

# Applicability of the effective index method for simulating ridge optical waveguides

D.V. Batrak, S.A. Plisyuk

**Abstract.** The accuracy of the effective index method in calculations of ridge optical waveguides is estimated by comparing the results of calculations with the results obtained by the finite element method. A comparison of the values of the refractive index for the fundamental mode and the corresponding near and far-field intensity distributions for the waveguide structure typical of semiconductor ridge lasers emitting at 0.98  $\mu\text{m}$  demonstrates a rather high accuracy of the effective index method in calculations of ridge waveguide structures and radiative characteristics of semiconductor lasers based on them.

**Keywords:** ridge waveguide, effective index method.

## 1. Introduction

Planar layered waveguide structures are widely used in optoelectronics [1], for example, in heterostructure semiconductor lasers and various integro-optical devices such as laser modulators [2], laser amplifiers [3], multiplexers [4], etc.

The waveguide effect in the vertical direction (perpendicular to the layers of the structure) is produced due to the difference between the refractive indices of the layers. As for another transverse direction (along the layers of the structure), the optical confinement in this, horizontal, direction is often achieved by varying the thickness of one or several layers. Commonly, the upper cladding layers of the waveguide are used for this purpose, so that the waveguide represents a ridge on a planar multilayer structure which is called a ridge waveguide.

Although at present there exist many numerical methods for calculating waveguides, including those used in commercial program products, the problem of calculation of such waveguides is still not solved completely. This is caused by a number of reasons. First, the method of specifying the boundary conditions in a numerical solution is by no means obvious, as well as the influence of these conditions on the accuracy of the solution obtained. This becomes especially

substantial in a practically important case of a single-mode waveguide. Second, the results obtained by such numerical methods cannot be always clearly physically interpreted, while the methods themselves and the corresponding program products are often difficult to use as a component of more complex simulation systems.

In this connection of particular interest is the effective index method (EIM) (see, for example, [5] and references therein). The calculation of a two-dimensional waveguide in this approximate method is reduced to the successive calculation of two one-dimensional (planar) waveguides. The approximation is based on the assumption that the wave amplitude in one of the transverse directions changes considerably slower than in the other. Although this requirement restricts the scope of problems that can be studied, the simplicity, clearness, and convenience of the EIM provide its wide applications. This especially concerns the optics of propagation and amplification of radiation in waveguides formed by layers of semiconductor heterostructures. The exact solution of the problem for a planar (one-dimensional) waveguide with an arbitrary number of layers, each of them being characterised by the complex dielectric constant  $\epsilon$ , can be readily obtained by numerical methods by using the theory of functions of complex variable [6]. The solution of the two-dimensional problem by the EIM can be also easily obtained both for the cases when the waveguide is formed by the real part of  $\epsilon$  (refractive index) and when a wave is confined due to the transverse profile of the imaginary part of  $\epsilon$  (amplification or absorption) and also for possible intermediate cases.

However, the accuracy of the obtained solutions remains the subject of discussions. The estimates of the EIM accuracy available in the literature concern mainly the so-called rib waveguide [7, 8], in which a ridge is formed by varying the thickness of the waveguide rather than cladding layer, thereby being involved in the waveguide formation both in the horizontal and vertical directions. In the case of ridge waveguides, which are no less widespread and important, the question of the EIM accuracy is not so clear. In this connection it is interesting to estimate the accuracy of this method for a ridge waveguide by comparing the results obtained by the EIM and some other standard numerical method applicable for any waveguide geometry, for example, the finite element method (FEM). In this paper, we present the results of such a comparison for a ridge waveguide based on a semiconductor heterostructure used in lasers emitting at 0.98  $\mu\text{m}$ .

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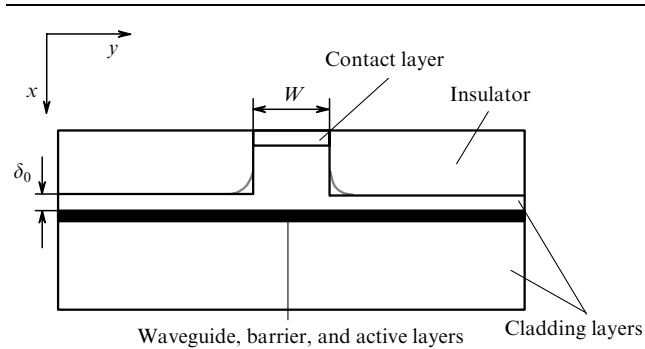
Received 1 February 2006

*Kvantovaya Elektronika* 36 (4) 349–352 (2006)

Translated by M.N. Sapozhnikov

## 2. Comparison method and results

Figure 1 shows one of the most typical waveguide geometries. The refractive indices and thickness of the heterostructure layers are presented in Table 1. Layers 3–9 are represented in Fig. 1 as a single dark layer. The refractive index of an insulator (ZnSe) is 2.4, which is considerably lower than the refractive indices of heterostructure layers. This results in a decrease in the effective refractive index outside the ridge and the appearance of the waveguiding effect in the ‘horizontal’ direction (along the  $y$  axis).



**Figure 1.** Geometry of the calculated structure. Grey lines show a variant with smoothed boundaries of the ridge.

**Table 1.** Thickness and refractive indices of the layers of the calculated structure (in the ridge region).

Layer number	Thickness/nm	Refractive index	Layer type
1	430	3.5235	contact
2	1660	3.3090	cladding
3	120	3.3327	waveguide
4	7	3.5235	barrier
5	5.3	3.63	active
6	10	3.5235	barrier
7	5.3	3.63	active
8	7	3.5235	barrier
9	120	3.3327	waveguide
10	3000	3.3090	cladding

We calculated the near- and far-field distributions of the zero guided mode and the mode refractive index  $n_m$  for the structures under study. The EIM calculations were performed as described in [5], while the FEM calculations were carried out by using the FlexPDE program [9]. In both cases, the scalar approximation for the field was used, i.e., the equation

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left( \frac{2\pi}{\lambda_0} \right)^2 [n^2(x, y) - n_m^2] \right\} E(x, y) = 0$$

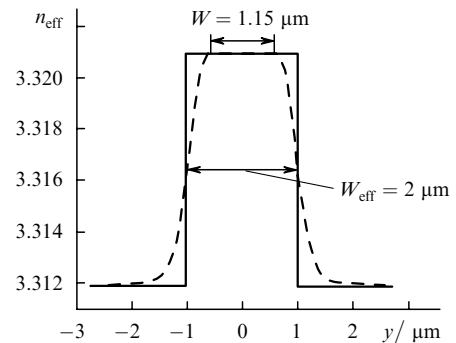
was solved.

The variable parameters in calculations were the ridge width  $W$  and the thickness  $\delta_0$  of the cladding layer outside the ridge. Calculations were performed for the following pairs of values: (1)  $W = 2 \mu\text{m}$ ,  $\delta_0 = 0.178 \mu\text{m}$ ; (2)  $W = 3 \mu\text{m}$ ,  $\delta_0 = 0.37 \mu\text{m}$ ; (3)  $W = 4 \mu\text{m}$ ,  $\delta_0 = 0.5 \mu\text{m}$ . All the three cases correspond to the cut off condition for the first mode of the waveguide calculated by the EIM, i.e., we can say that the ‘strength’ of the waveguide is the same in all these cases.

For the geometry described above, the effective refractive index experiences a jump at the ridge boundaries (for  $|y| = W/2$ ). To estimate the influence of this jump, we also performed calculations for the case of smoothed boundaries of the ridge (denoted by grey in Fig. 1). The residual thickness of the cladding layer was described by the expression

$$\delta(y) - \delta_0 = C \exp \left[ \left( \frac{W}{2} - |y| \right) \Delta^{-1} \right], \quad |y| > \frac{W}{2},$$

where  $C = 1.9 \mu\text{m}$  and  $\Delta = 0.173 \mu\text{m}$  (in this case, the ridge shape is close to that obtained in practice). The value of  $W$  was decreased in this case so that to preserve the effective width of the ridge, which was determined as the full width at half-maximum (FWHM) of the profile of the effective refractive index (Fig. 2).



**Figure 2.** Profile of the effective refractive index  $n_{\text{eff}}$  for sharp (solid curve) and smoothed (dashed curve) boundaries of the ridge. In both cases, the profile FWHM is set equal to  $2 \mu\text{m}$ .

Figure 3 presents the results of calculations for three combinations of parameters  $W$  and  $\delta_0$ . The widths of near ( $W_{\text{nf}}$ ) and far ( $W_{\text{ff}}$ ) fields were calculated as the FWHM. The horizontal intensity distribution was determined from the two-dimensional field distribution by the expression

$$I_{\text{nf}}(y) = \int E^2(x, y) dx = F^2(y),$$

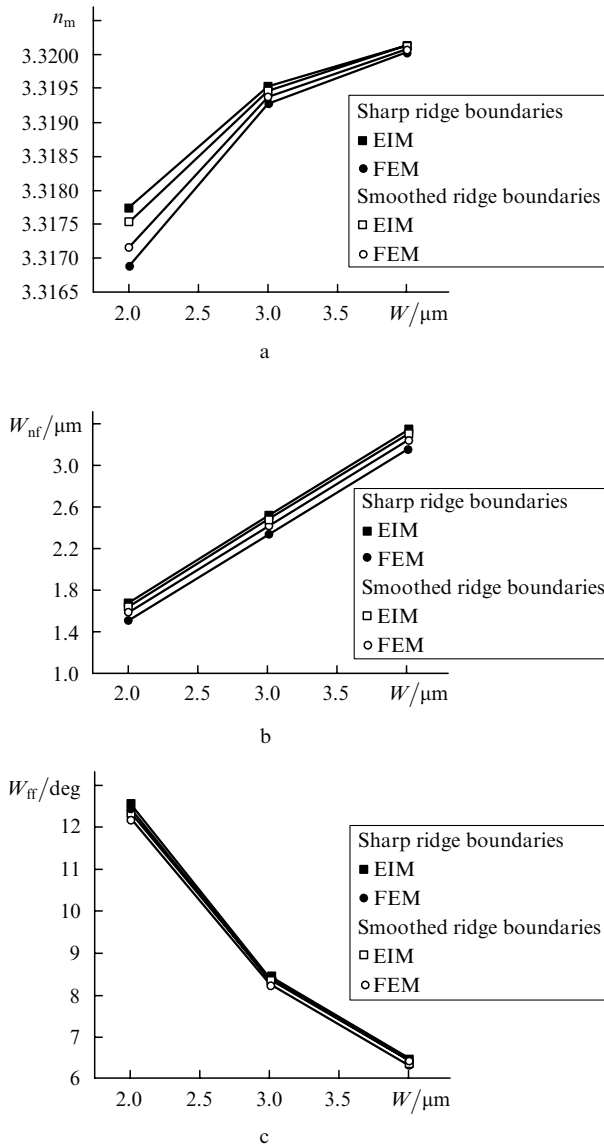
and the far-field intensity – from the expression

$$I_{\text{ff}}(\theta) = \left| \cos \theta \int F(y) \exp \left( i \frac{2\pi y}{\lambda_0} \sin \theta \right) dy \right|^2.$$

One can see from Fig. 3a that the EIM gives somewhat overestimated value of the mode refractive index. The error increases with decreasing the ridge width, and for the ridge with sharp boundaries the error is 2–2.5 times greater than that for the ridge with smoothed boundaries: in the former case, it changes from  $1 \times 10^{-4}$  to  $9 \times 10^{-4}$ , while in the latter case from  $0.5 \times 10^{-4}$  to  $3.5 \times 10^{-4}$ .

The values of  $W_{\text{nf}}$  determined by the EIM are also somewhat higher than the FEM values (Fig. 3b). In this case, the error weakly depends on the ridge width and is  $\sim 0.18$  and  $\sim 0.06 \mu\text{m}$  for the ridge with sharp or smoothed boundaries, respectively.

One can see from Fig. 3c that  $W_{\text{ff}}$  is calculated by the EIM with a high accuracy, and the relative and absolute errors for all configurations calculated in the paper do not exceed 1.5% and 0.15°, respectively.



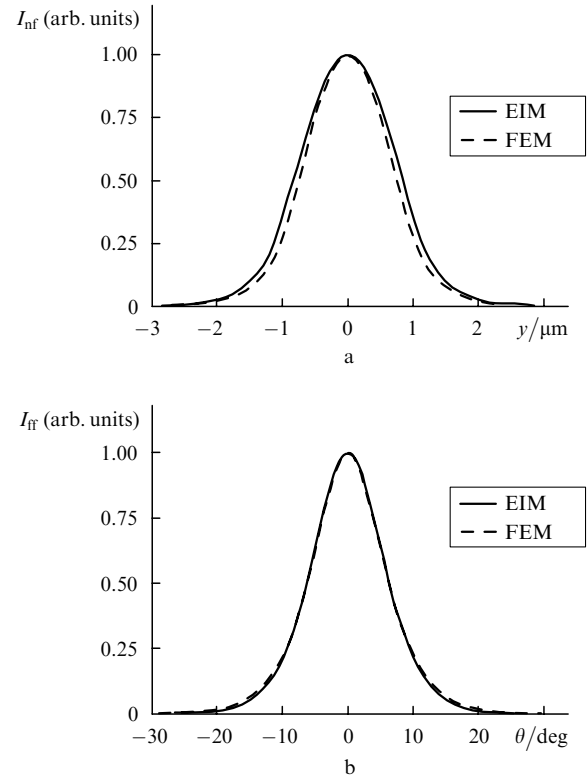
**Figure 3.** Calculated dependences of the mode refractive index  $n_m$  (a), the near-field width  $W_{\text{nf}}$  (b), and the far-field width  $W_{\text{ff}}$  (c) on the ridge width  $W$  for the constant ridge ‘strength’.

Figure 4 presents the dependences  $I_{\text{nf}}(y)$  and  $I_{\text{ff}}(y)$  calculated by the EIM and FEM for a ridge of width  $2\ \mu\text{m}$  with sharp boundaries (configuration for which the results differ the most). One can see that some difference between the near-field profiles leads to a noticeable change in the far-field profile only in the distribution wings and almost does not affect the FWHM.

Similar calculations performed for waveguides of other types, including waveguides with a more complicated horizontal structure (for example, of the W type) also demonstrate good agreement of the results obtained by these two methods.

### 3. Conclusions

The results obtained in our paper give the estimate of the EIM accuracy for a typical ridge waveguide. Even in the case of sharp boundaries of the ridge, when the EIM error is maximal, the difference in the mode refractive indices



**Figure 4.** Calculated near-field ( $I_{\text{nf}}$ ) (a) and far-field ( $I_{\text{ff}}$ ) (b) intensity profiles for a ridge of width  $2\ \mu\text{m}$  with sharp boundaries (configuration for which the EIM and FEM results differ the most).

does not exceed  $1 \times 10^{-3}$ , which is at the level of or below than the values characterising the scatter in this parameter for different devices manufactured by using the same technology. This means that such EIM accuracy is sufficient for adequate calculations of typical waveguides based on semiconductor heterostructures. The relative discrepancy between the EIM and FEM calculations for the smallest width of the ridge  $W = 2\ \mu\text{m}$  with smoothed boundaries considered in the paper did not exceed  $10^{-4}$  for the effective refractive index, 3% for the near-field width, and 1.5% for the far-field width. As the ridge width was increased, the discrepancy decreased and for  $W = 4\ \mu\text{m}$  it was  $1.5 \times 10^{-5}$  for the effective refractive index, 2% for the near-field width, and 0.5% for the far-field width.

Of course, one should bear in mind that good coincidence of the results obtained even in many cases does not mean that this will be always the case. It is possible that some ‘pathologic’ variants can be found when the EIM gives an inadequate result. Moreover, we can indicate the cases when this method cannot be used even in a simple variant. For example, this occurs when the waveguide problem in the vertical direction has no coupled solutions in some regions of the  $y$  axis.

Nevertheless, the positive result of our calculations is that for all the cases considered in the paper, which are of interest from the practical point of view (‘good’ variants), the EIM provides the required accuracy along with simplicity and convenience.

Thus, we conclude that the EIM accuracy is sufficiently high for determining the optical characteristics of ridge waveguides and for using this method to simulate the radiative characteristics of semiconductor lasers.

**Acknowledgements.** The authors thank A.P. Bogatov for useful discussions of the paper and also PDE Solutions Inc. for providing the test version of the FlexPDE program. This work was partially supported by the program ‘Coherent radiation of semiconductor compounds and structures’ of the Department of Physical Sciences of RAS and the program ‘Low-dimensional quantum structures’ of the Presidium of RAS.

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