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On stimulated VUV emission of atomic helium in a Bose – Einstein condensate: II

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***Abstract*. The scenario of the experiment on observation of stimulated VUV emission at a wavelength of 62 nm from metastable states of helium atoms in a Bose – Einstein condensate moving along an extended quantum-well trap is further considered taking into account the asymptotic behaviour of the current value of the stimulated emission cross section. Quantitative estimates are presented.**

Keywords: *Bose – Einstein condensate, stimulated emission, metastable states.*

1. Introduction

In section 10 of the first part of this paper [1], where the scenario and configuration of a possible experiment on the observation of stimulated VUV emission from metastable states of atomic helium in a Bose – Einstein condensate (BEC) were considered, the important features of the start transient process were pointed out. These are, first of all, a comparatively low level of the spontaneous photon background used in the resonator scheme for seeding self-excitation and the asymptotic behaviour of the current value of the stimulated emission cross section consisting in its gradual increase from zero at the initial moment to a stationary value for the time of the order of the natural radiative lifetime of the metastable state. Below, the role of these features is briefly estimated by using the scenario, notation, and numerical data from [1].

2. Spontaneous photon background level

At the initial stage of the self-excitation process, when stimulated emission can be neglected due to the asymptotic behaviour of its cross section, the spontaneous photon flux density $\Phi_{\rm sp}$ is determined by the competition between the influx of photons to the modes of the amplifying medium and losses in the medium (caused mainly by the photoelectric effect on metastable atoms).

The isotropic spontaneous decay of metastable states gives rise to the elementary spontaneous photon flux of density

$$\frac{\mathrm{d}\Phi_{\rm sp}(z)}{\mathrm{d}z} = \frac{n^*_{\rm BEC}(z_{\rm BEC})}{\tau}\frac{\Delta\Omega}{4\pi}\exp\left[-\frac{z - z_{\rm BEC}}{V(z_{\rm BEC})\tau}\right] \quad (1)$$

in the volume $z + \mathrm{d}z$ of the medium with the unit cross section, where τ is the radiative lifetime of the metastable state; $\Delta\Omega$ is the solid angle containing the modes of the medium; $n^*_{\rm BEC}(z_{\rm BEC})$ is the concentration of metastable atoms in a BEC at the input to the amplification region (region IV in [1]) with the coordinate $z_{\rm BEC}$; $V(z_{\rm BEC})$ is the transport velocity of the flux of helium atoms. The exponential takes into account a decrease in the concentration of metastable atoms due to spontaneous decay during their propagation in the positive direction of the z axis (see the scenario scheme in [1]). The density of the elementary photon flux propagating in the same direction up to the coordinate $z_x = z + x$ at the distance x from its appearance decays exponentially down to

$$\frac{\mathrm{d}\Phi_{\rm sp}(z_x)}{\mathrm{d}z} = \frac{n^*_{\rm BEC}(z_{\rm BEC})}{\tau}\frac{\Delta\Omega}{4\pi}$$
$$\times\exp\left[-\frac{z - z_{\rm BEC}}{V(z_{\rm BEC})\tau}\right]\exp[-\chi n(z_x - z)], \quad (2)$$

where n is the total concentration of atoms and χ is the total cross section for photon losses. In this case, $z_x - z_{\rm BEC} \leqslant L$ (L is the length of the region of interaction of atoms with the photon field of region IV).

The total density of the spontaneous photon flux in the modes of the amplifying medium at the point z_x can be obtained by integrating all the elementary fluxes from $z - z_{\rm BEC} = 0$ to $z - z_{\rm BEC} = z_x$:

$$\Phi_{\rm sp}(z_x) = \frac{n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})}{1 - V(z_{\rm BEC})\tau\chi n}\frac{\Delta\Omega}{4\pi}\{\exp[-\chi n(z_x - z_{\rm BEC})]$$
$$-\exp[-(V(z_{\rm BEC})\tau)^{-1}(z_x - z_{\rm BEC})]\}. \quad (3)$$

For $z_x - z_{\rm BEC} = L$, this expression gives the output spontaneous photon flux

$$\Phi_{\rm sp}(L) = \frac{n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})}{1 - V(z_{\rm BEC})\tau\chi n}\frac{\Delta\Omega}{4\pi}$$
$$\times\{\exp(-\chi nL) - \exp[-(V(z_{\rm BEC})\tau)^{-1}L]\}. \quad (4)$$

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Because usually $V(z_{\rm BEC})\tau\chi n \ll 1$, expressions (3) and (4) can be replaced by the approximate relations

$$\Phi_{\rm sp}(z_x) \approx n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\frac{\Delta\Omega}{4\pi}\exp[-\chi n(z_x - z_{\rm BEC})], \quad (5)$$

$$\Phi_{\rm sp}(L) \approx n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\frac{\Delta\Omega}{4\pi}\exp(-\chi nL). \quad (6)$$

Note that relations (5) and (6) are independent of the excited-state lifetime τ in the adopted transport scheme of metastable atoms and for numerical parameters used here. In addition, because photon losses are mainly determined by the photoelectric effect on metastable atoms with the cross section $\sigma_{\rm ph}$, we have $\chi n \approx \sigma_{\rm ph}n^* \ll 1$ (where $\sigma_{\rm ph} < 10^{-17}{\rm cm}^2$, n^* is the total concentration of metastable atoms), and the exponential factors in (5) and (6) are close to unity. Therefore, the spontaneous background level in the modes of the medium can be estimated from a simple expression

$$\Phi_{\rm sp} \sim n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\frac{\Delta\Omega}{4\pi}. \quad (7)$$

For example, for $n^*_{\rm BEC}(z_{\rm BEC}) = 10^{11}$ cm^{-3}, $V(z_{\rm BEC}) = 145$ cm s^{-1} and $\Delta\Omega/4\pi = 10^{-5}$ (these values are taken from the example in section 9 in [1]), the estimate from (7) gives $\Phi_{\rm sp} \sim 1.5\times10^8$ cm^{-2} s^{-1}, which is many orders of magnitude lower than the initial spontaneous background level in typical optical lasers which is sufficient for their stable self-excitation. A similar estimate is obtained for a spontaneous photon flux propagating in the negative direction of the z axis with the frequency different by the doubled Doppler shift from the frequency of photons propagating in the positive direction of the z axis [1].

Therefore, because of a low spontaneous background level, which cannot provide efficient self-excitation, the initial photon seed in the resonator should be apparently supplemented by the external injection of resonance photons to the modes of the medium.

3. Temporal evolution of the stimulated emission cross section

A resonance photon field begins to act on an atom instantly at the moment of its penetration into region IV occupied by the field, and then, in the case of the conditionally uniform distribution of the field along the trajectory of atoms [1], the evolution of the current value of the stimulated emission cross section $\sigma(t)$ determining the so-called laser lethargy effect [2–5] can be written in the form

$$\sigma(t) \approx \sigma\left[1 - \exp\left(-\alpha\frac{t}{\tau}\right)\right], \quad (8)$$

where $\sigma = \lambda^2/2\pi$. This process proceeds at the initial stage approximately linearly:

$$\sigma(t) \approx \sigma\alpha\frac{t}{\tau}. \quad (9)$$

Here, $\alpha = {\rm const}$ and it is assumed that any excess broadening of the radiative transition line of its natural width is absent.

4. Interaction of metastable atoms with a photon field

Taking into account the evolution of the current value of $\sigma(t)$ and a decrease in the concentration of metastable states due to spontaneous decay, the increase in the photon flux density Φ in region IV is described by the equation

$$\pm\frac{1}{\Phi}\frac{{\rm d}\Phi}{{\rm d}z} = \sigma n^*_{\rm BEC}(z_{\rm BEC})\left\{1 - \exp\left[-\alpha\frac{z - z_{\rm BEC}}{V(z_{\rm BEC})\tau}\right]\right.$$
$$\left.-\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}\right\}\exp\left[-\frac{z - z_{\rm BEC}}{V(z_{\rm BEC})\tau}\right] - \chi n(z_{\rm BEC}), \quad (10)$$

where the signs at the derivative correspond to the photon-flux propagation along the positive and negative directions of the z axis, i.e., parallel or anti-parallel to the atomic beam velocity.

The quantity $\Phi^{-1}|{\rm d}\Phi/{\rm d}z|$ in (10) achieves the maximum

$$\frac{1}{\Phi}\left|\frac{{\rm d}\Phi}{{\rm d}z}\right|_{\max} = \sigma n^*_{\rm BEC}(z_{\rm BEC})$$
$$\times\alpha\left\{\frac{1 - \sigma_{\rm ph}n^*(z_{\rm BEC})/[\sigma n^*_{\rm BEC}(z_{\rm BEC})]}{1+\alpha}\right\}^{1/\alpha+1} - \chi n(z_{\rm BEC}) \quad (11)$$

for

$$z_0 = z_{\rm BEC} + \frac{V(z_{\rm BEC})\tau}{\alpha}\ln\frac{1+\alpha}{1 - \sigma_{\rm ph}n^*(z_{\rm BEC})/[\sigma n^*_{\rm BEC}(z_{\rm BEC})]}. \quad (12)$$

Amplification appears when the inequality

$$\frac{1}{\Phi}\left|\frac{{\rm d}\Phi}{{\rm d}z}\right|_{\max} \geqslant 0 \quad (13)$$

is fulfilled, which specifies the critical concentration of metastable states

$$\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm cr} = \frac{\chi}{\sigma\alpha}$$
$$\times\left\{\frac{1+\alpha}{1 - \sigma_{\rm ph}n^*(z_{\rm BEC})/[\alpha n^*_{\rm BEC}(z_{\rm BEC})]}\right\}^{1/\alpha+1}, \quad (14)$$

below which even local amplification is impossible. In addition, Eqn (10) gives two limitations on the length L of the interaction site in region IV. Because at the initial instant of the arrival of atoms into region IV for $z = z_{\rm BEC}$, the current value of the stimulated emission cross section $\sigma(t)$ (8) is zero, even for

$$\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n} > \left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n}\right]_{\rm cr}$$

the photon flux at this stage does not increase and the sign of the derivative changes only at the coordinate $z_1 = z_{\rm BEC} + V(z_{\rm BEC})t_1$, where the equality between the influx of stimulated photons and their losses is achieved, t_1 and $z_1 < z_0$ being the amplification delay time and threshold coordinate, respectively.

The coordinate z_1 and the second characteristic coordinate $z_2 > z_0 > z_1$, which is related to the depletion of metastable states due to spontaneous decay, are determined as two roots of the right-hand side of (10) under the condition

$$\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})} > \left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm cr}$$

from the equation

$$\frac{1}{\alpha}\exp\left[-\frac{z_{1,2}-z_0}{V(z_{\rm BEC})\tau}\right]\left\{\alpha+1-\exp\left[-\alpha\frac{z_{1,2}-z_0}{V(z_{\rm BEC})\tau}\right]\right\}$$

$$=\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]^{-1}\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm cr}, \qquad (15)$$

which in the case of small moduli of the exponents gives the approximate values of the roots

$$z_{1,2}-z_0 \approx \mp V(z_{\rm BEC})\tau\left\{1-\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]^{-1}\right.$$

$$\left.\times\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm cr}\right\}^{1/2}. \qquad (16)$$

This means that it is reasonable to restrict the length L of the site of interaction of atoms with the field in region IV by the inequalities

$$z_1 \leqslant L + z_{\rm BEC} \leqslant z_2. \qquad (17)$$

The left inequality in (17) shows that the length L should exceed at least the threshold coordinate $z_1 - z_{\rm BEC}$ measured from the beginning of region IV, while the right inequality shows that the interaction site should not include the interval where the concentration of metastable states is already exhausted.

By integrating (10), we obtain the expression for the total gain G of the photon flux over the entire length L of the interaction site in region IV

$$\ln G = \sigma n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\tau\left[1-\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}\right]$$

$$\times\left\{1-\exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\}-\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\tau}{1+\alpha}$$

$$\times\left\{1-\exp\left[-\frac{1+\alpha}{V(z_{\rm BEC})\tau}L\right]\right\}-\chi n(z_{\rm BEC})L. \qquad (18)$$

The condition for the threshold single-pass amplification with $G \geqslant 1$ is determined by the inequality

$$\left.\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right|_{\rm th} \geqslant \frac{\chi L}{\sigma V(z_{\rm BEC})\tau}\times$$

$$\times\left\{\left\{1-\exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]\right\}\left[1-\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}\right]\right.$$

$$\left.-\frac{1}{1+\alpha}\left\{1-\exp\left[-L\frac{1+\alpha}{V(z_{\rm BEC})\tau}\right]\right\}\right\}^{-1}. \qquad (19)$$

The maximum value of the total gain G is obtained from (18) for $L = L_{\rm max}$ determined as the root of the equation

$$\left\{1-\frac{\sigma_{\rm ph}n^*(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}-\exp\left[-\frac{\alpha L_{\rm max}}{V(z_{\rm BEC})\tau}\right]\right\}$$

$$\times\exp\left[-\frac{L_{\rm max}}{V(z_{\rm BEC})\tau}\right]=\frac{\chi n(z_{\rm BEC})}{\sigma n^*_{\rm BEC}(z_{\rm BEC})}. \qquad (20)$$

5. Quantitative estimates

Numerical estimates were performed for $\alpha = 1$ using the data from section 9 in [1]:

$\sigma = 6.1\times10^{-12}$ cm^2, $\chi/\sigma = 10^{-3}$, $\sigma_{\rm ph}/\sigma \approx 10^{-4}$,

$n(z_{\rm BEC}) = 10^{13}$ cm^{-3}, $n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC}) \approx 0.01$,

$V(z_{\rm BEC}) \approx 145$ cm s^{-1}, $V(z_{\rm BEC})\tau \approx 0.435$ cm.

Then, we obtain $[n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC})]_{\rm cr} \approx 4\times10^{-3}$ from (14), $[n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC})]_{\rm th} \approx 5.6\times10^{-3}$ from (19), $L_{\rm max} =$ 1 cm from (20), $(L_{\rm max}/V(z_{\rm BEC})\tau = 2.3 > 1$, which satisfies criterion (7) in [1], $z_0 - z_{\rm BEC} \approx 0.3$ cm from (12), $z_1 - z_{\rm BEC} \approx 0.05$ cm and $z_2 - z_{\rm BEC} \approx 1$ cm from (16), and $G_{\rm max} \approx 1.05$ from (18).

6. Conclusions

The analysis of phenomena appearing upon stimulated VUV emission of metastable states of helium atoms in a Bose–Einstein condensate has shown that an extremely low level of the spontaneous photon background in the modes of the medium can lead to the necessity of the external injection of resonance photons to increase the self-excitation efficiency of stimulated emission in the resonator.

A gradual asymptotic increase in the current value $\sigma(t)$ of the stimulated emission cross section from $\sigma(t=0)=0$ to the stationary value $\sigma(t\to\infty)=\lambda^2/2\pi$ causes the time delay of the amplification onset (laser lethargy) and the corresponding decrease in the amplifying medium length. However, this increase does not prevent the observation of the total amplification with $G > 1$, although noticeably reduces $G_{\rm max}$ compared to its value estimated for the case $\sigma =$ const for the same initial parameters of the medium. This reduction can be compensated by increasing amplification parameters, for example, the concentration of atoms.

The maximum output power density P upon amplification of a weak signal is determined by the relation between the arrival rate of metastable atoms into region IV and their decay rate:

$$P \leqslant \hbar\omega V(z_{\rm BEC})n^*_{\rm BEC}(z_{\rm BEC})\exp\left[-\frac{L}{V(z_{\rm BEC})\tau}\right]. \qquad (21)$$

For a strong signal in the strong saturation regime, when stimulated radiation dominates over spontaneous radiation, we obtain similarly

$$P \leqslant \hbar\omega V(z_{\mathrm{BEC}}) n^{*}_{\mathrm{BEC}}(z_{\mathrm{BEC}}). \quad (22)$$

The estimates for the numerical data from section 9 in [1] give $P \leqslant 5 \times 10^{-6}$ W cm^{-2} (21) and $P \leqslant 5 \times 10^{-5}$ W cm^{-2} (22).

Note that the numerical estimated were performed for $\alpha = 1$. Variations in the value of α, in particular, caused by different multipolarities of the transition can change quantitative estimates by preserving the qualitative result.

Finally, a successful experiment with the VUV emission of metastable helium atoms in accordance with the scenario considered above would be the basis for a similar approach for observing stimulated gamma radiation upon direct transitions from the metastable states of nuclei in an atomic BEC.

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