for the exponential modulation function $U(t)$ by approximating the mean surface by a paraboloid of revolution when $L_s \gg L_c \gg q_N\sigma$. This condition is often fulfilled in practice. Taking into account that for $L_c \gg q_N\sigma$, we have

$$\rho_t(\boldsymbol{r}_1, \xi_1, \boldsymbol{\rho}_j, \boldsymbol{r}_2, \xi_2) \approx \exp\left\{-\frac{[r_1 - r_2 + (\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_j/r_c]^2}{L_c^2}\right\},$$

where $j = 1, 2$, we expand this function in a Taylor series in powers of $[(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_j/(r_c L_c)]^2$ in the vicinity of $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = 0$. As a result, we obtain the approximate relation

$$\rho_t(\boldsymbol{r}_1, \xi_1, \boldsymbol{\rho}_j, \boldsymbol{r}_2, \xi_2) \approx \exp\left[-\frac{(r_1 - r_2)^2}{L_c^2}\right] \times\left\{1 - \left[\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_j}{r_c L_c}\right]^2\right\}.$$

By using this relation, we obtain the approximation

$$B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) + \Re(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$$

for the correlation function $B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ of the time-averaged intensity distribution of the scattered field $\bar{I}(\boldsymbol{\rho})$, where

$$B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \sim \iint k_s(\boldsymbol{r}_1) k_s(\boldsymbol{r}_2) \exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] \times\left|G\left[-\frac{(r_1 - r_2)^2}{L_c^2}\right]\right|^2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2$$

is the stationary component of the correlation function; and

$$\Re(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \iint k_s(\boldsymbol{r}_1) k_s(\boldsymbol{r}_2) \exp\left[\mathrm{i}\frac{2\pi(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] \times\left|G\left[-\frac{(r_1 - r_2)^2}{L_c^2}\right]\right|^2 \left[\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{\rho}_1}{r_c L_c}\right]^2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2$$

is a small nonstationary addition. At the receiving-aperture centre ($\boldsymbol{\rho}_1 = 0$, $\boldsymbol{\rho}_2 = 0$) $\Re(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = 0$. Taking into account that in this case, $r \approx r_c + R^2/(2\rho_0)$ (where R is the distance from the paraboloid axis to a point on the surface and ρ_0 is the radius of curvature of the top of the surface under study) and the function $|G[-(r_1 - r_2)^2/L_c^2]|^2$ has a smaller width along the coordinate R compared to other functions in (10) and, hence, the approximation $r_1 - r_2 \approx R_1(R_1 - R_2)/\rho_0$ is valid, we can show that for a small value of $|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|$,

$$B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \sim \rho_0 L_c \int \left[1 - \frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2 d^2}{(\lambda_0 r_c)^2}\right] k_s^2(R) R \,\mathrm{d}R,$$

$$\Re(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \frac{(d\boldsymbol{\rho}_1)^2}{(r_c L_c)^2} B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2).$$

Then, for example, for a circular receiving aperture, the value of $\Re(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ increases upon removing from the centre to periphery, where $\rho_1 = d_\rho/2$, but the ratio $\Re/B_{a0} < 1/N_c^2$ for any $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, where $N_c = d_\rho d/(r_c L_c) = M^{1/2}\lambda_0/L_c$. For $L_c > 4\lambda_0 M^{1/2}$, we have $\Re/B_{a0} < 0.05$. Therefore, $B_a(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is approximately equal to $B_{a0}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$. The ratio $\Re/B_{a0}$ shows that the random distribution $\bar{I}(\boldsymbol{\rho})$ only slightly deviates from the stationary distribution.

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