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Spatiotemporal coherence of nonmonochromatic laser radiation in a turbulent atmosphere

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***Abstract*. The results of the analysis of coherence of nonmonochromatic radiation propagating in a turbulent atmosphere are presented. It is shown that the spatial and temporal coherence of the field of a nonmonochromatic spatially limited partially coherent Gaussian beam depends on the position and orientation of radius vectors of observation points in its cross section. Upon passing to unlimited plane and spherical waves, the temporal and spatial coherence of laser radiation becomes homogeneous over the entire transverse plane.**

Keywords: *spatiotemporal coherence, nonmonochromaticity, laser beam, atmospheric turbulence.*

The studies on the propagation of partially coherent laser beams in a turbulent atmosphere [1 – 6] usually assume that the light source is spectrally pure and the spatiotemporal coherence function of propagating radiation can be represented in a factorised form as a product of spatial and temporal coherence functions [7]. However, in the general case of nonmonochromatic sources this is not so. The influence of the width of the radiation spectrum on the far-field spatial coherence of the light source was studied in paper [8], the results of which are partly reproduced in fundamental monograph [9]. In our paper this problem is considered in a more general form and an uncertainty of the results presented in [8] is eliminated. This uncertainty has lead to not quite correct conclusions about the dependence of the spatial-coherence radius of an incoherent source on the distance.

We assume that in the initial plane $x' = x_0$ the spatiotemporal coherence function of the laser source field can be written as the product of the spatial and temporal coherence functions [7, 9]:

$$\Gamma_2(x_0, \boldsymbol{R}, \boldsymbol{\rho}, \tau) = \hat{\Gamma}_2(x_0, \boldsymbol{R}, \boldsymbol{\rho})\tilde{\Gamma}_2(x_0, \tau), \quad (1)$$

where

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$$\hat{\Gamma}_2(x_0, \boldsymbol{R}, \boldsymbol{\rho}) = U_0^2 \exp\left(-\frac{R^2}{a^2} - \frac{\rho^2}{4a^2} - \mathrm{i}k\frac{\boldsymbol{R\rho}}{F} - \frac{\rho^2}{4a_c^2}\right); \quad (2)$$

$$\tilde{\Gamma}_2(x_0, \tau) = \exp\left[-\left(\frac{\tau}{\tau_0}\right)^2\right]; \quad (3)$$

U_0 is the initial field amplitude on the beam axis; k is the wave number; a and F are the initial beam radius and the radius of its phase-front curvature; a_c is the radius of the spatial coherence of the initial field; τ_0 is the coherence time of the source; $\boldsymbol{R} = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2$; $\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$; $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ are radius vectors of observation points in the transverse plane.

Let us apply the Fourier transform in time

$$\Gamma_2(x_0, \boldsymbol{R}, \boldsymbol{\rho}, \omega) = G_2(x_0, \omega)\hat{\Gamma}_2(x_0, \boldsymbol{R}, \boldsymbol{\rho}) \quad (4)$$

to (1), where the spectral density $G_2(x_0, \omega)$ for coherence function (3) has the form

$$G_2(x_0, \omega) = \frac{1}{2\pi}\int \mathrm{d}\tau \exp(\mathrm{i}\omega\tau)\tilde{\Gamma}_2(x_0, \tau) = \frac{\tau_0}{2\sqrt{\pi}}\exp\left(-\frac{\omega^2\tau_0^2}{4}\right)$$

$$= \frac{1}{2\sqrt{\pi}\Delta\omega}\exp\left[-\left(\frac{\omega}{2\Delta\omega}\right)^2\right], \quad (5)$$

where $\Delta\omega$ is the width of the spectral density distribution. In the case of the stationary field, for each spectral component in the plane $x' = x$ we can write [9]

$$\Gamma_2(x, \boldsymbol{R}, \boldsymbol{\rho}, \omega) = G_2(x_0, \omega)\hat{\Gamma}_2(x, \boldsymbol{R}, \boldsymbol{\rho}). \quad (6)$$

Let us substitute (5) into (6) and apply the inverse Fourier transform to (6) taking into account that $k = \omega/c$. Then, for the function of the spatiotemporal coherence of the field of a laser beam propagating in a turbulent atmosphere we have the relation

$$\Gamma_2(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau) = \int \mathrm{d}\omega \exp(-\mathrm{i}\omega\tau)G_2(x_0, \omega)\hat{\Gamma}_2(x, \boldsymbol{R}, \boldsymbol{\rho})$$

$$= \frac{\tau_0}{2\sqrt{\pi}}\int \mathrm{d}\omega \exp(-\mathrm{i}\omega\tau)\exp\left(-\frac{\omega^2\tau_0^2}{4}\right)\frac{\omega^2}{4\pi^2c^2(x - x_0)^2}$$

$$\times \int \mathrm{d}\boldsymbol{R}'\mathrm{d}\boldsymbol{\rho}'\hat{\Gamma}_2(x_0, \boldsymbol{R}', \boldsymbol{\rho}')\exp\left[\mathrm{i}\frac{\omega}{c(x - x_0)}(\boldsymbol{R} - \boldsymbol{R}')(\boldsymbol{\rho} - \boldsymbol{\rho}')\right.$$

$$\left. - \frac{\pi\omega^2}{4c^2}\int_{x_0}^{x}\mathrm{d}\zeta H\left(\zeta, \boldsymbol{\rho}\frac{\zeta - x_0}{x - x_0} + \boldsymbol{\rho}'\frac{x - \zeta}{x - x_0}\right)\right], \quad (7)$$

where for $\hat{\Gamma}_2(x, \boldsymbol{R}, \boldsymbol{\rho})$ we used the expression [1, 2, 6] obtained in the Markovian approximation [10]; $H(x, \boldsymbol{\rho}) = 2\int d\boldsymbol{\kappa}\Phi_\varepsilon(|\boldsymbol{\kappa}|)(1 - \cos\boldsymbol{\kappa\rho})$; Φ_ε is the spectral density of fluctuations of the dielectric constant ε of the medium, which for the Kolmogorov turbulence has the from

$$\Phi_\varepsilon(|\boldsymbol{\kappa}|) = 0.033 C_\varepsilon^2 |\boldsymbol{\kappa}|^{-11/3};$$

C_ε^2 is the structural characteristic. The use of the Kolmogorov model of turbulence in (7) does not reduce the generality of the problem considered because taking into account inner and outer scales of turbulence will lead only to a change in the form of the function $H(x, \boldsymbol{\rho})$ [2, 6].

By selecting the central frequency ω_0 of the radiation spectrum in the integration variable $\omega = \omega_0 + \omega_1$ and by integrating (7) with the use of the quadratic approximation of the function $H(x, \boldsymbol{\rho})$ [2], for the modulus of the function $\Gamma_2(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau)$ we obtain the expression

$$|\Gamma_2(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau)| = \frac{a^2}{a_{\text{eff}}^2 D} \exp\left(-\frac{\tau^2}{\tau_0^2 D^2}\right) \exp\left(-\frac{\rho^2}{4\rho_a^2 D^2}\right) \times \exp\left(-\frac{R^2}{a_{\text{eff}}^2 D^2}\right) \exp\left(-\frac{\mu^2 \cos^2\varphi}{B\,2\omega_0^2\tau_0^2 D^2} \frac{R^2}{a_{\text{eff}}^2} \frac{\rho^2}{\rho_a^2}\right) \times \exp\left(\frac{\tau}{\tau_0} \frac{\mu\cos\varphi}{B\omega_0\tau_0 D^2} \frac{R}{a_{\text{eff}}} \frac{\rho}{\rho_a}\right), \quad (8)$$

where

$$D^2 = 1 + \frac{4R^2}{a_{\text{eff}}^2 \omega_0^2 \tau_0^2} + \frac{\rho^2}{\rho_a^2 \omega_0^2 \tau_0^2};$$

$$a_{\text{eff}}^2 = a^2\left[\left(1 - \frac{L}{F}\right)^2 + \Omega_0^{-2}\left(1 + \frac{a^2}{a_c^2} + \frac{4}{3}\frac{a^2}{\rho_p^2}\right)\right]$$

and

$$\rho_a^2 = a_{\text{eff}}^2 \left\{1 + \frac{a^2}{a_c^2} + 4\frac{a^2}{\rho_p^2}\left[1 - \frac{L}{F} + \frac{1}{3}\left(\frac{L}{F}\right)^2 + \frac{1}{3}\Omega_0^{-2}\left(1 + \frac{a^2}{a_c^2}\right)\right] + \frac{4}{3}\Omega_0^{-2}\left(\frac{a}{\rho_p}\right)^4\right\}^{-1}$$

are the effective radius and radius of the spatial coherence of a Gaussian beam at the frequency ω_0 in a turbulent atmosphere, respectively;

$$\mu = \Omega_0^{-1}\left(1 + \frac{a^2}{a_c^2} + 2\frac{a^2}{\rho_p^2}\right) - \frac{k_0 a^2}{F}\left(1 - \frac{L}{F}\right);\ B^2 = \left(\frac{\rho_a}{a_{\text{eff}}}\right)^{-2};$$

$\Omega_0 = k_0 a^2/L$ is the Fresnel number of the transmitting aperture; $L = x - x_0$; $\rho_p = (0.365 C_\varepsilon^2 k_0^2 L)^{-3/5}$ is the radius of the spatial coherence of a monochromatic plane wave in a turbulent atmosphere; $k_0 = \omega_0/c$; φ is the angle between vectors $\boldsymbol{R}$ and $\boldsymbol{\rho}$.

For the spatiotemporal coherence degree

$$\gamma(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau) = \frac{|\Gamma_2(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau)|}{I(x, \boldsymbol{R})}$$

[where $I(x, \boldsymbol{R})$ is the radiation intensity], we have from (8)

$$\gamma(x, \boldsymbol{R}, \boldsymbol{\rho}, \tau) = \frac{\tilde{D}}{D} \exp\left(-\frac{\tau^2}{\tau_0^2 D^2}\right) \exp\left(-\frac{\rho^2}{4\rho_a^2 D^2}\right) \times \exp\left[-\frac{\rho^2}{\rho_a^2} \frac{R^2}{a_{\text{eff}}^2} \frac{1}{\omega_0^2 \tau_0^2 D^2}\left(\frac{\mu^2\cos^2\varphi}{B^2} - \frac{1}{\tilde{D}^2}\right)\right] \times \exp\left(\frac{\tau}{\tau_0} \frac{R}{a_{\text{eff}}} \frac{\rho}{\rho_a} \frac{\mu\cos\varphi}{B} \frac{1}{\omega_0\tau_0 D^2}\right), \quad (9)$$

where

$$\tilde{D}^2 = 1 + \frac{4R^2}{a_{\text{eff}}^2 \omega_0^2 \tau_0^2}.$$

It follows from (9) that the temporal coherence radius (for $\rho = 0$) depends on the parameter $\tilde{D}$, and, therefore, on the radius vector $\boldsymbol{R}$ of the observation point and the product $\omega_0\tau_0$. In the optical wavelength range, $\omega_0 \sim 10^{14} - 10^{15}$ Hz. The coherence time of laser sources can vary from 10^{-3} to 10^{-14} s [6, 8, 9], which means that the parameter $\omega_0\tau_0$ can be of the order of unity. Therefore, the temporal coherence of radiation changes over the beam cross section, and for $\omega_0\tau_0 = 1$ the coherence time τ_c determined by the level $\gamma(\tau_c) = e^{-1}$ can achieve $\tau_0\sqrt{5}$ at the beam edge ($R = a_{\text{eff}}$), i.e., can exceed by $\sqrt{5}$ times the coherence time on the beam axis and the coherence time τ_0 of the source.

The spatial coherence (for $\tau = 0$) also depends on parameters D and $\tilde{D}$, the positions and mutual orientation of observation-point radius vectors. It follows from (9) in particular, that for $R = 0$ and $\rho \to \infty$ the index of the second exponential in (9) tends to the constant $\omega_0^2\tau_0^2/4$, and a decrease in the spatial coherence of nonmonochromatic radiation with increasing ρ is determined by the factor $D^{-1} \sim \rho^{-1}$ only. After passing in (9) to unlimited plane ($\Omega_0 \to \infty$) and spherical ($\Omega_0 \to 0$) waves, the dependence on the parameter $\omega_0\tau_0$ vanishes. For $\omega_0\tau_0 \gg 1$, expression (9) is reduced to formulas for a monochromatic laser beam [2, 6].

Figure 1 shows the results of calculations of the spatial coherence degree of a nonmonochromatic Gaussian beam for the symmetric location of observation points with respect to the beam axis ($\boldsymbol{R} = 0$) and $R = a_{\text{eff}}$, $\cos\varphi = 0$. For this geometry in dimensionless units ρ/ρ_a, we can compare the scales of spatial coherence of nonmonochromatic and monochromatic partially coherent sources under

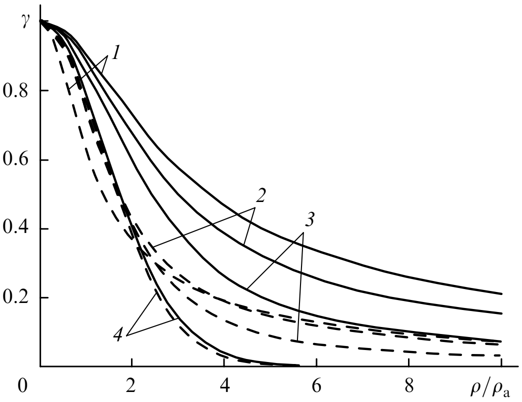

Figure 1. Degree of the spatial coherence of a nonmonochromatic radiation source for $\boldsymbol{R} = 0$ (dashed curve) and $R = a_{\text{eff}}$, $\cos\varphi = 0$ (solid curves) for $\omega_0^2\tau_0^2 = 1$ (*1*), 5 (*2*), 10 (*3*) and 100 (*4*).

the arbitrary diffraction and turbulent propagation conditions. It follows from Fig. 1 that the radius of the spatial coherence increases with decreasing the temporal coherence of the source (an increase in the width of the radiation spectrum), if the observation points are located with respect to the radius vector $\boldsymbol{R}$, which does not coincide with the beam axis ($\boldsymbol{R} \neq 0$). For $\omega_0^2\tau_0^2 = 1$, $R = a_{\text{eff}}$ and $\cos\varphi = 0$, the radius of the spatial coherence determined by the decrease in the degree of coherence down to the e^{-1} level by more than three times exceeds the coherence radius of a quasi-monochromatic source ($\omega_0^2\tau_0^2 > 100$). Upon symmetric (with respect to the beam axis) position of observation points, such an increase in the coherence radius does not occur. For $\omega_0^2\tau_0^2 = 1$, a weak increase and a decrease is observed for $\rho > 2\rho_{\text{a}}$ and $\rho < 2\rho_{\text{a}}$, respectively, compared to the coherence radius of monochromatic radiation.

In a particular case of a spatially incoherent source ($a_{\text{c}} \to 0$), we obtain from (9) the degree of spatial coherence

$$\gamma(x, \boldsymbol{R}, \boldsymbol{\rho}) = \frac{1}{D}\exp\left[-\frac{\rho^2}{4a_{\text{d}}^2 D^2}\left(1 + 4\frac{R^2\rho_{\text{a nc}}^2 k_0^2\cos^2\varphi}{\omega_0^2\tau_0^2 L^2}\right)\right], \quad (10)$$

where

$$\hat{D}^2 = 1 + \frac{\rho^2}{\rho_{\text{a nc}}^2\omega_0^2\tau_0^2};$$

$$\rho_{\text{a nc}}^2 = \left(\frac{4}{3}\rho_{\text{p}}^{-2} + a_{\text{d}}^{-2}\right)^{-1}$$

is the spatial-coherence radius of a monochromatic incoherent source at the frequency ω_0 in a turbulent atmosphere; $a_{\text{d}} = L/(k_0 a)$ is the radius of the spatial coherence of an incoherent source in vacuum. It follows from (10) that, unlike the symmetric position of observation points ($\boldsymbol{R} = 0$) and the case $\cos\varphi = 0$ (Fig. 1), for $\cos\varphi \neq 0$ the spatial coherence of nonmonochromatic incoherent source decreases upon a displacement of the radius vector $\boldsymbol{R} = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2$ from the beam axis and for $\omega_0^2\tau_0^2 = 1$, $R/a = 2$, $\cos\varphi = 1$ in the absence of turbulence ($\rho_{\text{a nc}} \equiv a_{\text{d}}$) becomes four times smaller than a_{d} (Fig. 2). However, this is not caused by a deviation from a linear dependence of the spatial coherence radius of an incoherent nonmonochromatic source on the distance beginning with some critical distance [8]. One can see from Fig. 2 that the displacement of the vector $\boldsymbol{R}$ from the beam axis by a distance smaller that $a/2$, does not lead to any noticeable decrease in the spatial-coherence radius of an incoherent nonmonochromatic source because the calculation curves for γ at $R = a/2$ and $R = 0$ almost coincide.

In [8], the expression was obtained for the degree of spatial coherence of a nonmonochromatic source for the observation-point radius-vector geometry $\boldsymbol{\rho}_1 = 0$, $\boldsymbol{\rho} = \boldsymbol{\rho}_2$. In this case, an inaccuracy was committed: the difference vector $\boldsymbol{\rho} = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1$ equal to $\boldsymbol{\rho}_2$ for $\boldsymbol{\rho}_1 = 0$ and the observation-point vector $\boldsymbol{\rho}_2$ were identified. In (9) and (10), it would correspond to a change of R by ρ for $\cos\varphi = 1$. As a result, an incorrect conclusion was made in [8, 9] about a change in the power dependence of the spatial coherence radius on the distance beginning with some critical propagation path length and 'violation' of the Van Cittert – Zernicke theorem for an incoherent source.

In the general case when $\cos\varphi \neq 0$, a dependence on parameters μ and B, and, therefore, on diffraction parameters of a Gaussian beam and turbulent propagation conditions appears in (9). Figure 3 shows as an example the results of calculations of the spatial coherence degree of a focused beam propagating in a homogeneous medium

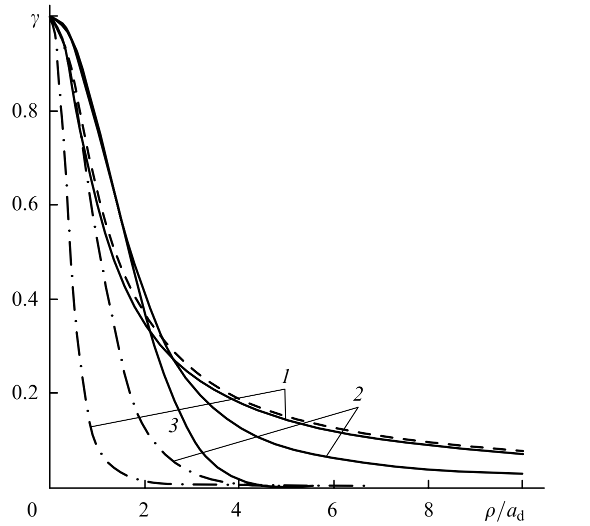

Figure 2. Degree of the spatial coherence of an incoherent nonmonochromatic radiation source for $R/a = 0$ (dashed curve), 1/2 (solid curves) and 2 (dash-and-dot curves) for $\omega_0^2\tau_0^2 = 1$ (*1*), 10 (*2*) and 10^4 (*3*). Curves (*3*) coincide for $R/a = 1/2$ and 2, and curves (*1, 2, 3*) – for $R/a = 0$.

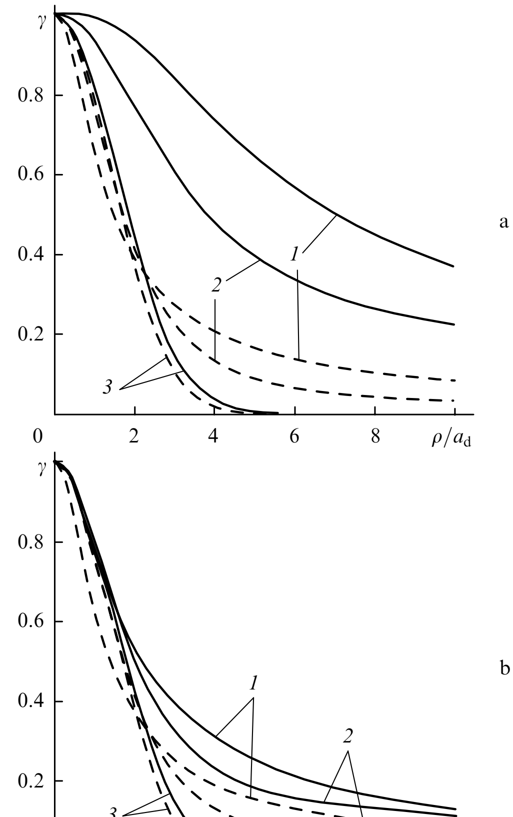

Figure 3. Degree of the spatial coherence of a focused beam for $a/a_{\text{c}} = 0$ (solid curves) and 10 (dashed curves) for $\omega_0^2\tau_0^2 = 1$ (*1*), 10 (*2*) and 10^4 (*3*), $R/a_{\text{d}} = 2$ (a), and 1/2 (b).

($C_\varepsilon^2 = 0$) for $\cos\varphi = 1$. One can see that as in the case of an incoherent source, the spatial coherence degree of a nonmonochromatic focused beam depends on the radius vector $\boldsymbol{R}$. However, in this case, the spatial-coherence radius increases at the beam edge and for $R = 2a_\mathrm{d}$, $\omega_0^2\tau_0^2 < 10$, $a_\mathrm{c} = \infty$ exceeds considerably the spatial-coherence radius of the field of a monochromatic ($\omega_0\tau_0 = 100$) focused beam. Note that the parameter a_d for a focused monochromatic beam is both a spatial coherence radius and a beam diffraction radius [1, 2, 6]. For $R < a_\mathrm{d}/2$ (Fig. 3b) and partial spatial coherence of the source, the calculated curves for γ are close to curves in the case of the symmetric position of observation points (Fig. 1).

Therefore, it follows from the above results of the analysis of the field coherence of nonmonochromatic laser radiation propagating in a turbulent atmosphere that the spatial and temporal coherence of a nonmonochromatic field of a spatially limited Gaussian beam depends on the location and orientation of observation-point radius vectors in the beam cross section. The coherence time increases upon a displacement of the observation point from the beam centre and for small coherence times τ_0 of the source, such that $\omega_0\tau_0 = 1$ at the beam edge exceeds by $\sqrt{5}$ times the coherence time τ_0 on the beam axis.

For $\omega_0\tau_0 \to 1$, the spatial coherence radius of the field of the propagating nonmonochromatic radiation also increases by several times compared to the coherence radius of monochromatic radiation for an asymmetric position of observation point with respect to the beam axis. The exception is an incoherent source. The nonmonochromatism of the radiation spectrum in this case results, on the contrary, in a decrease in the spatial-coherence radius if the observation points are located at a considerable distance from the beam axis in the direction perpendicular to the radial one.

The results obtained can be treated as a manifestation of a dispersion dependence of the diffraction of a nonmonochromatic spatially limited beam on a transmitting aperture and random inhomogeneities of the refractive index in atmosphere. Upon passing to unlimited plane and spherical waves the temporal and spatial coherence of laser radiation becomes homogeneous over the entire transverse plane.

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