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LIGHT BEAMS

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Peculiarities of propagation of quasi-diffraction-free light beams in strongly scattering absorbing media

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***Abstract*. Based on the relation between the theory of light field coherence and theory of radiation transfer in scattering media, a method is proposed for calculating the illumination distribution produced by coherent quasi-diffraction-free beams at different penetration depths of radiation into scattering media such as biological tissues. The method uses the optical transfer function or the point spread function (PSF) of the medium. A simple and convenient analytic PSF model is described. Examples of the illumination distribution produced by a Bessel light beam in a medium with optical parameters typical of real biological tissues are presented. It is shown that the half-width of the axial maximum of a Bessel light beam scattered due to scattering almost does not increase up to optical depths where the contribution of multiple scattering is already considerable.**

Keywords: *coherence, quasi-diffraction-free beam, Bessel light beam, scattering medium, biological tissues.*

1. Introduction

The use of quasi-diffraction-free beams, for example, Bessel light beams (BLBs) for studying biological tissues [1, 2], in particular, in optical coherence tomography attracts considerable recent interest. This is explained by the specific features of the propagation of BLBs, which are usually formed with the help of a circular aperture [3] or a conic lens [4, 5]. Recently, efficient methods were proposed for BLB formation in anisotropic crystal media [6, 7]. Unlike conventional optical systems (spherical lenses, mirrors, etc.), in which the increase in the spatial resolution in the focal region leads to the decrease in the length of this region and unlike Gaussian beams, a high transverse spatial resolution in BLBs is preserved at large distances. This gives promise that the probe depth in optical tomography can be considerably increased by preserving a high spatial resolution. The efficiency of such an approach concerning the propagation of light beams in a turbulent atmosphere was experimentally demonstrated in [8].

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However, it is known that biological tissues are strongly scattering media [9 – 12]. In this connection of special interest is the study of the propagation of coherent quasi-diffraction-free beams in scattering media because multiple scattering spreads the beam and destroys its spatial coherence. The complexity of this problem is that it is located at the border of the two fields: coherence optics dealing with concepts of wave optics and neglecting, as a rule, multiple scattering, and the photometric theory of radiation transfer dealing with multiple scattering by neglecting the wave nature of light.

In particular, the attempt to describe the multiply scattered light field by summing up the fields scattered from individual randomly arranged scatterers by taking phase relations into account shows little promise because of its awkwardness and complexity.

A similar problem concerning the description of the interference component of coherent inverse scattering from a multiply scattering medium was considered in papers [13 – 15], where the original approach was proposed based on the correspondence of the stochastic Monte Carlo method to the iteration procedure for solving the Bethe – Salpeter equation for the correlation function of the field.

One of the simplest and obvious methods for theoretical description of the propagation of quasi-diffraction-free beams in scattering media is the representation of the photometric illumination $E(\boldsymbol{r}, z)$ produced at the point of a medium with coordinates $\boldsymbol{r}, z$, by a sum of two components

$$E(\boldsymbol{r}, z) = E_0(\boldsymbol{r}, z) + E_s(\boldsymbol{r}, z), \quad (1)$$

where

$$E_0(\boldsymbol{r}, z) = E_0^{fr}(\boldsymbol{r}, z)\exp(-\tau) \quad (2)$$

and $E_s(\boldsymbol{r}, z)$ are the contributions of the unscattered and scattered radiation components, respectively; $E_0^{fr}(\boldsymbol{r}, z)$ is the illumination distribution produced by the beam in a free space; $\tau = \varepsilon z$ is the optical depth; z is the geometrical depth; and ε is the attenuation factor.

However, a number of questions appear in this approach, in particular, whether the above separation is justified, taking into account the coherence of scattered and unscattered fields? What can be considered a source of the illumination distribution $E_s(\boldsymbol{r}, z)$ and how can this distribution be calculated?

If we assume that a source of scattered light field is initial wave fields and scattered fields are added as photometric intensities, i.e., by neglecting their phases, this means that radiation coherence is neglected in the multiply scattered light field. As an example, consider the illumination of a medium by two plane waves incident at some angle to each other. It is known that in this case a classical interference pattern appears. At the same time, under the above assumption, the illumination distribution produced by multiply scattered radiation proves to be homogeneous and independent of $\boldsymbol{r}$.

As an alternative, we can assume that the source of the scattered light field is the illumination distribution $E_0(\boldsymbol{r}, z=0)$ at the input to a scattering medium, and the formation of the component $E_s(\boldsymbol{r}, z)$ can be described by the theory of radiation transfer. In this case, the radiation coherence is taken into account to some extent in the function $E_0(\boldsymbol{r}, z=0)$. However, this requires the knowledge of the angular structure of radiation forming the distribution $E_0(\boldsymbol{r}, z=0)$, and in addition, this approach cannot be considered justified enough, as the previous one.

In this paper, we propose a method describing the propagation of quasi-diffraction-free beams in a scattering medium taking into account multiple scattering and coherence of radiation and also present some results obtained within the framework of this approach.

2. Description of the method

Before answering the questions formulated above, note that from the point of view of optics of scattering media the soft biological tissues are strongly scattering media, as a rule, with small specific absorption and strong scattering anisotropy. In particular, the typical variations of optical parameters in the therapeutic transparency window 0.6 – 1.3 μm are: the scattering coefficient $\sigma \sim 10-100\ \mathrm{mm}^{-1}$ [11, 12, 16, 17], the absorption coefficient $\varkappa \sim 0.001-1\ \mathrm{mm}^{-1}$ [11, 12], and the average cosine of the scattering indicatrix $g \sim 0.8-0.95$ [11, 12]. It is known that the scattering of light in media with the strongly elongated scattering indicatrix is often described in the small-angle approximation of the radiation transfer theory, which can be applied for the optical thickness of the unabsorbing medium $\tau \leqslant 6-8$ [18]. In the presence of absorption, this region becomes broader.

The approach proposed here to describe the propagation of quasi-diffraction-free beams in a scattering medium taking multiple scattering into account is based on the known interrelation between the theory of light field coherence in a scattering medium and the radiation transfer theory [19 – 26]. Consider the reciprocal function of the spatial light field coherence $u(\boldsymbol{r}, z)$ in the plane $z=$ const [27]

$$\Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z) = \overline{u(\boldsymbol{r}-\boldsymbol{\rho}/2, z)u^*(\boldsymbol{r}+\boldsymbol{\rho}/2, z)}. \tag{3}$$

Here, $\boldsymbol{r} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$; $\boldsymbol{\rho} = \boldsymbol{r}_2 - \boldsymbol{r}_1$; $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the coordinates of points in the medium. It was shown in paper [26] that if the inhomogeneity scale of the field coherence function $\Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z)$ over the variable $\boldsymbol{r}$ is large compared to the radiation wavelength, this function can be represented in the region outside sources in the form

$$\Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z) = \int I(\boldsymbol{r}, \boldsymbol{n}_\perp, z)\exp(-\mathrm{i}k\boldsymbol{n}_\perp\boldsymbol{\rho})\mathrm{d}\boldsymbol{n}_\perp, \tag{4}$$

where the function

$$I(\boldsymbol{r}, \boldsymbol{n}_\perp, z) = \frac{1}{\lambda^2}\int \Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z)\exp\left(-\mathrm{i}\frac{2\pi}{\lambda}\boldsymbol{n}_\perp\boldsymbol{\rho}\right)\mathrm{d}\boldsymbol{\rho}, \tag{5}$$

which is the angular spectrum of the field coherence function $\Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z)$ satisfies the radiation transfer equation at least in the region of applicability of the small-angle approximation of the radiation transfer theory. Here, $\boldsymbol{n}_\perp$ is the projection of the unit vector $\boldsymbol{n}$ on the plane $z=$ const; λ is the radiation wavelength; and $k = 2\pi/\lambda$ is the wave number.

The solution of the radiation transfer equation in the small-angle approximation has the form [18]

$$I(\boldsymbol{v}, \boldsymbol{p}, z) = I_0^{\mathrm{fr}}(\boldsymbol{v}, \boldsymbol{p}, z)J(\boldsymbol{v}, \boldsymbol{p}, z), \tag{6}$$

where $I(\boldsymbol{v}, \boldsymbol{p}, z)$ and $I_0^{\mathrm{fr}}(\boldsymbol{v}, \boldsymbol{p}, z)$ are the Fourier transforms of the field $I(\boldsymbol{r}, \boldsymbol{n}_\perp, z)$ in the medium and of the spatial-angular brightness distribution produced by a radiation source in a free space, respectively; $J(\boldsymbol{v}, \boldsymbol{p}, z)$ is the Fourier transform of the normalised brightness $J(\boldsymbol{r}, \boldsymbol{n}_\perp, z)$, which is the Green function of the problem and depends on the optical characteristics of the medium but is independent of the characteristics of the source; $\boldsymbol{v}$ and $\boldsymbol{p} = k\boldsymbol{\rho}$ are the parameters of the Fourier transform in the coordinates $\boldsymbol{r}$ and $\boldsymbol{n}_\perp$, respectively.

Thus, based on the relation between the theory of light field coherence in a scattering medium and the radiation transfer theory, we can assert that relation (6) describes not only transfer of the photometric brightness of radiation but also the propagation of the light field coherence function. In this case, the functions $I_0^{\mathrm{fr}}(\boldsymbol{v}, \boldsymbol{p}, z)$ and $I(\boldsymbol{v}, \boldsymbol{p}, z)$ should be considered as Fourier transforms of the coherence function of the source field $\Gamma_0^{\mathrm{fr}}(\boldsymbol{r}, \boldsymbol{p}/k, z)$ and the field in the medium $\Gamma(\boldsymbol{r}, \boldsymbol{p}/k, z)$, respectively, over the coordinate $\boldsymbol{r}$. Note that the function $J(\boldsymbol{v}, \boldsymbol{p}, z)$ in (6) describes the decrease in the spatial coherence of the light field during its propagation in the scattering medium.

It is important to emphasise that the functions $I_0^{\mathrm{fr}}(\boldsymbol{r}, \boldsymbol{n}_\perp, z)$ and $I(\boldsymbol{r}, \boldsymbol{n}_\perp, z)$ are not equivalent to the photometric brightness of radiation [26]. In particular, they may take negative values. However, if the coherence function changes over the coordinate $\boldsymbol{r}$ slower than over $\boldsymbol{\rho}$, then upon averaging over regions considerably exceeding the coherence region, the angular spectrum $I(\boldsymbol{r}, \boldsymbol{n}_\perp, z)$ of the coherence function $\Gamma(\boldsymbol{r}, \boldsymbol{\rho}, z)$ becomes essentially positive and can be treated as the photometric brightness of radiation.

Let us now assume that $\boldsymbol{p} = 0$ in (6). One can see from (4) that this corresponds to passing to the Fourier spectra $E(\boldsymbol{v}, z) = I(\boldsymbol{v}, \boldsymbol{p}=0, z)$ and $E_0^{\mathrm{fr}}(\boldsymbol{v}, z) = I_0^{\mathrm{fr}}(\boldsymbol{v}, \boldsymbol{p}=0, z)$ of the illumination distributions $E(\boldsymbol{r}, z)$ and $E_0^{\mathrm{fr}}(\boldsymbol{r}, z)$ in the plane z:

$$E(\boldsymbol{v}, z) = E_0^{\mathrm{fr}}(\boldsymbol{v}, z)S(\boldsymbol{v}, z). \tag{7}$$

Here, $S(\boldsymbol{v}, z) = J(\boldsymbol{v}, \boldsymbol{p}=0, z)$ is the optical transfer function (OTF) of the medium [18], i.e., the Fourier spectrum of the point spread function (PSF) $S(\boldsymbol{r}, z)$ representing the illumination distribution produced in an arbitrary plane z of the medium with a point monodirectional source.

It is important that the illumination distribution $E(\boldsymbol{r}, z)$ in the plane z depending on the distribution $E_0^{\mathrm{fr}}(\boldsymbol{r}, z)$ (i.e., also in the plane z) is independent of the method by which the distribution $E_0^{\mathrm{fr}}(\boldsymbol{r}, z)$ was obtained. This result is a direct consequence of one of the properties of the so-called aspect invariance of the system [18], which can be described in the small-angle approximation of the radiation transfer theory.

Thus, to find the illumination distribution $E(\boldsymbol{r}, z)$ produced by a quasi-diffraction-free beam in the plane z, it is sufficient to know the illumination distribution $E_0^{\text{fr}}(\boldsymbol{r}, z)$ produced by the radiation source in the same plane z in a free space and the PSF $S(\boldsymbol{r}, z)$ or OTF $S(\boldsymbol{v}, z)$ of the medium. Note that in this case, both multiple scattering in the medium and coherence of radiation are automatically taken into account.

3. OTF and PSF of a medium

In the small-angle approximation of the radiation transfer theory, the expression for the function $J(\boldsymbol{v}, \boldsymbol{p}, z)$ for a homogeneous medium has the form [18]

$$J(\boldsymbol{v}, \boldsymbol{p}, z) = \exp\left[-\varepsilon z + \sigma \int_0^z x(\boldsymbol{p} + \boldsymbol{v}\xi) \mathrm{d}\xi \right], \tag{8}$$

where $x(\boldsymbol{p})$ is the Fourier spectrum of the scattering indicatrix $x(\beta)$ and β is the scattering angle. From expression (8), we obtain the OTF for a homogeneous medium

$$S(\boldsymbol{v}, z) = \exp\left[-\varepsilon z + \sigma \int_0^z x(\boldsymbol{v}\xi) \mathrm{d}\xi \right]. \tag{9}$$

Expression (9) has a very simple form; however, it is not always convenient for calculations because, first, the scattering indicatrix only rarely can be approximated so that to obtain the analytic expression for the function $x(\boldsymbol{p})$ and, second, it is difficult to make an analytic transfer from the OTF $S(\boldsymbol{v}, z)$ to the PSF $S(\boldsymbol{r}, z)$.

To describe the PSF analytically, the small-angle diffusion approximation (SADA) is often used in the radiation transfer theory [18]. The SADA gives the analytic expression for the zero and second moments of the azimuthally symmetric PSF. Therefore, the PSF is often represented as a sum of the unscattered and scattered radiation components:

$$S(\boldsymbol{r}, z) = S_0(\boldsymbol{r}, z) + S_{\text{s}}(\boldsymbol{r}, z), \tag{10}$$

where the illumination produced by the scattered component is approximated by the Gaussian function

$$S_{\text{s}}(\boldsymbol{r}, z) = \frac{S_{\text{s}}(z)}{2\pi V_{\text{s}}(z)} \exp\left[-\frac{r^2}{2V_{\text{s}}(z)} \right]; \tag{11}$$

$S_{\text{s}}(z)$ is the scattered radiation flux through the plane z; and $V_{\text{s}}(z)$ is the variance of the scattered PSF component.

It is known [18] that the function $S_{\text{s}}(\boldsymbol{r}, z)$ in the small-angle approximation tends to the Gaussian distribution of type (11) for $\sigma z \to \infty$; however, it can noticeably differ from this distribution for small σz. To overcome this difficulty, a multicomponent method was proposed in paper [28]. Within the framework of this method, the scattering indicatrices of real objects, having a strongly 'forward' elongated peak, are represented by a sum of several components:

$$x(\beta) = \sum_i a_i x_i(\beta). \tag{12}$$

Correspondingly, the PSF of the medium is also approximately represented by a sum of several components

$$S(\boldsymbol{r}, z) = \sum_i S_i(\boldsymbol{r}, z), \tag{13}$$

where

$$S_i(\boldsymbol{r}, z) = \frac{S_i(z)}{2\pi V_i(z)} \exp\left[-\frac{r^2}{2V_i(z)} \right]. \tag{14}$$

The functions $S_i(z)$ and $V_i(z)$ can be easily calculated analytically in terms of the optical characteristics of the medium [18].

The accuracy of the multicomponent method is illustrated in Fig. 1 by the example of the PSF for ocean water with the scattering indicatrix having elongation similar to that of the scattering indicatrix of biological tissues. One can see that this method provides almost the same accuracy as that of the small-angle approximation by preserving the SADA simplicity.

Expression (7), relating the Fourier spectra of the illumination distributions $E(\boldsymbol{r}, z)$ and $E_0^{\text{fr}}(\boldsymbol{r}, z)$, is not always convenient for practical calculations. Let us find the relation that directly relates these functions in the space of originals.

Taking into account (7) and the azimuthal-symmetry properties of the functions $E_0^{\text{fr}}(\boldsymbol{r}, z)$ and $S(\boldsymbol{r}, z)$, we have

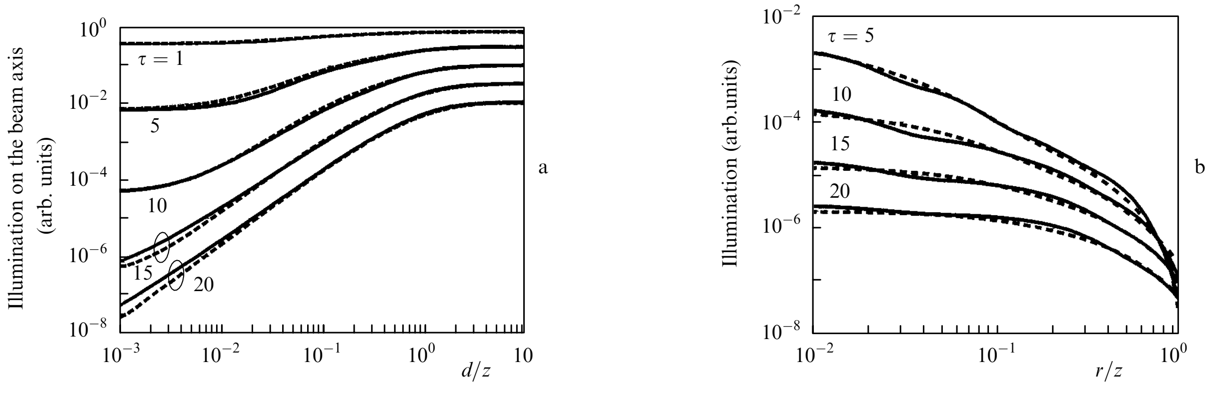

Figure 1. Illumination on the beam axis as a function of the beam diameter d (a), and the PSF as a function of the ratio r/z (b) for ocean water with $\varepsilon = 0.1$ m^{-1} and $\sigma = 0.08$ m^{-1} for different optical thicknesses τ of the layer. The solid curves correspond to solution (13), (14), the dashed curves correspond to the small-angle approximation.

$$E(\boldsymbol{r},z) = \frac{1}{2\pi}\int_0^\infty E(\boldsymbol{v},z)J_0(vr)v\mathrm{d}v$$
$$= \frac{1}{2\pi}\int_0^\infty E_0^{\mathrm{fr}}(\boldsymbol{v},z)S(\boldsymbol{v},z)J_0(vr)v\mathrm{d}v, \quad (15)$$

or

$$E(\boldsymbol{r},z) = \int_0^\infty S(\boldsymbol{v},z)J_0(vr)v\mathrm{d}v \int_0^\infty E_0(r',z)J_0(vr')r'\mathrm{d}r', \quad (16)$$

where $J_0(vr)$ is the Bessel function. By changing the integration order in (16), we obtain

$$E(\boldsymbol{r},z) = \int_0^\infty E_0(r',z)r'\mathrm{d}r' \int_0^\infty J_0(vr)S(\boldsymbol{v},z)J_0(vr')v\mathrm{d}v. \quad (17)$$

If the PSF can be represented in the form (13), (14), we obtain from (17) [29]

$$E(\boldsymbol{r},z) = \sum_i \frac{S_i(z)}{V_i(z)}\int_0^\infty E_0(r',z)\exp\left[-\frac{(r-r')^2}{2V_i(z)}\right]$$
$$\times \exp\left[-\frac{rr'}{V_i(z)}\right] I_0\left[\frac{rr'}{V_i(z)}\right] r'\mathrm{d}r', \quad (18)$$

where $I_0[rr'/V_i(z)]$ is the modified Bessel function. In a number of cases, relation (18) is more convenient for numerical calculations than (7).

4. Scattering indicatrices of biological tissues

As mentioned above, from the point of view of optics of scattering media, soft biological tissues are strongly scattering media with a high scattering anisotropy. In optical diffusion tomography of biological tissues (brain tomography, mammography) aimed, as a rule, at the detection of absorbing inhomogeneities of the type of various tumours, the transmission of an optically thick tissue layer is commonly measured [12]. In these cases, due to multiple scattering of radiation, the fine structure of the scattering indicatrix is 'blurred', i.e., is not observed. Because of this, the real scattering indicatrix $x(\beta)$ can be replaced by its approximate representation by a combination of the small-angle $[x_1(\beta)]$ and isotropic scattering indicatrix

$$x(\beta) = Fx_1(\beta) + (1 - F). \quad (19)$$

Here, the indicatrices $x(\beta)$ and $x_1(\beta)$ are normalised as usual:

$$2\pi\int_0^\pi x(\beta)\sin\beta\,\mathrm{d}\beta = 2\pi\int_0^\pi x_1(\beta)\sin\beta\,\mathrm{d}\beta = 1; \quad (20)$$

the parameter $F \leqslant 1$ determines the fraction of energy falling within the small-angle part $x_1(\beta)$ of the scattering indicatrix, and $(1 - F)$ is the energy fraction of the isotropic scattering indicatrix.

A strongly elongated small-angle part $x_1(\beta)$ is often replaced by the delta function. Then, it follows from (19) that $F = g$, and we pass to the known transport approximation of the radiation transfer theory [18]. In this case, the propagation of light is usually considered in the diffusion approximation of the radiation transfer theory, and the optical characteristics of the medium are described by the transport scattering coefficient $\sigma_{\mathrm{tr}} = \sigma(1-g)$ [12, 30–32] [which is often denoted by $\mu_s' = \mu_s(1-g)$ in the literature] and the absorption coefficient $\varkappa$. In this case, the scattering indicatrix of a new 'transport' medium is isotropic.

However, for optical coherent tomography and a number of other problems related to the diagnostics of relatively thin layers of strongly scattering biological tissues, where an increase in the transverse and longitudinal spatial resolution is very important, such an approach is obviously insufficient. The matter is that radiation scattered within small angles can rather long preserve its spatial coherence even upon multiple scattering and provide a high spatial resolution. For this reason, the small-angle part $x_1(\beta)$ of the scattering indicatrix cannot be replaced by the delta function in the description of radiation transfer in this class of problems, and it is necessary to take it into account more correctly. This can be performed within the framework of small-angle methods of the radiation transfer theory, in particular, by using the small-angle approximation and SADA. These methods use either detailed information on the small-angle scattering indicatrix or information on the integrated parameters (SADA), in particular, the average square of the scattering angle

$$\beta_{21} = \frac{\int_0^\infty \beta^2 x_1(\beta)\beta\,\mathrm{d}\beta}{\int_0^\infty x_1(\beta)\beta\,\mathrm{d}\beta}. \quad (21)$$

Unfortunately, the measurements of the small-angle part of the scattering indicatrix are technically rather complicated, especially in optically dense biological tissues, and information on such measurements is scarce in the literature [9, 10]. To make up for this lack of data to some degree, we can use some correlations between the parameters of strongly elongated scattering indicatrices known in the hydrooptics and optics of clouds.

Analysis of many strongly elongated scattering indicatrices of sea water [33] and clouds [34] shows that there exists a sufficiently stable correlation between the average cosine of the scattering indicatrix g, on the one hand, and a fraction F of light falling within the small-angle part of the scattering indicatrix $x_1(\beta)$ in the angular interval $0-\beta_0$ as well as the parameter β_{21}, on the other hand. For $\beta_0 = 45°$, we have [18]

$$1 - F \approx \frac{2}{3}(1-g), \quad (22)$$

$$\beta_{21} = \frac{1-g}{2}. \quad (23)$$

Thus, the parameters of the scattering indicatrix (19) are expressed in terms of the average cosine of the scattering indicatrix g. If more detailed specification of the small-angle scattering indicatrix is necessary, it can approximated, for example, by the exponential function [18]

$$x_1(\beta) = 2p^2\mathrm{e}^{-p\beta}, \quad (24)$$

where

$$p = 6/\beta_{21}. \quad (25)$$

5. Calculation of the illumination distribution in a Bessel light beam

Figure 2 shows the example of calculations of the illumination distribution, including its scattered and unscattered components, produced by a BLB at different depths in a scattering medium with optical parameters typical for real biological tissues [35]: the attenuation coefficient $\varepsilon = 12.6\ \text{mm}^{-1}$, the absorption coefficient $\varkappa = 0.033\ \text{mm}^{-1}$, and the average cosine of the scattering indicatrix $g = 0.9$.

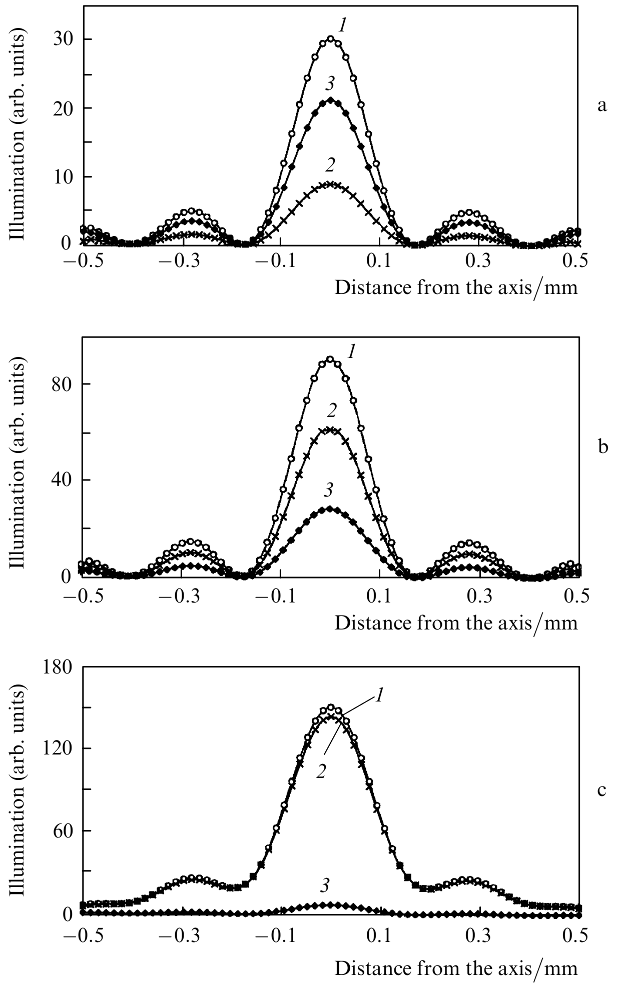

Figure 2. Distribution of the total illumination (*1*) and the scattered (*2*) and unscattered components (*3*) produced by the Bessel beam at different depths in the scattering medium for $\tau = 0.378$ (a), 1.26 (b), and 3.78 (c).

It is known [1–4] that the illumination distribution in a Bessel beam in some region behind a conic lens can be approximately described by the function

$$E_0^{\text{fr}}(\boldsymbol{r}, z) = (a + bz) J_0^2(\alpha r), \tag{26}$$

where a and b are some constants; $J_0(\alpha r)$ is the Bessel function; $\alpha = (2\pi/\lambda)\sin\gamma$; and 2γ is the divergence angle of the beams forming the Bessel beam. For the case presented in Fig. 2, $\alpha = 13.66\ \text{mm}^{-1}$, $a = 0$, and $b = 0.02\ \text{mm}^{-1}$. It is interesting to note that, when a conic lens is used to form a Bessel beam [4, 5], the parameter $b > 0$ in (26). This can result in an increase in the illumination on the beam axis with increasing depth z (cf. Figs 2a, b, c) despite its attenuation due to absorption and scattering of light in the medium. This increase is explained by the additional energy transfer from peripheral regions of the conic lens.

Note first of all that the distribution of the scattered and unscattered radiation components has the interference structure. This reflects the fact that scattered radiation preserves partially its spatial coherence. One can also see from Fig. 2 that in this case the half-width of the axial maximum of the Bessel beam almost does not increase up to the optical depths $\tau \approx 3 - 4$, where the contribution of multiple scattering is already significant. At first glance this result can appear rather strange. However, it can be simply explained. The matter is that in this case the half-width Δr of the zero maximum of the function $E_0^{\text{fr}}(\boldsymbol{r}, z)$ equal to ~ 0.1 mm cannot be considered small because for the attenuation coefficient of the medium $\varepsilon = 12.6\ \text{mm}^{-1}$, we have the optical size $\varepsilon\Delta r \approx 1.3$. For this reason, the PSF half-width at depths under consideration is noticeably narrower than that of the zero maximum. Therefore, scattered light almost does not leave the direct-light region and weakly broadens the light beam.

6. Conclusions

The method proposed in this paper gives a simple and efficient description of the spatial structure of quasi-diffraction-free beams propagating in scattering and absorbing media taking into account multiple scattering and coherence of radiation. Based on the relation between the classical theory of coherence and radiation transfer theory, this method allows one to use the well developed apparatus of the modern radiation transfer theory to solve this problem, to develop the efficient algorithms of computer simulation, etc. In addition, it opens up the possibilities for estimating the coherence function of the light field upon multiple scattering during the propagation of quasi-diffraction-free beams in a scattering medium.

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References

1. Goldfain E. *Proc. SPIE Int. Soc. Opt. Eng.*, **3917**, 119 (2000).
2. Ding Z., Ren H., Zhao Y., Nelson J.S., Chen Z. *Opt. Lett.*, **27** (4), 243 (2002).
3. Durnin J., Miceli J.J. Jr., Eberly J.H. *Opt. Lett.*, **13**, 79 (1988).
4. Herman R.M., Wiggins T.A. *J. Opt. Soc. Am. A*, **8**, 932 (1991).
5. Herman R.M., Wiggins T.A. *Appl. Opt.*, **33**, 7297 (1994).
6. Belyi V.N., Kazak N.S., Khilo N.A. *Kvantovaya Elektron.*, **30**, 753 (2000) [*Quantum Electron.*, **30**, 753 (2000)].
7. King T.A., Hogervorst W., Kazak N.S., Khilo N.A., Ryzhevich A.A. *Opt. Commun.*, **187**, 407 (2001).
8. Aruga T., Li S.W., Yoshikado S., Takabe M., Li R. *Appl. Opt.*, **38** (15), 3152 (1999).
9. Van Gemert M.J.C., Jacques S.L., Sterenborg H.J.C.M., Star W.M. *IEEE Trans. Biomed. Eng.*, **36** (12), 1146 (1989).
10. Marchesini R., Bertoni A., Andreola S., Melloni E., Sichirollo A.E. *Appl. Opt.*, **28**, 2318 (1989).
11. Tuchin V.V. *Lazery i volokonnaya optika v biomeditsinskikh issledovaniyakh* (Lasers and Fibre Optics in Biomedical Studies) (Saratov: Saratov University, 1998).
12. Zimnyakov D.A., Tuchin V.V. *Kvantovaya Elektron.*, **32**, 849 (2002) [*Quantum Electron.*, **32**, 849 (2002)].

13. Kuz'min V.L., Meglinskii I.V. *Pis'ma Zh. Eksp. Teor. Fiz.*, **79**, 139 (2004).
14. Kuz'min V.L., Meglinskii I.V. *Opt. Spektrosk.*, **97**, 108 (2004).
15. Meglinskii I.V., Churmakov D.Yu. *Opt. Spektrosk.*, **98**, 673 (2005).
16. Wilson B.C., Jacques S.L. *IEEE J. Quantum Electron.*, **26** (12), 2186 (1990).
17. Cheong W.F., Prahl S.A., Wlch A.J. *IEEE J. Quantum Electron.*, **26** (12), 2166 (1990).
18. Zege E.P., Ivanov A.P., Katsev I.L. *Image Transfer Through a Scattering Medium* (Heidelberg: Springer-Verlag, 1991).
19. Dolin L.S. *Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz.*, **7**, 471 (1964).
20. Dolin L.S. *Izv. Vyssh. Uchebn. Zaved. Ser. Radiofiz.*, **11**, 840 (1968).
21. Barabanenkov Yu.N. *Zh. Eksp. Teor. Fiz.*, **56**, 1262 (1969).
22. Ovchinnikov G.I. *Radiotekh. Radioelektron.*, **18**, 2044 (1973).
23. Furutsu K. *Radio Sci.*, **10**, 29 (1975).
24. Wolf E. *Phys. Rev. D*, **13**, 869 (1976).
25. Ishimaru A. *Wave Propagation and Scattering in Random Media, Vol. 2* (New York: Academic Press, 1978; Moscow: Mir, 1981).
26. Dolin L.S. *Izv. Vyssh. Uchebn. Zaved. Ser. Radiofiz.*, **23**, 332 (1980).
27. Born M., Wolf E. *Principles of Optics, 4th ed.* (Oxford: Pergamon Press, 1969; Moscow: Nauka, 1973).
28. Zege E.P., Katsev I.L., Polonsky I.N. *Appl. Opt.*, **32** (15), 2803 (1993).
29. Gradshtein I.S., Ryzhik I.M. *Tablitsy integralov, sum, ryadov i proizvedenii* (Tables of Integrals, Series and Products) (Moscow: Nauka, 1971).
30. Kienle A., Patterson M.S. *J. Opt. Soc. Am. A*, **14** (1), 246 (1997).
31. Wabnitz H., Rinneberg H. *Appl. Opt.*, **36** (1), 64 (1997).
32. Bevilacqua F., Marquet P., Coquoz O., Depeursinge C. *Appl. Opt.*, **36** (1), 44 (1997).
33. Kopelevich O.V., in *Optika okeana. Tom 1. Fizicheskaya optika okeana* (Optics of Ocean. Vol. 1: Physical Optics of Ocean) (Moscow: Nauka, 1983) pp 166 – 208.
34. Deirmendjian D. *Electromagnetic Scattering on Spherical Polydispersion* (New York: Elsevier, 1969).
35. Ivanov A.P., Katsev I.L. *Kvantovaya Elektron.*, **35**, 670 (2005) [*Quantum Electron.*, **35**, 670 (2005).