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Preliminary training of a self-pumped loop phase-conjugate mirror based on a photorefractive crystal

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***Abstract*. It is shown by the example of a loop self-pumped phase-conjugate (SPPC) mirror based on a photorefractive crystal (PRC) $BaTiO_3$ that formation of a phase-conjugate (PC) wave in a SPPC mirror can be considerably accelerated by using a preliminary training of the mirror. For this purpose, it is necessary to direct preliminary an auxiliary (training) optical field on the SPPC mirror, which contains some information on the properties of the input signal whose wave front will be conjugated later. This procedure provides the writing of static refractive-index gratings in the PRC already at the training stage. The presence of these gratings ensures a much more rapid (by 6 – 20 times) production of volume refractive-index gratings required for the efficient conjugation of the signal radiation. Several variants of static and dynamic SPPC mirror training procedures are simulated and their efficiencies are compared.**

Keywords: *loop PC mirror, photorefractive nonlinearity, dynamic holograms, nonlinear response formation.*

1. Introduction

Phase distortions appearing during the propagation of laser radiation in inhomogeneous paths can be compensated by means of phase-conjugate (PC) mirrors based on photorefractive crystals (PRCs) [1], which can operate at cw radiation intensities up to a few mW cm^{-2} [2]. The so-called self-pumped PC (SPPC) mirrors do not even require any auxiliary pump sources [3, 4], being in fact threshold-free (with respect to the radiation intensity but not to the nonlinear interaction constants) optical parametric oscillators. For example, in double SPPC mirrors [5 – 7], two incoherent [8], orthogonally polarised [9] or two light waves with different carrier frequencies [10] are conjugated simultaneously.

Lasing is developed due to the self-organisation (phase transition) in the nonlinear medium – light field system [11, 12]. In this sense, the situation is similar to the formation of PC radiation in SRS and SBS SPPC mirrors [13]. It is quite natural therefore that, although the stationary values of the nonlinear reflection coefficient R and the overlap integral H in SPPC mirrors achieve in this case 0.8 – 0.9 and more than 0.9, respectively, the formation time of the conjugated wave proves to be rather long, exceeding tens of seconds (see, for example, [14]).

Note that the well-known methods for increasing the writing rate of dynamic holograms in PRCs based on variations in the parameters and geometry of the problem [14], an increase in the seed noise level [13, 15], the use of frequency shifts [16, 17], and external constant [18, 19] or alternating [20 – 22] electric fields prove to be in fact inapplicable in this case. They either drastically reduce the SPPC mirror efficiency (R or H) or cause the development of instabilities and self-excitation resulting in the development of complicated self-oscillation lasing regimes with characteristic times up to a few hours and days [2].

Below, we will show by the example of a loop SPPC mirror based on a photorefractive $BaTiO_3$ crystal by using the calculation procedures described in [23, 24] that the formation of a conjugated wave in a SPPC mirror can be considerably accelerated by employing the preliminary training of the mirror. This is achieved by preliminary irradiating the SPPC mirror by an auxiliary (training) optical field to write some dynamic holograms in the PRC. The presence of these holograms provides later a much more rapid formation of volume refractive-index gratings required for the efficient conjugation of the signal radiation. Obviously, such a SPPC mirror training procedure cannot be versatile, and to realise it in practice (to select the optimal spatial structure of the training field), it is necessary to have some preliminary information on the properties of signal radiation whose wave front will be conjugated later.

The concept of training considered below is closely related to the pattern recognition problems [25, 26], the development of optical correlators [27] and elements of the associative memory [28 – 30] based on SPPC mirrors. In one of the variants of such devices considered earlier, the authors [30] studied the possibility of simultaneous holographic writing of several time-separated (and spatially separated, in the opinion of authors [30]) input images in a PRC. The SPPC mirror based on a Cu : KNSBN photorefractive crystal was used in experiments. At the writing stage (training, in our terms), two optical images (positive and negative) produced by means of a liquid-crystal transparency and different from each other by a complete contrast inversion, were interchanged at the SPPC mirror input at the frequency $f_m = 30$ frames per second. The image

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interchange period $\tau_{\rm m} = 1/f_{\rm m}$ was specially selected to be much shorter than the writing and erasing time of dynamic holograms in the PRC. After the end of the transient writing stage, the reading stage was started (in our terms, the start of conjugation of the signal radiation) during which some new input image formed by the same transparency was projected onto the SPPC.

The two results obtained by the authors [30] are of the main interest to us. First, they have found that at the writing stage itself, after the end of all the transient processes, the spatial structure of the output SPPC mirror signal does not coincide at each instant of time with alternating input images. Second, they have shown that the formation time of a stationary nonlinear SPPC mirror response at the reading stage drastically depends on the degree of correlation of a new input image with images used for writing. Although the spatial structure of the input SPPC mirror signal after the end of the transient process always exactly corresponded to the phase conjugation of a new input image, in the cases when the spatial structure of the latter coincided with that of one of the writing fields, the nonlinear SPPC mirror response was formed faster by two–three orders of magnitude. However, the natural question about the required degree of correlation between a new input image and the writing (training) fields and about the optimal SPPC-mirror training procedure itself was not solved in [30].

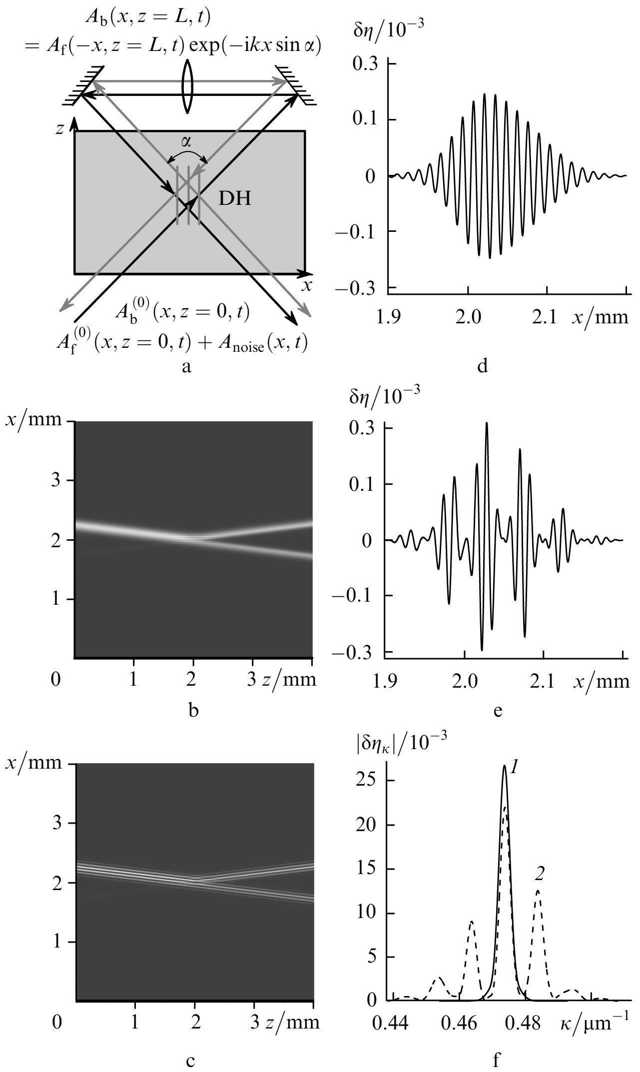


Figure 1. Loop SPPC mirror. The interaction geometry of the waves $A_{\rm f,b}$ (a) and maps (in linear shades of grey) of distributions $I_{\rm b}(x, z)$ (b, c), $\delta\eta(x)$ (d, e), and their spectra $\delta\eta_\kappa(\kappa)$ in the section $z = L/2$ of dynamic holograms upon stationary conjugation of a Gaussian beam without the harmonic modulation of the input field $A_{\rm f}(x, z = 0)$ with the spatial period $\Lambda_{\rm m} = 100$ μm [$I_{\max} = 35$ mW cm^{-2}, Figs b, d and curve (*1*) in Fig. f] and upon modulation [$I_{\max} = 55$ mW cm^{-2}, Figs c, d and curve (*2*) in Fig. f]. Hereafter, the beam diameter is $2\rho_0 = 230$ μm, $\alpha = 14°$, $\langle I_{\rm noise}\rangle/I_{\max} = 10^{-4}$, and $E_0 = 1$ V cm^{-1}.

2. Model of a loop SPPC mirror and numerical calculations

Figure 1a illustrates the geometry of a model problem under study. As in [23], we assume that the forward and backward light waves with the amplitudes $A_{\rm f,b}$ and wave vectors $\boldsymbol{k}_{\rm f,b} = \{k_x, \pm k_z\}$ propagate from the opposite faces of a PRC (planes $z = 0$ and $z = L$) at a small angle $\alpha/2$ ($k_z \gg k_x$) to the positive (negative) direction of the z axis, respectively. It is assumed that the wave $A_{\rm b}$ is formed by an optical system consisting of two fold mirrors and a lens. The mirrors change the propagation direction of the wave $A_{\rm f}$ by the angle $(\pi - \alpha)$, while the lens projects without scaling the distribution of the field of the wave $A_{\rm f}$ on the input face of the PRC. This provides the fulfilment of the boundary condition $A_{\rm b}(x, z = L, t) = A_{\rm f}(-x, z = L, t)\exp(-{\rm i}kx\sin\alpha)$.

As in [23, 24], the kinetics of the nonlinear PRC response was calculated by using the system of microscopic equations [31] obtained for the two-dimensional case (the so-called slit beams [32]) taking into account only transmission dynamic holograms [the vector $\boldsymbol{\kappa}$ of the refractive-index gratings $\delta\eta(x, z, t)$ written in the PRC is directed along the axis x] by neglecting the photovoltaic effect [1, 14, 18]. It was assumed that the external electrostatic field E_0 in the PRC was directed along the x axis and is comparatively weak ($E_0 = 1$ V cm^{-1}). The problem to be solved was transferred to the class of self-consistent problems taking into account the relation of the intensity distributions $I_{\rm f,b}(x, z, t) = |A_{\rm f,b}(x, z, t)|^2$ with the crystal field $E_{\rm sc}(x, t) \propto \delta\eta(x, z, t)$, which is specified by standard truncated wave equations for the amplitudes $A_{\rm f,b}(x, z, t)$ of the interacting waves. The truncated equations were written in the paraxial approximation by neglecting the spatially homogeneous addition to the refractive index η of the PRC caused by the external field E_0. As in [23, 24], we considered the situation in which the total intensity was determined by the sum of intensities $I(x, z, t) = I_{\rm f}(x, z, t) + I_{\rm b}(x, z, t)$, i.e., the case of incoherent or orthogonally polarised counterpropagating waves.

We solved the self-consistent problem by calculating numerically the evolution of distributions $A_{\rm f,b}(x, z, t)$ and $\delta\eta(x, z, t)$ in time. All the variables were described on the grid with the number of nodes on the PRC aperture equal to 8192 (along the width $h = 4$ mm) and 512 (along he length $l = 4$ mm). The initial conditions corresponded to the 'switching on' of the PC mirror at the instant $t = 0$. After that ($t \geqslant 0$), the input field $A_{\rm f}(x, z = 0, t)$ was defined as a superposition of the useful (training or information) signal $A_{\rm f}^{(0)}(x, z = 0, t)$ and delta-correlated (taking into account a step over x and t) noise $A_{\rm noise}(x, t)$, whose average intensity $\langle I_{\rm noise}\rangle = \langle|A_{\rm noise}(x, t)|^2\rangle$ in all realisations was 10^{-4} of the maximum intensity $I_{\max} = 35$ or 55 mW cm^{-2} of the useful signal (see below). The problem was solved within the framework of the adiabatic approximation by using the method of separation over physical factors and the fast Fourier transform [33, 34]. As in [23, 24], the step in time ($\Delta t \simeq 0.15$ s) was much smaller than the evolution time of the PRC state, and most of the problem parameters were not varied and their values were determined by the PRC type ($BaTiO_3$, see Table 1 in [23, 24]). The period of dynamic holograms written in the PRC was specified by the convergence angle $\alpha = 14^{\circ}$ of the beams, which was also fixed.

The spatial distribution of the 0.514-nm radiation at the PRC input was defined as

$$A_{\mathrm{f}}^{(0)}(x, z=0, t) = G(x)M(x, t), \tag{1}$$

where the functions $G(x)$ and $M(x, t)$ describe the Gaussian envelope of the light beam (the beam diameter is $2\rho_0 = 230$ μm) and its spatial (information) modulation. It is the instantaneous variation in $M(x, t)$ at the instants $t = 0$ corresponding to the beginning $[M(x, t < 0) \equiv 0 \rightarrow M(x, t \geqslant 0) \neq 0]$ and end $[M(x, t < 0) \neq 0 \rightarrow M(x, t \geqslant 0)]$ of the SPPC mirror training stage (this notation takes into account that the time reading was started at these instants anew) that simulated the transient process whose acceleration is the main goal of this paper. The form of the function $M(x, t)$ was different for different realisations and will be described below.

Because we optimised earlier [23] all the parameters of the problem, the SPPC mirror performed phase conjugation of the input beam both in the absence of harmonic information modulation $[M(x, t \geqslant 0) = 1]$ and in its presence $[M(x, t \geqslant 0) = \sin(2\pi\kappa_{\mathrm{m}}x)]$. Figures 1b, c show the maps of the distribution $I_{\mathrm{b}}(x, z)$ in shades of grey (darker sites correspond to the lower values of I) in the PRC after the end of the transient process ($t = 150$ s) in both these cases. The distribution $I_{\mathrm{b}}(x, z)$ in Fig. 1c corresponds to the period of spatial modulation $\Lambda_{\mathrm{m}} = \kappa_{\mathrm{m}}^{-1} = 100$ μm. Stationary dynamic holograms were formed in the region of self-crossing of the forward and backward beams (Fig. 1a). Their distributions $[\delta\eta(x, z)$, Figs 1d, e] and spatial spectra $[\delta\eta_{\kappa}(\kappa)$, Fig. 1f] in the plane $z = L/2$ in the absence of the harmonic modulation [Fig. 1d and solid curve (*1*) in Fig. 1f] and in its presence [$\Lambda_{\mathrm{m}} = 100$ μm for Fig. 1e and dashed curve (*2*) in Fig. 1f] are presented in Figs 1d–f. The positions of the maxima for dependences $\delta\eta_{\kappa}(\kappa)$ correspond to $\alpha = 14^{\circ}$ and to the indicated frequency κ_{m} of the spatial modulation of the input signal.

The kinetics of the nonlinear reflection coefficient of the SPPC mirror

$$R(t) = \int_0^{H/2} |A_{\mathrm{b}}(x, z=0, t)|^2 \mathrm{d}x \Big/ \int_0^{H/2} |A_{\mathrm{f}}(x, z=0, t)|^2 \mathrm{d}x \tag{2}$$

and of the overlap integral

$$H(t) = \left| \int_0^{H/2} A_{\mathrm{f}}(x, z=0, t) A_{\mathrm{b}}^{*}(x, z=0, t) \mathrm{d}x \right|^2$$

$$\times \left[\int_0^{H/2} |A_{\mathrm{f}}(x, z=0, t)|^2 \mathrm{d}x \int_0^{H/2} |A_{\mathrm{b}}(x, z=0, t)|^2 \mathrm{d}x \right]^{-1}, \tag{3}$$

as well as of their product RH characterising the instant fraction of power of the conjugated component of the output field on the input face of the PRC normalised to its maximum value $(RH)_{\max}$ is illustrated for both these cases in Fig. 2. One can easily verify that, although the maximum values of these parameters are $R_{\max} \simeq 0.83$ и $H_{\max} > 0.95$, the time interval required for the SPPC mirror to pass to stationary lasing proves to be very long. By defining it as the time τ_{t} required for the parameter RH to achieve 90 % of $(RH)_{\max}$, we obtain $\tau_{\mathrm{t}} = 60$ and 65 s in the absence (Figs 2a–c) and presence (Figs 2d–f) of the harmonic information modulation. Note that, as in [23, 24], the dependences $R(t)$ and $H(t)$ were calculated by passing preliminary the output field $A_{\mathrm{b}}(x, z=0, t)$ through a spatial filter separating half the linear aperture of the PRC corresponding to the position of the intensity maximum of the input beam A_{f} on the input plane [see (2) and (3)]. As a result, the parameter R characterised the power of the part of the input wave which was reflected at the given instant t from a dynamic hologram, and the parameter H characterised the degree of its correlation with the input field.

3. Static training of the SPPC mirror

As mentioned above, the SPPC mirror training procedure cannot be versatile because the choice of the optimal spatial

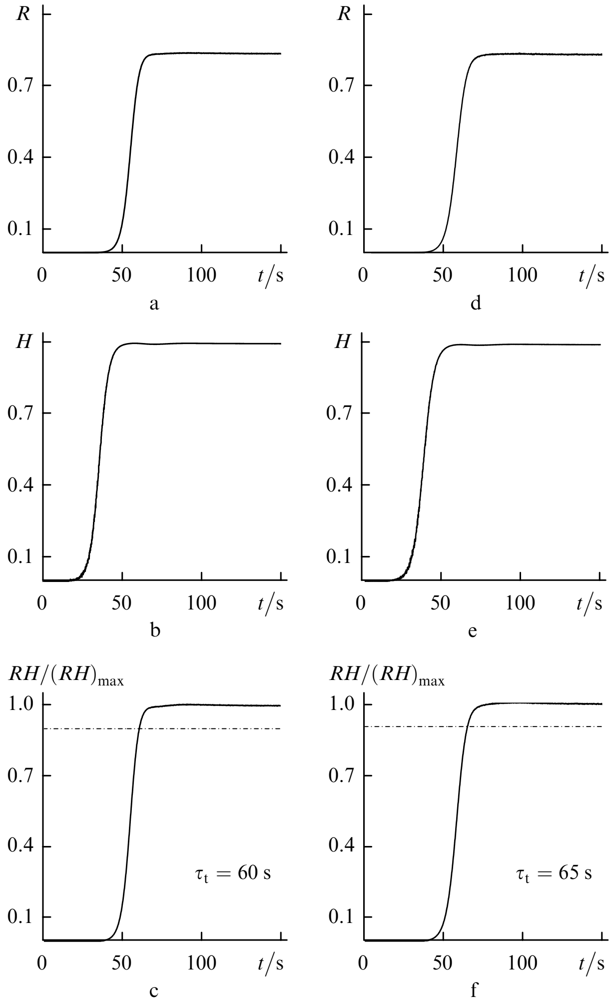


Figure 2. Transient processes for R (a, d), H (b, e), and RH (c, f) upon conjugation of a Gaussian beam without the spatial harmonic modulation $M(x)$ of the field $A_{\mathrm{f}}(x, z=0)$ ($I_{\max} = 35$ mW cm^{-2}, a–c) and upon modulation with the period $\Lambda_{\mathrm{m}} = 100$ μm ($I_{\max} = 55$ mW cm^{-2}, d–f).

structure of the training field requires the consideration of a number of properties of the information signal whose wave front will be conjugated later. Therefore, we assume below that almost all characteristics of the signal are known. We assume that this signal after the start is the above-described Gaussian beam with the harmonically modulated amplitude. However, the initial phase φ and spatial period $\Lambda_{\rm m} = \kappa_{\rm m}^{-1}$ of the function $M(x, t \geqslant 0) = \sin(2\pi\kappa_{\rm m}x + \varphi)$ (where $t = 0$ is the start instant) in (1) are unknown. Note here that all the results presented below will be related to the worst case of the choice of φ from the point of view of the realised value of τ and to the three possible values of $\Lambda_{\rm m} = 100$, 90, and 80 μm after the start.

From this point of view, the final stationary state of the PRC, i.e., the distribution $\delta\eta(x, z)$ for $\Lambda_{\rm m} = 100$ μm at the instant $t = 150$ s, shown in Fig. 1f, can be treated itself as a new initial state (with respect to the start instant $t = 0$ of phase conjugation) of the SPPC mirror obtained due to mirror training by a Gaussian beam with the harmonically modulated amplitude. Because such training neglects the possibility of existence of different phases φ of the information signal, it was unlikely that these procedure would be efficient. This was confirmed by the results of our simulations. Transient processes for R, H, and RH in the case of a beam for which the phase φ of the information modulation $M(x)$ shifts jumpwise by $\pi/2$ at the instant $t = 0$ and $\Lambda_{\rm m}$ simultaneously changes [$\Lambda_{\rm m} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*)] are illustrated in Figs 3a–c. It is easy to verify that, although the time required for the parameter RH to achieve 90 % of $(RH)_{\max}$ is somewhat shorter than before ($\tau_{1,2,3} = 39$, 27, and 20 s for $\Lambda_{\rm m} = 100$, 90, and 80 μm), this advantage in τ achieved due to preliminary SPPC mirror training is insignificant. However, it is much more important that this advantage noticeably decreases when the values of $\Lambda_{\rm m}$ for the training and conjugated signals coincide. This clearly indicates that there is no sense to use this procedure from the point of view of preliminary writing the initial information on the expected value $\Lambda_{\rm m}$ of the information signal in the SPPC mirror.

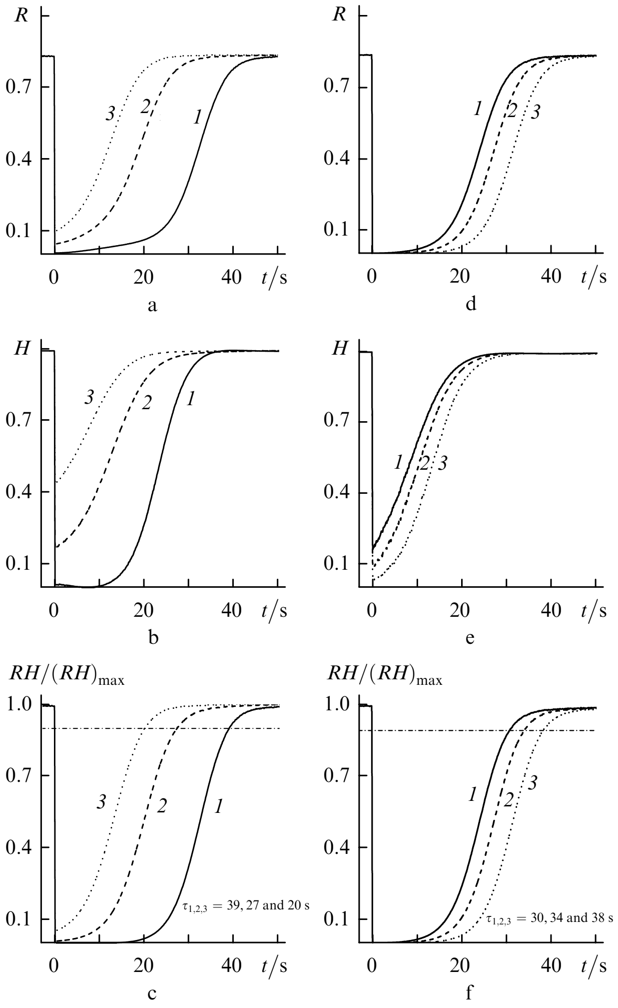


Figure 3. SPPC mirror training by a spatially modulated beam ($\Lambda_{\rm m} =$ 100 μm, a–c) and a beam without modulation (d–f). Transient processes for R (a, d), H (b, e), and RH (c, f) after training. During the start at the instant $t = 0$, the phase of $M(x)$ shifts by $\pi/2$ and $\Lambda_{\rm m}$ changes [$\Lambda_{\rm m} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*), respectively].

Because preliminary information on the expected value of $\Lambda_{\rm m}$ in the above-described SPPC mirror training procedure proved to be unfavourable, it would be expected that, all other conditions being equal, its elimination during the training stage should play a positive role. This was confirmed by our calculations. Figures 3d–f [curves (*1*), (*2*), and (*3*)] illustrate transient processes for R, H, and RH in the case of a beam with a smooth Gaussian envelope for which the value of $I_{\max}$ changes jumpwise from 35 to 55 mW cm^{-2} at the instant $t = 0$ (the total beam power being preserved) and the harmonic modulation $M(x)$ of the light field amplitude with the period $\Lambda_{\rm m} = 100$, 90, and 80 μm simultaneously appears. One can see that the general character of the dependence $\tau(\Lambda_{\rm m})$ becomes opposite in this case ($\tau_{1,2,3} = 30$, 34, and 38 s for $\Lambda_{\rm m} = 100$, 90, and 80 μm). However, the advantage obtained due to SPPC mirror training proves to be very small in this case as well, which suggests that it is necessary to use some other procedures to write information in the PRC about the expected value of $\Lambda_{\rm m}$ (but not φ) upon its training.

4. Dynamic SPPC mirror training

The general scheme of dynamic training can be readily constructed taking into account the following simple considerations. Because upon SPPC mirror training we should write in a PRC the information on the expected value of $\Lambda_{\rm m}$, the PRC should be irradiated at this stage with the spatial modulation frequency corresponding to $\Lambda_{\rm m}$. At the same time, because information on the phase φ of the spatial modulation of the signal that should be later conjugated is absent, it is impossible to memorise the value of the phase φ of the function $M(x)$ during training. Taking into account that the PRC is inertial, these two requirements can be satisfied in the simplest way by rapidly changing the phase of the function $M(x)$ in time, so that the phase φ takes many times all the possible values during training.

It is this situation that will be realised, for example, if the input signal amplitude is modulated by the function $M(x, t) = \sin[2\pi(\kappa_{\rm m}x + f_{\rm m}t)]$, where the modulation frequency $f_{\rm m}$ characterising the rate of a linear increase in the phase φ in time is chosen so high that the refractive-index gratings $\delta\eta(x, z)$ written in the PRC have no time to follow its variations. Figure 4 illustrates the results of this procedure. As follows from Figs 4a–c, the SPPC mirror training stage ($\Lambda_{\rm m} = 100$ μm, the phase φ changes by 2π for 2 s) proves to be rather long in this case. The transient

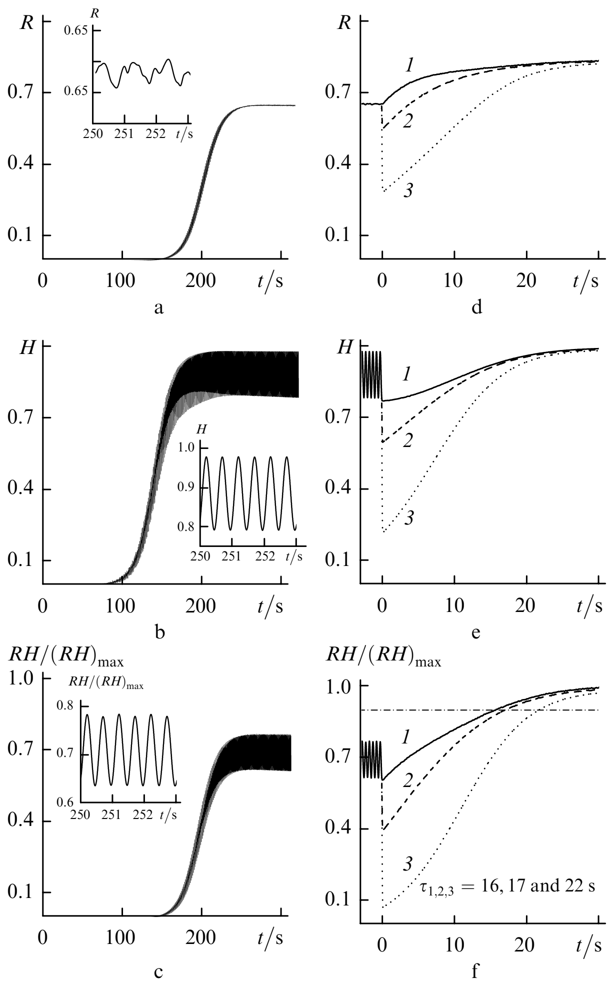


Figure 4. SPPC mirror training by a beam with the phase of $M(x)$ linearly increasing in time ($\Lambda_{\mathrm{m}} = 100$ μm, the phase changes by 2π for 2 s). Transient processes for R (a, d), H (b, e), and RH (c, f) at the training stage (a, b, c) and after the start (d–f). The insets in Figs a, b, c show the established forced oscillations of R, H, and RH. During the start, the phase of $M(x)$ shifts by $\pi/2$ and Λ_{m} changes [$\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*) in Figs d–f, respectively].

process for R (Fig. 4a), H (Fig. 4b), and RH (Fig. 4c) lasts now almost for 200 s, and, which is most important, even after its end the values of R, H, and RH continue to oscillate rapidly in time (see insets in Figs 4a–c). In fact, forced oscillations of the values of all the above-mentioned parameters are observed at the frequency f_{m} of variations of the external 'force' $M(x,t)$. Note here that the amplitude R of forced oscillations is very small in this case.

Transient processes for R, H, and RH after such a training in the case of a Gaussian beam, for which the phase of the information modulation $M(x,t)$ at the start instant ($t = 0$) shifts sharply by $\pi/2$ and Λ_{m} simultaneously changes [$\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*)], are illustrated in Figs 4d–f. One can see that the parameter RH now achieves 90 % of $(RH)_{\max}$ for a considerably shorter time ($\tau_{1,2,3} = 16$, 17, and 22 s for $\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm), and the advantage in τ achieved due to a preliminary SPPC mirror training is much greater.

It is clear that the dynamic training procedure described above is not the only one at least because the phase φ of information modulation of the signal can be varied in time in different ways. As another variant of such training, we considered the situation with rapid oscillations of φ. The input signal amplitude was modulated at this SPPC mirror training stage by the function $M(x,t) = \sin\{2\pi[\kappa_{\mathrm{m}}x + \sin(f_{\mathrm{m}}t)/2]\}$ with the same values $f_{\mathrm{m}} = 0.5$ Hz and $\Lambda_{\mathrm{m}} = 100$ μm. The results of simulations are presented in Fig. 5. One can see (Figs 5a–c) that in this case the SPPC training stage is much shorter. The transient process for R (Fig. 5a), H (Fig. 5b), and RH (Fig. 5c) lasts now less than 120 s. However, the values of R, H, and RH continue to oscillate after the end of the transient process in this case as well. The amplitude of these forced oscillations proves to be even larger, and the oscillations are no longer harmonic (see insets in Figs 5a–c). We will discuss this question below.

Transient processes for R, H, and RH observed after such training for a beam in which the phase φ of modulation $M(x,t)$ shifts by $\pi/4$ at the start instant ($t = 0$) and Λ_{m} changes simultaneously [$\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*)] are shown in Figs 5d–f. One can see that now the parameter RH achieves 90 % of $(RH)_{\max}$ for even a

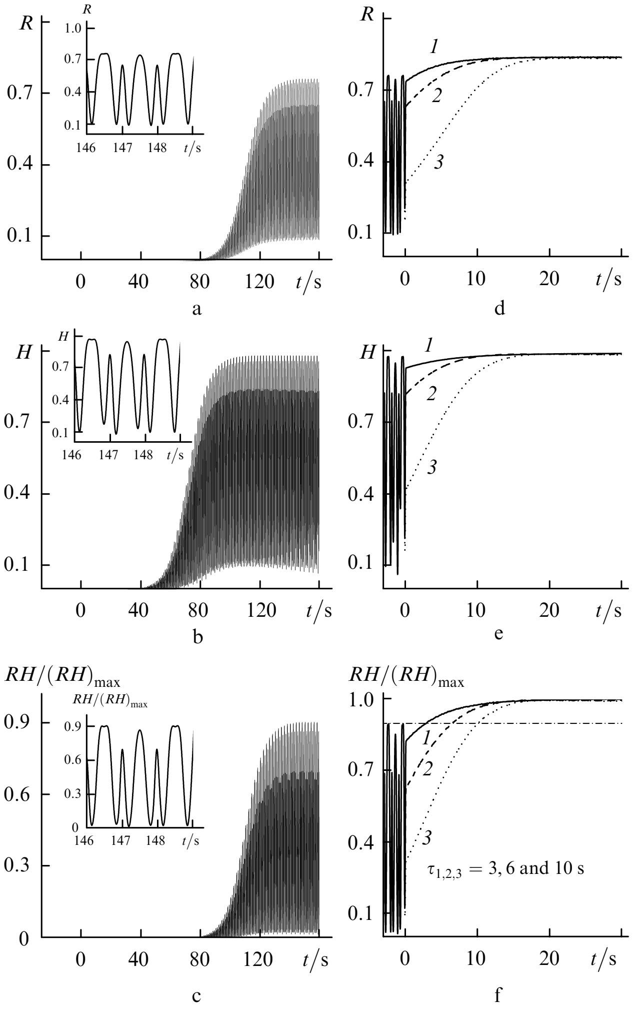


Figure 5. SPPC mirror training by a beam with the phase of $M(x)$ oscillating in time ($\Lambda_{\mathrm{m}} = 100$ μm, the amplitude and period of phase oscillations are 2π and 2 s, respectively). Transient processes for R (a, d), H (b, e), and RH (c, f) at the training stage (a, b, c) and after the start at the instant $t = 0$ (Figs d–f). The insets in Figs a, b, c show the established forced oscillations of R, H, and RH. During the start, the phase of $M(x)$ shifts by $\pi/4$ and Λ_{m} changes [$\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm for curves (*1*), (*2*), and (*3*) in Figs d–f, respectively].

shorter time τ ($\tau_{1,2,3} = 3$, 6, and 10 s for $\Lambda_{\mathrm{m}} = 100$, 90, and 80 μm), and the advantage in τ obtained due to SPPC training is still larger.

To understand why the two procedures described above give substantially different results, we discuss in more detail what occurs with the output field of the SPPC mirror during the training. Figure 6 shows periodic variations in the spatial distributions of the amplitudes $A_{\mathrm{f,b}}(x, z = 0)$ (Figs 6a, c, e) and phases $\varphi_{\mathrm{f,b}}(x, z = 0)$ (Figs 6b, d, f) for the input and output SPPC mirror fields (solid and dashed curves) related to the forced oscillations of R, H, and RH for three successive instants of time corresponding to the minimum (Figs 6a, b), maximum (Figs 6c, d), and the next minimum (Figs 6e, f) of the dependence $H(t)$ in the case of the linearly increasing phase $\varphi(t)$. One can see that the spatially modulated part of the field $A_{\mathrm{b}}(x, z = 0)$ follows the shifts of the maxima of the distribution $A_{\mathrm{f}}(x, z = 0)$. At the instants when $H(t)$ has minimal values, the wave front of the field $A_{\mathrm{b}}(x, z = 0)$ is tilted and the modulation depth of the distribution of its amplitude decreases. Taking into account that the amplitude of forced oscillations of R is small, this means that two components are present constantly in the field reflected from the dynamic hologram. One of these components – a Gaussian beam with a smooth envelope and a plane wave front, 'oscillates' in the propagation direction, while the second one, which is spatially modulated, exactly follows the distribution of amplitudes $A_{\mathrm{f}}(x, z = 0)$. The ratio of the contributions of these two components also oscillates in time. All this demonstrates a very small angular selectivity of the refractive-index grating $\delta\eta(x, z)$ written in the PRC during training.

The case of rapid harmonic oscillations of $\varphi(t)$ (Fig. 5) proves to be much simpler from his point of view. The character of the time evolution of the spatial distributions of $A_{\mathrm{f,b}}(x, z = 0)$ (Figs 7a, c, e) and $\varphi_{\mathrm{f,b}}(x, z = 0)$ (Figs 7b, d, f) for the input and output fields (solid and dashed curves, respectively) for the three successive instants of time [at the maximum (Figs 7a, b), minimum (Figs 7c, d), and the next maximum (Figs 7e, f) of the dependence $H(t)$], taking into account the large amplitude of the forced oscillations of R,

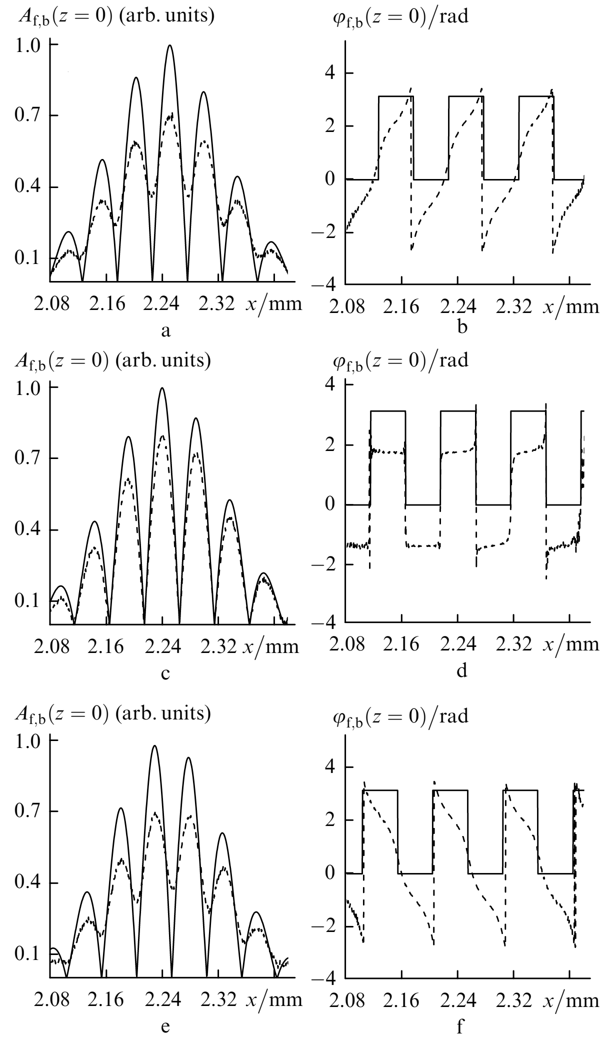


Figure 6. SPPC mirror training by a beam with the phase of $M(x)$ linearly increasing in time ($\Lambda_{\mathrm{m}} = 100$ μm, the phase changes by 2π for 2 s). The distributions $A_{\mathrm{f,b}}(x, z = 0)$ (a, c, e) and $\varphi_{\mathrm{f,b}}(x, z = 0)$ (b, d, f) (solid and dashed curves) for three successive instants of time corresponding to the minimum (a, b), maximum (c, d), and the next minimum (e, f) of the dependence $H(t)$.

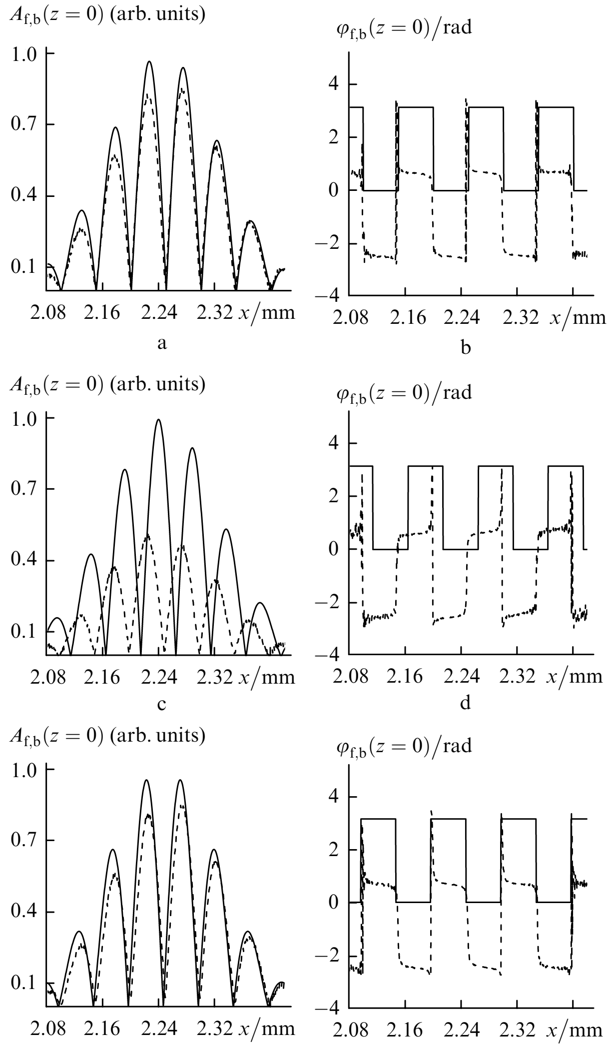
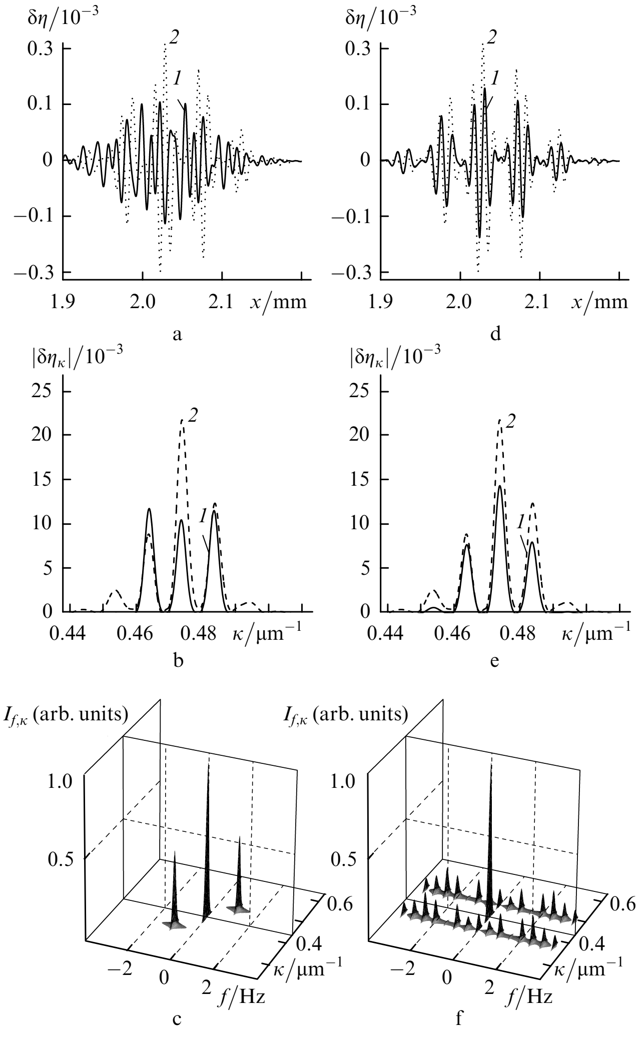


Figure 7. SPPC mirror training by a beam with the phase of $M(x)$ oscillating in time ($\Lambda_{\mathrm{m}} = 100$ μm, the amplitude and period of the phase oscillations are 2π and 2 s, respectively). The distributions $A_{\mathrm{f,b}}(x, z = 0)$ (a, c, e) and $\varphi_{\mathrm{f,b}}(x, z = 0)$ (b, d, f) (solid and dashed curves) for three successive instants of time corresponding the minimum (a, b), maximum (c, d), and the next maximum (e, f) of the dependence $H(t)$.

on the contrary, demonstrates an extremely high angular selectivity of the refractive-index grating $\delta\eta(x,z)$ written in the PRC upon SPPC training.

A substantial difference between the two dynamic training procedures considered above is illustrated in Fig. 8. The distributions of the refractive index $\delta\eta(x,z)$ (Figs 8a, d) in the central plane $z = L/2$ of dynamic holograms written in the PRC and their spatial spectra $\delta\eta_\kappa(\kappa)$ (Figs 8b, e) after the end of the transient process of SPPC mirror training by a beam with a linearly increasing (changing by 2π for 2 s, Figs 8a, b, c) and oscillating (the oscillation amplitude and period are 2π and 2 s, respectively, Figs 8d, e, f) phase $\varphi(t)$ of the spatial modulation $M(x)$ of the input signal are shown here by solid curves (*1*). Dashed curves (*2*) correspond to the same dependences after the end of the transient process upon conjugation of the signal radiation with $\Lambda_\mathrm{m} = 100$ μm. It is easy to verify that these two procedures do give quite different results (Figs 8a, b, d, e). Upon SPPC mirror training by a Gaussian beam with the oscillating phase of the spatial modulation $M(x)$, the dynamic holograms written in the PRC correspond almost perfectly to those that should be produced in it at the phase conjugation stage [see dependences (*1*) and (*2*) in Figs 8d, e].



a d

b e

c f

Figure 8. SPPC mirror training by a beam with the linearly increasing phase (changing by 2π for 2 s; a, b, c) and oscillating phase of $M(x)$ (the amplitude and period of phase oscillations are 2π and 2 s, respectively; d, e, f) for $\Lambda_\mathrm{m} = 100$ μm. The distributions $\delta\eta(x, z = L/2)$ (a, d) and spectra $\delta\eta_\kappa(\kappa)$ (b, e) established after the end of transient processes at the training stage (*1*) and after the start (*2*) (a, b, d, e). The spatiotemporal intensity spectra of the training field $I_{f,\kappa}(f,\kappa)$ (c, f); the position of the point $\kappa = 0$ is shifted by the frequency corresponding to the convergence angle of the waves $A_\mathrm{f,b}$.

In our opinion, this is explained by the fact that the spatial and temporal harmonics of the input signal in a beam with the oscillating phase are well 'mixed', allowing a considerably greater amount of useful information to be written in the PRC. This is demonstrated by the spatiotemporal spectra of the training field $I_{f,\kappa}(f,\kappa)$ presented for the two considered cases in Figs 8c, f. The position of the point $\kappa = 0$ is shifted by the frequency corresponding to the convergence angle of the waves $A_\mathrm{f,b}$. One can see that for a Gaussian beam with the oscillating phase, the dependence $I_{f,\kappa}(f,\kappa)$ contains much more spatial and temporal harmonics, which probably explains a considerably higher angular selectivity of the stationary refractive-index grating $\delta\eta(x,z)$ written in the PRC during training. However, more important is the fact that due to the efficient 'mixing' of the spatial and temporal harmonics in the training input field, the PRC inertia (the selection of harmonics for which $f = 0$) does not prevent in this case the writing of information on the expected value of Λ_m (see Fig. 8f).

5. Conclusions

The simulation of a loop self-pumped PC mirror based on a $\mathrm{BaTiO_3}$ photorefractive crystal performed in the paper has shown that the time required for the formation of a conjugated wave in the SPPC mirror can be considerably reduced by using the preliminary training. This is achieved by irradiating preliminary the SPPC mirror by an auxiliary (training) spatially modulated optical field, which writes static dynamic holograms in the PRC before the arrival of the signal radiation. These holograms provide a rapid subsequent formation (6 – 20 times faster) of the volume refractive-index gratings required for efficient phase conjugation. Of course, to realise such procedures (to choose the optimal spatial structure of the training field), it is necessary to know beforehand some properties of the signal radiation whose wave front is to be conjugated.

Our study of several variants of static and dynamic procedures of SPPC mirror preliminary training has show that, other conditions being equal, the efficiency of dynamic procedures base on the temporal averaging (i.e., the use of the inertia of the nonlinear response of the PRC) should be considerably higher than that for static procedures. However, it should be taken into account in these procedures that the temporal averaging should efficiently 'mix' the spatial and temporal harmonics of the training field to transfer information on the expected spatial period of the signal radiation to the zero frequency.

In our opinion, the approach used in the paper can be also applied for solving pattern recognition problems [25, 26], in the development of optical correlators [27] and elements of the associative memory [28 – 30] based on SPPC mirrors.

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