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Calculations of a fibre amplifier with the hexagonal waveguide structure

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Abstract. **An ytterbium-doped fibre amplifier with the hexagonal structure containing seven waveguides is simulated by using a numerical program based on the scalar paraxial optics approximation. The influence of the scatter in parameters of active fibre cores on spontaneous phasing discovered earlier is studied numerically for the first time. The critical scatter in the core parameters at which the phase locking of radiation is preserved is found. It is shown that a decrease in the numerical aperture of fibre cores facilitates the stabilisation of phase locking.**

Keywords: *fibre laser, coupled waveguides, optical phase locking, three-dimensional simulation.*

1. Introduction

Recently impressive results have been obtained demonstrating high-power emission of a single-mode fibre laser [1]. Nevertheless, the possibility of generating high-power emission of high optical quality in fibre lasers containing a system of active diffraction-coupled waveguides [2 – 6] still attracts great attention. Investigations in this direction can provide an increase in the aperture of a laser with phase-locked channels. An increase in the total area of active fibre cores will result in the absorption of pump radiation at a shorter length, thereby decreasing the role of nonlinear effects related to the development of stimulated Raman and Brillouin scattering. In addition, the distribution of radiation over the total aperture area exceeding the total area of fibre cores will reduce the optical load on them. All this will increase the maximum brightness of the output radiation.

The problem of phase locking of radiation propagating in a system of parallel waveguides was studied earlier for semiconductor lasers [7 – 9]. The study of schemes with the distributed radiation exchange or with external optical coupling has shown that the diffraction exchange between waveguides cannot solve this problem. A more promising is the scheme in which the main amplification occurs in regions with a lower refractive index [8]. The radiation exchange between these regions can be efficient for a proper geometry.

There exists a difference between semiconductor laser heterostructures and waveguide structures of a fibre laser which can play an important role for solving the problem of phase locking. The matter is that it is technologically very difficult to produce the spatial modulation of inversion in quantum wells in a semiconductor laser. The amplifying medium in a fibre laser is located in fibre cores whose shape and location can be easily changed. It was pointed out in review [10] that the inhomogeneous spatial distribution of the gain in a system of lasers plays itself the role of a spatial filter selecting certain optical modes. However, so far the role of the geometry of arrangement of lasers in the system was studied only for schemes with external coupling.

Cheo et al. [3] proposed and studied experimentally a fibre laser based on a fibre with the hexagonal structure containing seven diffraction-coupled cores doped with ytterbium ions. The authors of [3] obtained 100 W of output radiation with the far-field radiation distribution corresponding to the in-phase mode field. Later [11], a fibre amplifier was developed which contained 19 fibre cores in the hexagonal structure with a gain of 20 dB and had the output radiation quality close to the theoretical limit. Note that the improvement of the far-field radiation quality was observed in [3] only in the case of strong enough pumping. This gave grounds to the authors of [12] to explain, by using the method of coupled modes [13], the spontaneous selection of the in-phase mode by the dependence of the contribution to the refractive index of glass caused by transitions between ytterbium levels on the pump and lasing intensities. It was also assumed that the nonlinear part of the refractive index was positive, which provided the self-focusing condition. Later [14], it was found also by the method of coupled modes that the resonance nonlinear dependence of the refractive index gives rise to the ‘passive’ phase locking of the field even in the case of a negative nonlinear addition to the refractive index (the negative nonlinearity was experimentally proved in [15]).

Numerous studies of phase locking in various lasers have shown that the technological scatter in the properties of individual elements destroys the single-mode regime already slightly above the lasing threshold. Only a system with strongly coupled elements can preserve the coherence of a total field upon a noticeable excess over the lasing threshold. However, in the strong coupling limit, the traditional theoretical analysis by the method of coupled modes becomes invalid. In this case, it is necessary to calculate numerically the propagation of the radiation field in a

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composite structure. We developed a complex of programs for simulations of waveguide structures in the scalar and paraxial approximations based on the three-dimensional method for calculations of beam propagation [16]. This complex was tested by comparing experimental data with calculations of the propagation of monochromatic light in a fibre of large radius containing 38 single-mode cores with axes located on one circle [17].

In [18], the complex developed in [16] was used for the calculation of propagation of a monochromatic field in a waveguide structure [3] with identical active cores. It was shown that in the case of the initial phase scatter $\sim$0.3 rad, the tendency to the levelling of the relative optical phases in cores exists only in the presence of amplification. The nonlinearity of the refractive index cannot be the only reason for spontaneous phase locking. In addition, the rate of phase levelling considerably increases with decreasing a drop in the refractive index.

In this paper, we considered the influence of a random scatter in core parameters, namely, variations in the refractive index of waveguides on the phase locking effect. The critical scatter in the values of the refractive index destroying phase locking is determined for the first time. It is shown that a decrease in the average waveguide parameter of active cores considerably increases the admissible scatter in their parameters.

2. Mathematical model of a fibre amplifier

The calculation method used here was described in detail earlier [16], and therefore we consider it only briefly. The three-dimensional calculation is based on the method of splitting [19] into operators of field diffraction and refraction/amplification at each propagation step. Diffraction effects are calculated by using the two-dimensional fast Fourier transform. The refractive index profile determines the integration step along the propagation direction (z axis), the upper limit of the step size along this axis is determined by the variation of the radiation field phase. For the construction under study, the network step was set equal to 2 μm. Calculations were performed in the transverse plane on a square 512×512 network. To avoid the artificial distortion of the field due to periodic boundary conditions imposed in the method of the fast Fourier transform, additional absorption was introduced in the network nodes outside the pump region according to recommendations proposed in [20].

The simulated structure from [3] contained seven cores in a hexagonal lattice. The diameter of each of the cores was 7 μm and the waveguide parameter was $V = 1.73$ (the corresponding step of the refractive index was $\Delta n = 2.57 \times 10^{-3}$). The distance between the axes of adjacent cores was 10.5 μm. The nonlinear response of the medium was provided by the simplest Rigrod amplification model [21] $g = g_0/(1 + I/I_s)$ and the nonlinearity of the refractive index approximated by the dependence $\delta n_{nl} = n_2 I$. The small-signal gain corresponded to the pump radiation intensity 482 MW cm^{-2} [12] and was 0.26 cm^{-1}, the radiation wavelength was set equal to 1079 nm, the nonlinearity coefficient in the core was $n_2 = 2 \times 10^{-12}$ cm^2 W^{-1}, and the saturation intensity was $I_s = 64.4$ kW cm^{-2} [22]. The pump depletion during the propagation of radiation was neglected. The input field consisted of seven beams with identical intensities and phases, each of them being the fundamental mode of a single fibre (Fig. 1). The core parameters were randomly varied and the evolution of the amplified radiation along the fibre length was studied.

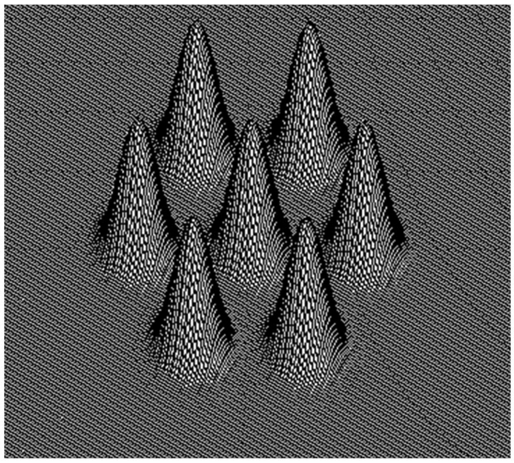

Figure 1. Intensity distribution of the input laser beam.

3. Simulation of a fibre amplifier with $V = 1.73$

It is technologically difficult to fabricate an ideal fibre structure containing seven waveguides. The most commonly encountered distortions of such a structure are the displacements of axes and variations in the shapes of fibre cores [17]. The consideration of such perturbations in a numerical model is complicated due to a large number of random parameters. It is known [23] that the overlap of field distributions and refractive-index profiles plays the main role in processes of the diffraction exchange between the cores. The method of changing this overlap is of minor importance, and therefore we used the simplest method to produce perturbations in the structure by introducing a random scatter in the values of waveguide increments of the refractive index.

The direct diffraction calculation does not contain concepts widely used in the method of coupled modes such as the mode of a single waveguide and the strength of optical coupling. In the diffraction calculation method, supermodes are linear combinations of modes of single waveguides. Because within the framework of direct calculations only optical modes of the whole structure can be considered, the evolution of the field being amplified can be described by expanding the field in these modes. In this paper, we did not attempt to find established optical modes (such modes were found for ideal hexagonal structures with 7 and 19 waveguides in [24]). Our aim was to determine the possibility of preserving the effect of spontaneous phase locking upon distortions of the waveguide structure parameters, and therefore our main concern was to study the levelling rate of field phases in cores.

Calculations performed in [18] showed that in a system of identical cores with a step in the refractive index realised in experiments, a combination of two axially symmetric modes was selected. The amplified field exhibits spatial beats caused by the difference in the propagation constants of the modes. It was found that the phase difference of the fields for both modes in the central and side cores was small. As a result, the beam quality over the length changes insignificantly and is preserved close to the diffraction limit in the total aperture size.

A random scatter of the refractive index in each of the cores was introduced by means of a random-number generator with the Gaussian statistics, which generated an infinite sequence of random numbers with the given dispersion and zero mathematical expectation. The sampling of seven numbers from this sequence was performed with the normalisation providing the required scatter of the refractive index. A small number of elements in the sampling produce irregular variations in the average value and dispersion for different realisations. We performed calculations both for different realisations and for a fixed sampling with a regular variation of normalisation.

Table 1 presents the parameters of different samplings with the same normalisation and presents the results of simulation of an amplifier of length 1 m with the output power providing a noticeable saturation of the gain from the very beginning of radiation propagation. The input field distribution in the form of the in-phase combination of modes of individual cores is shown in Fig. 1. The radiation quality was characterised by calculating the phase-locking parameter Σ introduced in [22]. This parameter is similar to the Strehl number and is defined by the expression $\Sigma = 1 - |\sum A_i(z)|/\sum |A_i(z)|$, where A_i is the field amplitude on the axis of the ith core. For the in-phase field distribution, $\Sigma = 0$. If the field distribution tends to the in-phase mode, the phase-locking parameter should tend to zero.

Table 1. Parameters of random realisations of a fibre amplifier with the waveguide core parameter $V = 1.73$.

Number of random realisation	$\langle \Delta \rangle/10^{-5}$	$\delta n/10^{-5}$	L_Σ/cm	α/cm^{-1}
1	3.316	6.381	41	0.0244
2	−0.7942	8.150	64.8	0.0154
3	−2.603	6.902	49.6	0.0202
4	−3.382	6.573	52.7	0.0190
5	−1.666	7.389	66.6	0.0150
6	2.582	9.239	71.1	0.0141

Notes: $\langle \Delta \rangle$ is the realised core-averaged variation in the refractive index; δn is the corresponding root-mean-square scatter; L_Σ is the phase-locking length; α is the phase-locking rate.

Our calculations showed that due to the scatter of core parameters, radiation acquires different phase shifts in different cores immediately after coupling into the fibre, which causes a rapid increase in the phase-locking parameter (Fig. 2, realisation no. 3). After propagation over a distance of ~20 cm, the phase locking of radiation begins, and Σ tends to zero (Fig. 3). The value of Σ decreases against the background of oscillations with a period of ~2 mm and it can be quite accurately described on average by an exponential law with the phase-locking length L_Σ. Along with the decay length of the parameter Σ, it is also convenient to use the parameter α – the phase-locking rate (which is inverse to L_Σ). The length L_Σ and phase-locking rate α depend on the specific variations of the refractive index in cores (see Table 1).

Oscillations with a period of ~2 mm are caused by the beats of two modes. It was found earlier [18] that similar beats of two axially symmetric modes with the field phase difference in the central and side cores equal to 0 and π, respectively, are observed in the ideal structure. The beat

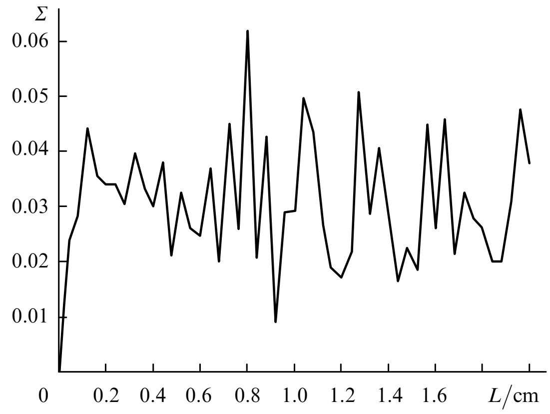

Figure 2. Behaviour of the phase-locking parameter at the beginning of propagation of radiation in the amplifier; $\delta n = 1.53 \times 10^{-4}$, $V = 1.73$.

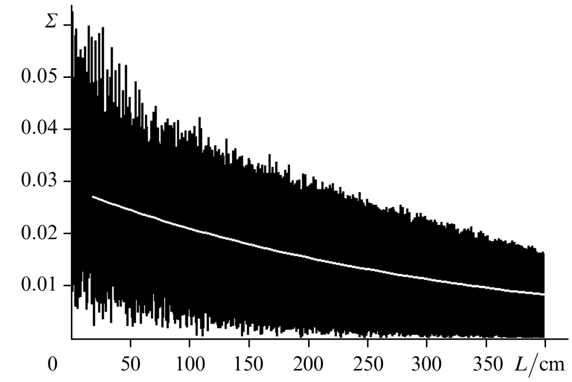

Figure 3. Dependence of the phase-locking parameter on the propagation length in the amplifier. The light curve is the approximation of the exponential decay $\Sigma(L)$; $\delta n = 1.53 \times 10^{-4}$, $\Delta n = 2.57 \times 10^{-3}$.

amplitude for the phase-locking parameter is comparable with its average value. After propagation over a large distance, the field distribution weakly varies on the beat period, and the central peak is maximal in this case. Correspondingly, the far-field output radiation distribution is rather stable (the beam quality parameters along the x and y axes are $M_x^2 = 2.1$ and $M_y^2 = 1.8$). We performed control calculations of the amplifier by varying the value and sign of the nonlinear part $\delta n_{\rm nl}$ of the refractive index. As in the case of the ideal structure [18], nonlinearity does not play any noticeable role in phase locking.

From the practical point of view, the key question is the maximum possible scatter of the core parameters at which the mode selection property is preserved. One can see from Table 1 that variations in the samplings of random quantities allow the dispersion δn to be changed approximately by one and a half. To obtain the dependence of the field-phase-locking rate of the scatter of parameters in a broader range, we performed calculations for two fixed samplings of random numbers (realisations no. 1 and no. 3) by regularly changing dispersion. Figure 4 shows the phase-locking rate as a function of δn for realisations no. 1 and no. 3 and all other realisations from Table 1. As expected, the phase-locking rate α decreases with increasing the scatter of the refractive index in cores and is well approximated by a parabola for the given realisation. However, for different samplings of random numbers, the convergence rate is, generally speaking, different for the same scatter, so that it is

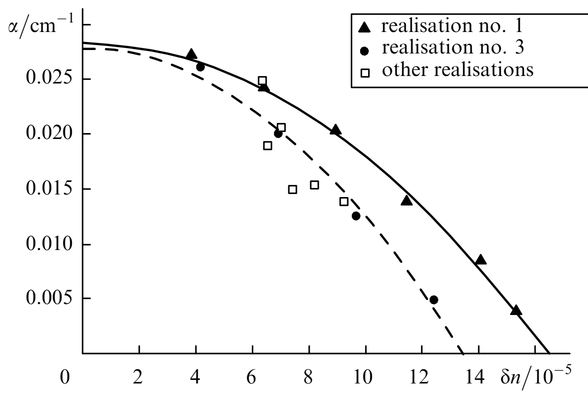

Figure 4. Dependence of the decay decrement of the phase-locking parameter on the variable dispersion of the refractive index of cores; $\Delta n = 2.57 \times 10^{-3}$.

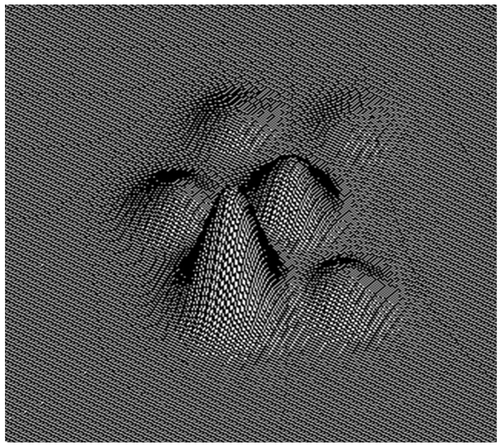

Figure 5. Intensity distribution at the amplifier output with the scatter of the refractive index of cores presented in Table 2.

impossible to find one critical value of the scatter of the refractive index above which the field phase locking disappears. Note here that realisation no. 1 proved to be not typical. The results of simulation show that the admissible scatter of the refractive index in a structure with experimental parameters [3] lies within $(1.3 - 1.6) \times 10^{-4}$. In other words, the relative critical scatter of the refractive index is $\delta n/\Delta n \geqslant 5\,\% - 6\,\%$. This is a reasonable value which can be achieved in practice.

The results of calculation of the amplifier with variations of the refractive index presented in Table 2 and the scatter close to the critical value demonstrate the redistribution of radiation between cores and the appearance of a side maximum (Fig. 5). This redistribution is explained by the relative increase in the refractive-index step in side channel no. 1 (see Table 2). Nevertheless, the far-field radiation distribution changes weakly, almost without any deterioration of the beam quality (Fig. 6).

Table 2. Deviations of the refractive index of cores for random realisation no. 1; $\Delta n = 2.57 \times 10^{-3}$, $\delta n = 1.53 \times 10^{-4}$.

Core number	$\delta n_i/10^{-3}$
1	0.394
2	0.0161
3	−0.0851
4	0.174
5	0.114
6	−0.0468
7 (Centre)	−0.00948

4. Simulation of a fibre amplifier with $V = 1.22$

We showed in [18] that a decrease in the refractive-index step results in a considerable acceleration of phase levelling when a set of beams with random phases is supplied to the amplifier input. It can be expected that similar effect will be also observed in the case of the scatter of parameters of individual cores. To verify this assumption, we calculated the propagation of radiation in the amplifier with the geometry described above, but with the refractive-index step $\Delta n = 1.27 \times 10^{-3}$ reduced by half, which corresponds to the waveguide parameter $V = 1.22$.

Calculations were performed for a set of numbers from random realisation no. 1 by changing regularly the dispersion of the refractive index. Figure 7 shows the dependence of the phase-locking rate on δn. A comparison with the results presented in Fig. 4 shows that the system with a lower waveguide parameter of the core ($V = 1.22$) is

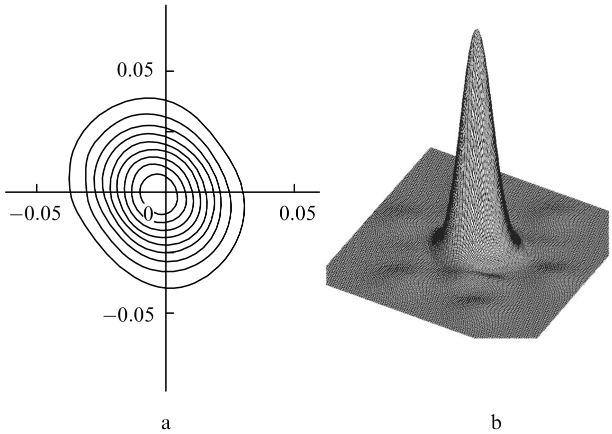

Figure 6. Far-field intensity distributions for the output beam of a fibre with the scatter of the refractive index of cores presented in Table 2 in the form of the contour map (a) and in the three-dimensional representation (b).

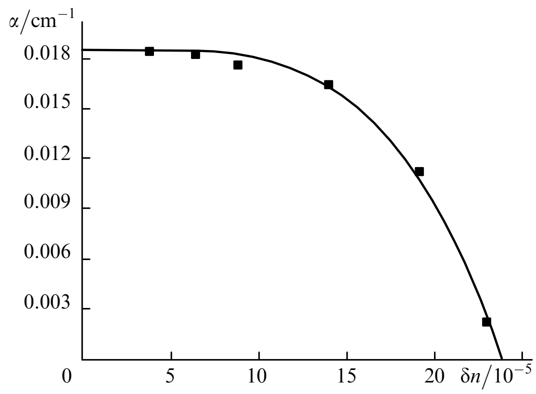

Figure 7. Dependence of the decay decrement of the phase-locking parameter on the sample dispersion of the refractive index of cores (squares) and its approximation by a four-order parabola (curve); $\Delta n = 1.27 \times 10^{-3}$.

much more stable to the scatter of parameters than the system with the refractive-index step $\Delta n = 2.57 \times 10^{-3}$ ($V = 1.73$). The dependence of the phase-locking rate on the dispersion δn also changes. The calculated points in Fig. 7 are well approximated by a fourth-order parabola. The absolute value of the admissible scatter in the refractive index in the system with a lower waveguide parameter is larger by a factor of 1.5 compared to the system studied in [3], while the relative critical dispersion of the refractive index $\delta n/\Delta n$ is 15%.

It is obvious that this effect is explained by a more intense radiation exchange between cores. We also found that the total gain in the system with a smaller average step of the refractive index was lower by a factor of 1.6 than that in a fibre with the waveguide core parameter $V = 1.73$ for the same values of scatter δn, input intensity, and amplifier length. This is explained by a decrease in the overlap of the radiation field with the cross section of active waveguides. Note that this effect plays a positive role in amplifiers from the point of view of a smoother distribution of the field over the total aperture reducing the far-field intensity of side orders and also due to a decrease in the peak intensity at the fixed total radiation power.

Figure 8 presents the results of numerical simulation of the amplifier with the scatter of the refractive index in cores close to the critical value (deviations of the refractive index are presented in Table 3). Despite a strong redistribution of the field compared to the case of identical cores, the far-field radiation distribution corresponds to the high-quality beam ($M_x^2 = 1.76$, $M_y^2 = 1.3$). In addition, an elliptic beam has no side orders and is somewhat narrower than the beam for a fibre with the larger refractive-index step (see Fig. 6).

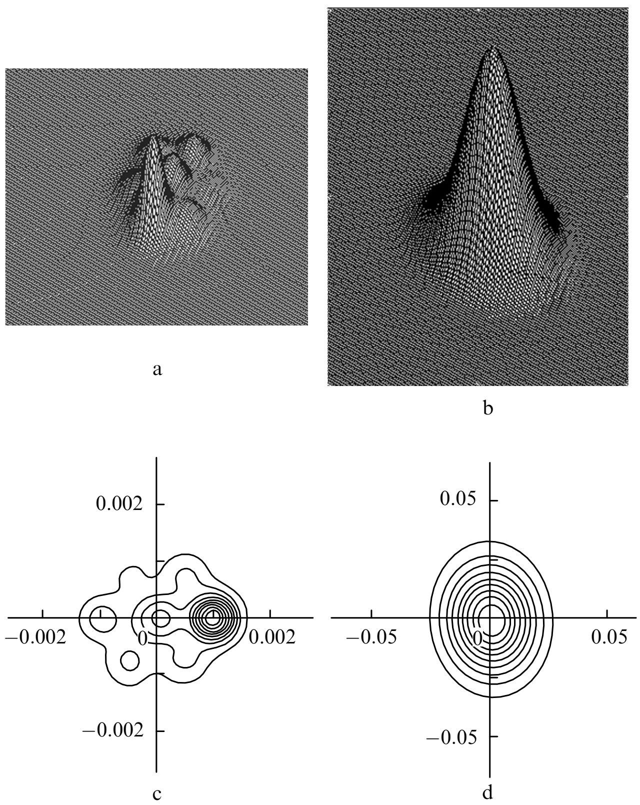

Figure 8. Radiation intensity at the input of the amplifier with the scatter of the refractive index of cores presented in Table 3: three-dimensional near-field (a) and far-field (b) intensity distributions and contour maps of near-field (c) and far-field (d) intensity distributions.

Table 3. Deviations of the refractive index of cores for random realisation no. 1; $\Delta n = 1.27 \times 10^{-3}$, $\delta n = 1.914 \times 10^{-4}$.

Core number	$\delta n_i/10^{-3}$
1	0.492753
2	0.0201045
3	−0.1063755
4	0.217986
5	0.142266
6	−0.0584535
7 (Centre)	−0.011856

5. Conclusions

The diffraction three-dimensional calculation of the propagation of beams in a fibre amplifier with a hexagonal lattice containing seven different active cores doped with ytterbium has shown that the effect of phase levelling over the cross section is preserved when the scatter in the core parameters is small. The role of random variations in the refractive-index step in cores has been studied numerically for the first time and the admissible values of the dispersion of this step have been found. It has been shown that the admissible values of the scatter are not too small and such a stability of n can be probably achieved in practice. The quality of the output beam of the amplifier remains high even when the scatter in the core parameters results in a strong redistribution of the radiation field over the laser aperture.

A decrease in the refractive-index step maintaining radiation in cores considerably improves the stability of phase locking with respect to random variations in propagation constants in individual cores. The expected admissible relative scatter of the refractive-index step can achieve 15 %. Control calculations by varying the value and sign of the nonlinear part of the refractive index have shown that nonlinearity does not play any significant role in the selection of the in-phase distribution.

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