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Scenario of the experiment on gamma-ray amplification upon stimulated transitions in isomeric nuclei in a Bose – Einstein condensate

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***Abstract*. The scenario of the experiment on the amplification of gamma rays upon stimulated transitions from metastable states of isomeric nuclei in a Bose – Einstein condensate (BEC) is presented. The experiment involves the successive transport of atoms along the so-called quantum conveyer. It is shown that if the excess broadening of the gamma-ray emission line is eliminated, the total gain $G > 1$ can be achieved despite the presence of negative factors such as the 'laser lethargy', spontaneous decay of metastable states, etc.**

Keywords: *gamma emission, isomeric nuclei, Bose – Einstein condensate*.

1. Introduction

The possibility of performing experiments on the observation of stimulated gamma emission of isomeric nuclei is based on two statements [1]:

(i) If the broadening of a nuclear gamma-transition line exceeding its natural radiative width $\Delta\omega_\gamma$ is eliminated, the stimulated emission cross section $\sigma = \lambda^2/2\pi$ is proportional to the square of the radiation wavelength λ and is independent of the matrix element, transition multipolarity, and the degree of the transition forbiddeness.

(ii) The excess broadening of the emission line can be completely eliminated by introducing atoms containing emitting nuclei into a Bose – Einstein condensate (BEC).

The first statement follows directly from the Einstein's derivation of the radiation laws based on the most general thermodynamic approach which contains no above-mentioned details characterising the transition.

The second statement is based on the assumption that BEC atoms, described by one wave function and forming in a sense a common 'megaatom', lose to a great extent the ability to perform individual chaotic motion, which results in the excess inhomogeneous broadening of the gamma line. The degree of the quantum coherence of the condensate and the corresponding minimal linewidth are determined first of all by the inverse lifetime $\Theta_{\rm BEC}$ of atoms in the condensate, which are in the dynamic equilibrium with the rest of the gas atoms. The lifetime $\Theta_{\rm BEC}$ has yet to be determined both theoretically and experimentally. It seems that the existing theoretical estimates of $\Theta_{\rm BEC}$ [2] are more likely too optimistic rather than reliable, while the known experimental confinement times of a Bose condensate in a trap characterise only the quality of the experimental technology rather than the value of $\Theta_{\rm BEC}$. The value of $\Theta_{\rm BEC}$ can be estimated based on the following simple consideration.

To undergo a transition from the zero-momentum state and evaporate from a condensate, an atom should receive a finite momentum and energy, which can be imparted to the condensate atom in a simple model only during its collision with a gas atom with the nonzero momentum. The mean time between such collisions is

$$\Delta t_{\rm col} = \frac{[\sigma_{\rm col} u(T)]^{-1}}{n - n_{\rm BEC}} = 0.32\,\frac{(2J_{\rm a}+1)^{1/3} M}{\hbar\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2$$

$$\approx 500\,\frac{(2J_{\rm a}+1)^{1/3} A}{\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2, \qquad (1)$$

where $\sigma_{\rm col}$ is the collision cross section; $u(T) = (3k_{\rm B}T/M)^{1/2}$ is the root-mean-square velocity of gas atoms with the mass M at temperature T;

$$n_{\rm BEC} = n\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right] \qquad (2)$$

is the concentration of condensate atoms; n is the total concentration of gas; $J_{\rm a}$ is the angular momentum of the atom;

$$T_0 = 3.3\,\frac{\hbar^2 n^{2/3}}{k_{\rm B} M (2J_{\rm a}+1)^{2/3}} \qquad (3)$$

is the degeneracy and condensation temperature of atoms; $k_{\rm B}$ is the Boltzmann constant; and A is the isotopic number of the atom.

If we assume that other factors limiting the lifetime of condensate atoms stronger than the collision process are absent, the estimate $\Theta_{\rm BEC} \approx \Delta t_{\rm col}$ is admissible, and then

$$\Theta_{\rm BEC} = 0.32\,\frac{(2J_{\rm a}+1)^{1/3} M}{\hbar\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2$$

$$\approx 500\,\frac{(2J_{\rm a}+1)^{1/3} A}{\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2. \qquad (4)$$

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Thus, for $n = 10^{12}$ cm^{-3}, $A = 100$, $T/T_0 = 1.3$, and $\sigma_{col} = 10^{-16}$ cm^2, we have $\Theta_{BEC} \approx 10^5$ s.

Our further analysis is based on the assumption that the lifetime Θ_{BEC} of atoms in the condensate exceeds the metastable-state lifetime τ of the isomer ($\Theta_{BEC} > \tau$), while the experimental technique can provide the confinement time of the condensate in the trap that is at least no shorter than Θ_{BEC}.

The same considerations about the necessity of eliminating the line broadening caused by collisions of condensate atoms with other atoms outside the condensate restrict the total concentration of atoms and their temperature [3]:

$$n < 0.43\left(\frac{M}{\hbar\sigma_{col}\tau}\right)^{3/4}(2J_a+1)^{1/4}\left(\frac{T_0}{T}\right)^{3/2}$$

$$\approx 107\left(\frac{A}{\sigma_{col}\tau}\right)^{3/4}(2J_a+1)^{1/4}\left(\frac{T_0}{T}\right)^{3/2}, \quad (5)$$

$$T < 1.9\hbar^{3/2}k_B^{-1}[(2J_a+1)M\sigma_{col}\tau]^{-1/2}$$

$$\approx 3.6\times 10^{-13}(\sigma_{col}A\tau)^{-1/2}(2J_a+1)^{-1/2}. \quad (6)$$

For example, by assuming that $T/T_0 = 1.3$ and $\sigma_{col} = 10^{-16}$ cm^2, we obtain the restrictions $n < 2.4\times 10^{14}$ cm^{-3}, $T < 4\times 10^{-7}$ K and $n < 1.4\times 10^{13}$ cm^{-3}, $T < 5\times 10^{-8}$ K for $^{111}_{47}$Ag and $^{135}_{55}$Cs, respectively.

When the collision broadening is excluded due to a low enough concentration of atoms in the atomic ensemble, this ensemble can be treated as a gas of noninteracting atoms, and in this case the excess broadening caused by interaction with neighbouring atoms and their inhomogeneous distribution is also virtually absent.

The time-of-flight broadening is eliminated by restricting from below the length L of the region of interaction between nuclei and gamma-ray field [3]:

$$L > V\tau, \quad (7)$$

where V is the transport velocity of atoms.

The fulfilment of all these requirements provides almost complete elimination of the excess broadening of gamma-ray emission lines. In this case, the stimulated emission cross section is equal to its asymptotic value $\sigma = \lambda^2/2\pi$. [However, it is not inconceivable that inequality (7) cannot be fulfilled under restricted laboratory conditions and the value of L, which can be unacceptable in the case of long-lived isotopes, should be decreased; as a result of this compromise, the stimulated emission cross section σ will decrease proportionally to the ratio $L/V\tau < 1$.]

However, the elimination of the excess broadening of a gamma-ray emission line does not warrant at all that the total gain $G > 1$ of gamma rays per pass in the isomeric medium will be higher than unity.

To gain a quantitative impression about the situation, it is useful to estimate some artificial test parameter G_A, which determines in fact the maximum real value of the total gain $G < G_A$ that can be achieved:

$$G_A = \exp(\sigma nL). \quad (8)$$

If, for example, we assume that $\sigma = 10^{-18}$ cm^2, $L = 10^3$ cm, and $n = 10^{13}$ cm^{-3} according to limitation (5), then the test parameter G_A will be 1.01, i.e. slightly higher than unity. This follows from the low value of the cross section σ in the short-wavelength region and limiting inequality (5) and considerably complicates the obtaining of the gain $G > 1$ in the isomeric medium.

In this case, it is important to take into account that we estimated the test parameter G_A by neglecting the factors that can considerably strengthen the inequality $G < G_A$. Thus, instead of the concentration n^* of active isomeric nuclei directly involved in the amplification, the total concentration $n > n^*$ was used in expression (8) for G_A. Moreover, we neglected a decrease in the concentration n^* of isomers due to the spontaneous decay of metastable states, a decrease in the real value of the stimulated emission cross section due to the asymptotic behaviour of its current value $\sigma(t) < \sigma = \lambda^2/2\pi$ (the so-called laser lethargy [3–6]), nonresonance losses of gamma rays in the medium, etc.

Thus, the real value of the total gain $G < G_A$ is always lower than G_A. Moreover, the neglect of the above-mentioned negative factors casts some doubt whether the value of $G > 1$ can exceed unity at all. Therefore, the elucidation of the possibility of achieving even small but exceeding unity value of G in the gamma range within the framework of our scenario is important because it would open up the principal opportunity for obtaining stimulated gamma-ray emission from metastable states of isomeric nuclei. The answer to the question of whether the total gain $G > 1$ exceeding unity can be achieved in nuclear isomers is the aim of our analysis.

2. Criteria for the choice of an isomeric nucleus

It is known that the lifetime of nuclear metastable states varies from fractions of microseconds to many thousand-year periods. The choice of an isomer with the lifetime τ acceptable for experiments is determined by several mutually contradictory considerations.

On the one hand, it seems attractive to use long-lived isomers, which can be rather easily prepared (pumped). Moreover, in the case of natural isomers, pumping is not needed at all. However, the asymptotic behaviour of the current value $\sigma(t)$ of the stimulated emission cross section, which delays the beginning of amplification by the time close to the lifetime τ of the metastable state (laser lethargy [3–6]), although does not result in the theoretical prohibition, but taking (7) into account, makes long-lived isomers unattractive for researchers.

On the other hand, very short lifetimes τ require very intense and fast pumping to prepare the isomer and exclude, of course, the possibility of using natural isomers.

It seems that the most efficient method for obtaining isomers with the moderate lifetime τ is the radiative neutron-capture reaction. Thus, thermal neutrons with the typical flux density of 10^{13} cm^{-2} s^{-1} can form on the linear part of the capture reaction the isomers with the rate 10^6 cm^{-3}s^{-1} from parent isotopes with the capture cross section $\sigma(n,\gamma) = 10^{-22}$ cm^2 and concentration $n_m = 10^{15}$ cm^{-3} up to the achievement of the saturated concentration of isomers of the order of $10^6\tau$ cm^{-3}. The passage from the linear region to the saturation region, in which the reaction efficiency decreases due to the spontaneous decay of metastable states of the isomer, occurs for the time of the order of τ during which the isomer concentration is

Table 1.

Isomer	τ	$\hbar\omega$/keV	Parent isotope in the (n,γ) reaction and its lifetime	$\sigma(n,\gamma)/10^{-24}$ cm^2 for the 0.0253-eV neutrons	$\sigma/$ 10^{-20} cm^2	$\chi/$ 10^{-20} cm^2
$^{91}_{39}$Y	49.7 min	**555**	$^{90}_{39}$Y, 64.1 h	< 6.5	0.8	$\sim 1.4 \times 10^{-3}$
$^{95}_{41}$Nb	86.6 h	**235**	$^{94}_{41}$Nb, 2×10^4 years	14.9	4.5	$\sim 2.5 \times 10^{-3}$
$^{105}_{45}$Rh	45 s	**129**	$^{104}_{45}$Rh, 42.3 s	40	14.5	$\sim 10^{-2}$
$^{111}_{47}$Ag	64.8 s	**60**	$^{110}_{47}$Ag, 250 days	82	68.5	~ 0.1
$^{135}_{55}$Cs	53 min	**781**, 840	$^{134}_{55}$Cs, 2 years	140	0.405	$\sim 2 \times 10^{-3}$
$^{178}_{72}$Hf	4.0 s	**89**, 213, 326, 426	$^{177}_{72}$Hf, stable	365	31.2	$\sim 4 \times 10^{-2}$
$^{180}_{72}$Hf	5.5 h	**58**, 215, 333, 444	$^{179}_{72}$Hf, stable	45	73	$\sim 9 \times 10^{-2}$
$^{190}_{76}$Os	9.9 min	**187**, 361, 502, 616	$^{189}_{76}$Os, stable	23	7.1	$\sim 2 \times 10^{-2}$
$^{193}_{77}$Ir	10.6 days	**80**	$^{192}_{77}$Ir, 74.2 days	1100	38.5	$\sim 3 \times 10^{-1}$

accumulated, approaching the saturation value. This means that the use of short-lived isotopes produced in the radiative neutron-capture reaction is undesirable.

The estimate of incoherent X-ray excitation schemes, which could be of interest for producing rapidly decaying metastable states ($\tau < 1$ s), show that these schemes are hopeless because the spectral density of radiation emitted by available radiation sources is many orders of magnitude lower than that required for level pumping. Therefore, the compromise is the choice of isomers with moderate values of τ lying in the interval from tens of seconds to tens of minutes.

Table 1 [7] presents the examples of isomers which are contained in boson atoms and can be produced from parent nuclei in the radiative thermal neutron-capture reaction. These examples demonstrate broad ranges of metastable-state lifetimes τ, gamma-ray energies $\hbar\omega$, cross sections $\sigma(n,\gamma)$ for the radiative neutron-capture, stimulated emission cross sections σ, and averaged photon loss cross sections χ (energies $\hbar\omega$ in bold were used in calculations). The values of antagonistic parameters presented in Table 1 prevent the use of many corresponding isomers in experiments on stimulated gamma emission.

If we also consider the atomic characteristics, which are important for efficient laser manipulation by atoms containing nuclei, it is unlikely that we can hope to obtain the optimal combination of the atomic and nuclear parameters. However, the statement of leading scientists in the field of laser manipulation by neutral atoms that 'in principle laser cooling can be used for any atom' ([8], p. 273) can arouse some optimism.

It follows from the above discussion that certain voluntarism exists in the choice of the object for calculations to make estimates. Such estimates will be performed below, not pretending to any optimisation, for silver and cesium isomers $^{111}_{47}$Ag and $^{135}_{55}$Cs with lifetimes corresponding to both limits of the above-accepted interval of admissible lifetimes τ.

3. Experimental configuration ('quantum conveyer for atoms')

The experimental configuration under study is similar to that in [3, 9] and represents the sequence of operations with an atomic beam propagating as in a conveyer in an extended quantum well consisting of five zones with different functions: zone I: preparation of an isomer; zone II: deep cooling and formation of an atomic beam; zone III: deceleration and compression of the atomic beam, Bose condensation of atoms; zone IV: stimulated gamma emission from metastable states of isomers in the Bose condensate; zone V: collection of waste nuclei with an atomic collector. The simulation of stimulated VUV emission from the metastable $2\,^3S_1$ state of the atomic helium [3, 9] showed the possible logical consistency of operations in this chain.

The main physical mechanism controlling sequential operations with atoms is the kinematic transformation of the parameters of the atomic beam (longitudinal velocity, concentration, etc.) during its propagation along the two-dimensional quantum well (trap) with a varying transverse potential [10]. Such an experimental configuration can be treated in some sense as a quantum conveyer for atoms.

4. Quantitative estimates of the sequence of operations in a quantum conveyer

The course of events during the propagation of atoms along a quantum conveyer and quantitative estimates of the results of each of the successive operations are as follows.

Zone I. The isomeric nuclei $^{111}_{47}$Ag or $^{135}_{55}$Cs are prepared in zone I by irradiating gaseous isotopes of silver-110 or cesium-134 at concentrations $n_m = 10^{15}$ cm^{-3} by a flux of thermal neutrons of density 10^{13} cm^{-2} s^{-1} in radiative-neutron capture reactions $^{110}_{47}\mathrm{Ag}(n,\gamma)\,^{111}_{47}$Ag or $^{134}_{55}\mathrm{Cs}(n,\gamma)\,^{135}_{55}$Cs [cross sections $\sigma(n,\gamma)$ of the reactions are presented in Table 1] with rates 0.82×10^6 and 1.4×10^6 cm^{-3} s^{-1}, respectively. The neutron exposure time Δt_n is determined by the atom residence time in zone I, which is assumed small compared to the isomer lifetime τ ($\Delta t_n \ll \tau$), which allows the spontaneous decay of metastable states to be neglected. Then, the resulting concentrations n_I^* of the $^{111}_{47}$Ag and $^{135}_{55}$Cs isomers are 0.82×10^6 cm^{-3} for $\Delta t_n = 1$ s and 0.84×10^8 cm^{-3} for $\Delta t_n = 60$ s, respectively.

After the capture of a thermal neutron of energy 0.0253 eV, the nucleus acquires the momentum equal to 2.3×10^{-7} eV cm^{-1} s. If the neutron flux is directional, after the absorption of a neutron the nuclei acquire the anisotropic additional velocity V_n against the background of the isotropic root-mean-square thermal velocity $V(T) \approx 1.56 \times 10^4 \times (T/A)^{1/2}$ cm s^{-1}, which is directed along the

neutron flux and is equal to $2.2 \times 10^5 A^{-1}$ cm s^{-1}. At temperature $T = 300$ K, we have $V(T) \approx 2.5 \times 10^4$ cm s^{-1} and $V_n \approx 2 \times 10^3$ cm s^{-1} for $^{111}_{47}$Ar and $V(T) \approx 2.3 \times 10^4$ cm s^{-1} and $V_n \approx 1.6 \times 10^3$ cm s^{-1} for $^{135}_{55}$Cs.

Then, the gaseous mixture with the root-mean-square velocity of atoms $V(T)$ and temperature $T = 300$ K is admitted into zone II.

Zone II. In this zone, the parent isotope and its isomer differing in the mass of one neutron (0.9 % for silver and 0.7% for cesium) are separated by the laser method [11] (for example, by two-step selective photoionisation [12, 13]). The ionised parent isotopes are extracted from the zone by an external electric field to enrich the ensemble with atoms containing working isomers (for example, 100-% photoionisation of rubidium atoms and electric extraction of up to 10^{13} ions per pulse were achieved in [13]) and, finally, the deep laser cooling of the ensemble is performed down to temperature not exceeding the limiting value (6). Such a procedure was already proposed in [14, 15]. If these operations require the time that is noticeably shorter than τ, the spontaneous decay of isomers can be neglected.

If the gaseous atomic mixture is not strongly mixed during all these operations, i.e. the thermalisation of atomic ensembles has no time to occur, the isotropic thermal background $V(T)$ drastically decreases upon laser cooling, while the directional velocity V_n appearing after neutron capture remains constant, i.e. a directional flux of cooled atoms with the velocity V_n is formed. However, according to estimates, such a situation is unlikely. Therefore, to produce directional beams of cold atoms containing isomeric nuclei, more efficient laser methods for manipulation by neutral atoms are required (for example, a directional cooled flux of 10^9 sodium atoms per second with a transport velocity of 5×10^4 cm s^{-1} was obtained in experiment [16]).

Then, the directional beam of cold silver atoms with temperature $T = 3.5 \times 10^{-7}$ K, the isomer concentration $n_I^* = n^*(z_{III}) = 0.82 \times 10^6$ cm^{-3}, the relative isomer concentration $n^*(z_{III})/n(z_{III}) = 0.95$, the transport velocity $V_{III} = 4 \times 10^4$ cm s^{-1} and the corresponding energy $E_{III} \approx 9 \times 10^{-2}$ eV enters at the point z_{III} into zone III. In the case of cesium atoms, $T = 4.9 \times 10^{-8}$ K, $n_I^* = n^*(z_{III}) = 0.84 \times 10^8$ cm^{-3}, $n^*(z_{III})/n(z_{III}) = 0.95$, $V_{III} = 465$ cm s^{-1}, and $E_{III} \approx 1.5 \times 10^{-5}$ eV. In this case, the temperature of cooled atoms does not exceed the limit determined by inequality (6).

Zone III. The atomic beam is loaded into an extended trap with a potential well along the transverse coordinates x and y with the typical parabolic potential

$$U(x, y) = a\left(x^2 + y^2\right) \equiv a\rho^2 \tag{9}$$

[10] and free movement along the longitudinal coordinate z, the depth of the transverse potential well increasing with the coordinate z:

$$\frac{\mathrm{d}a(z)}{\mathrm{d}z} > 0\,, \quad \frac{\mathrm{d}U}{\mathrm{d}z} > 0\,. \tag{10}$$

The kinematics of the atomic beam in such a trap studied in [10] and simulated for the metastable atomic helium [3, 9] has properties that are important for the scenario under study. Among them, in particular, are the transformation of the kinetic energy of atoms during their propagation along the z axis and the increasing energy of the transverse quantum state (the total energy E of the atom being constant), which is accompanied by the slowing down of atoms and an increase in their concentration. The slowing down of atoms is characterised by a decrease in the modulus of the longitudinal component of their wave vector

$$|p_{km}| = [2M(E - E_{km})]^{1/2} \tag{11}$$

due to the increase in the energy eigenvalue

$$E_{km}(z) = \hbar\left(\frac{2a}{M}\right)^{1/2}(k + m + 1), \tag{12}$$

caused by the increase in the coefficient $\mathrm{d}a/\mathrm{d}z > 0$ (k and m are integers). The simultaneous increase in $E_{km}(z)$ reduces the effective area $S_{km}(z)$ of the cross section of the channel in which the atomic beam propagates. For example, the cross section for the lower quantum state with $k = m = 0$ decreases as

$$S_{00}(z) = \sqrt{2}\pi\hbar[a(z)M]^{-1/2} = 2\pi\hbar^2[ME_{00}(z)]^{-1}. \tag{13}$$

As a result, if we assume that the total atomic flux is invariable, the concentration of atoms increases during their propagation from z_1 to $z_2 > z_1$ with the compression coefficient [10]

$$\Xi_{km}(z_2, z_1) \equiv \frac{n(z_2)}{n(z_1)} = \left|\frac{p_{km}(z_1)}{p_{km}(z_2)}\right| \frac{S_{km}(z_1)}{S_{km}(z_2)} > 1. \tag{14}$$

For example, for the lowest state with $k = m = 0$, we have

$$\Xi_{00}(z_2, z_1) = \frac{E/E_{00}(z_1) - 1}{E/E_{00}(z_2) - 1} > 1. \tag{15}$$

Simultaneously, the velocity of atoms decreases according to (11) with the slowing down coefficient

$$\Xi_{00}^V(z_2, z_1) = \frac{V(z_2)}{V(z_1)} = \frac{1 - E_{00}(z_2)/E}{1 - E_{00}(z_1)/E} < 1 \tag{16}$$

and the effective cross section S_{00} of the channel decreases according to (13) with the transverse compression coefficient

$$\Xi_{00}^S(z_2, z_1) = \frac{S_{00}(z_2)}{S_{00}(z_1)} = \frac{E_{00}(z_1)/E}{E_{00}(z_2)/E} < 1, \tag{17}$$

so that

$$\Xi_{00}(z_2, z_1) = \left[\Xi_{00}^V(z_2, z_1)\Xi_{00}^S(z_2, z_1)\right]^{-1}. \tag{18}$$

Thus, if the ratio of the energy of the lowest quantum state to the total energy of the silver atom is increased from $E_{00}(z_{III})/E = 10^{-4}$ at the coordinate $z_1 = z_{III}$ to $E_{00}(z_{IV})/E = 0.99995$ at the coordinate $z_2 = z_{IV}$ in zone III, this means, according to (16), the slowing down of atoms with the coefficient $\Xi_{00}^V(z_{IV}, z_{III}) = 5 \times 10^{-5}$ from the

velocity $V(z_{III}) = 4 \times 10^4$ cm s^{-1} to $V(z_{IV}) = 2$ cm s^{-1} and the decrease, according to (17), in the effective cross section of the quantum channel with the coefficient $\Xi_{00}^S = 10^{14}$. As a result, the total concentration increases according to (18) with the coefficient $\Xi_{00} = 2 \times 10^8$ from $n(z_{III}) = n^*(z_{III})/0.95 = 0.9 \times 10^6$ cm^{-3} to $n(z_{IV}) = 1.8 \times 10^{14}$ cm^{-3}.

A similar increase in the ratio of the energy of the lowest quantum state to the total energy of the cesium atom from $E_{00}(z_{III})/E = 10^{-3}$ to $E_{00}(z_{IV})/E = 0.9934$ means the slowing down of atoms with the coefficient $\Xi_{00}^V(z_{IV}, z_{III}) = 6.6 \times 10^{-3}$ from the velocity $V(z_{III}) = 465$ cm s^{-1} to $V(z_{IV}) = 3$ cm s^{-1} and the decrease in the effective cross section of the quantum channel with the coefficient $\Xi_{00}^S = 10^{-3}$. This results in the increase in the total concentration of atoms with the coefficient $\Xi_{00} = 1.5 \times 10^5$ (18) from $n(z_{III}) = n^*(z_{III})/0.95 = 0.88 \times 10^8$ cm^{-3} to $n(z_{IV}) = 1.3 \times 10^{13}$ cm^{-3}. In this case, both the concentration $n(z_{IV})$ and temperature T of both isomers do not exceed the limits defined by inequalities (5) and (6).

The above-described evolution of the atomic beam in zone III continues until the high concentration of atoms is achieved at the point $z_2 = z_{IV} = z_{BEC}$ where the atoms form a Bose condensate.

It is assumed usually that such a phase transition is caused by a decrease in the temperature T of the atomic ensemble below the critical degeneracy temperature T_0. Here, on the contrary, the degeneracy temperature increases up to $T_0 > T$, while T does not change. This increase in the critical temperature

$$T_0 = 3.3 \frac{\hbar^2 n^{2/3}}{(2J_a + 1)^{2/3} k_B M} \approx 1.6 \times 10^{-14} \frac{n^{2/3}}{A} \tag{19}$$

occurs due to the increase in the atomic concentration n according to (14).

For silver and cesium with concentrations $n(z_{IV}) = 1.8 \times 10^{14}$ and 1.3×10^{13} cm^{-3} obtained above for $n(z_{IV}) = n(z_{BEC})$, this gives $T_0 = 4.6 \times 10^{-7}$ and 6.5×10^{-8} K, respectively, which exceeds the values $T = 3.5 \times 10^{-7}$ and 4.9×10^{-8} K adopted previously by a factor of $T_0/T = 1.3$ (6).

It is important to note that expression (19) and subsequent estimates of the Bose condensation process were made for free atoms with a continuous spectrum. However, in the case under study the atoms are in a potential well with discrete states. It is assumed, however, that this causes no principal differences, introducing only some quantitative corrections that should be analysed.

The condensation of atoms and the formation of a BEC is the final event in zone III, after which a mixture of atoms with isomeric and other nuclei, both in the Bose condensate and in a usual gas, is directed to zone IV. If the time of all the previous operations is small compared to τ, a decrease in the concentration of isomers caused by their spontaneous decay can be neglected and the ratio $n^*/n = 0.95$ adopted earlier in zone III can be assumed constant. Then, the ratio of the concentration $n_{BEC}^*(z_{BEC})$ of isomers in the condensate to the total concentration $n(z_{BEC})$ of the gas at the input to zone IV is

$$\frac{n_{BEC}^*(z_{BEC})}{n(z_{BEC})} = 0.95\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right]. \tag{20}$$

For example, for $T_0/T = 1.3$ [see (6)], we have the ratio $n_{bec}^*(z_{BEC})/\ n(z_{BEC}) \approx 0.315$.

Zone IV. A beam of the atomic mixture with the concentration ratio (20) propagates in zone IV in a quantum trap with a constant depth of the potential well along the z axis and a constant velocity, which are equal to the values at which atoms formed a Bose condensate at the output from zone III. This atomic mixture containing isomers and other nuclei is the active medium for amplification of gamma rays, which has the following properties.

It is assumed that the fulfilment of conditions (5)–(7) in zone IV means that the excess broadening of the gamma line is completely eliminated and the stimulated emission cross section is equal to its asymptotic value $\sigma = \lambda^2/2\pi$.

The resonance amplification frequency for isomers moving at the transport velocity $V(z_{BEC})$ proves to be shifted with respect to the nuclear transition energy due to the Doppler effect [3].

In addition, amplification occurs under the condition of hidden inversion when the mutual shift of the gamma-ray emission and absorption lines by the doubled recoil energy of the nucleus excludes the resonance absorption of photons by unexcited nuclei, which is manifested in the absence of the term corresponding to their concentration in the rate equation for photons.

Then, according to expression (8) from [9], the asymptotic behaviour of the current value of the stimulated emission cross section (laser lethargy [3–6])

$$\sigma(t) = \sigma\left[1 - \exp\left(\frac{\alpha t}{\tau}\right)\right] \tag{21}$$

is taken into account, which increases from zero up to $\sigma(t \to \infty) = \sigma$ during the penetration of nuclei moving at the velocity $V(z_{IV})$ to the region of their interaction with the photon field of zone IV (the coefficient $\alpha = \text{const}$).

Finally, by using expressions (12), (14), (15), (17)–(20) from [9] with omitted terms containing the cross section σ_{ph} for photoelectric absorption by metastable helium atoms, which are absent in the case of isomers under study, we can obtain the basic expressions describing processes in an isomeric amplifying medium. The requirement that the maximum value of the gain achieved at the coordinate

$$z_0 = z_{BEC} + \frac{V(z_{BEC})\tau}{\alpha} \ln(1 + \alpha) \tag{22}$$

should be positive determines the critical concentration of isomers at the input to zone IV below which even local amplification is impossible:

$$\left.\frac{n_{BEC}^*(z_{BEC})}{n(z_{BEC})}\right|_{crit} = \frac{\chi}{\sigma\alpha}(1 + \alpha)^{1/\alpha + 1}. \tag{23}$$

For silver and cesium for $\alpha = 1$, we have

$$\left.\frac{n_{BEC}^*(z_{BEC})}{n(z_{BEC})}\right|_{crit} \approx 6 \times 10^{-3}$$

and

$$\left.\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right|_{\rm crit} \approx 2\times 10^{-2},$$

respectively.

The asymptotic behaviour of the current value of the stimulated emission cross section (21) causes the displacement of the coordinate of the amplification onset to $z_3 > z_{\rm BEC}$ (laser lethargy), while the decrease in the concentration of isomers due to their spontaneous decay specifies the coordinate $z_4 > z_0 > z_3 > z_{\rm BEC}$, where amplification ceases. The characteristic points $z_{3,4}$ are determined as the roots of the equation

$$\frac{1}{\alpha}\exp\left(-\frac{z_{3,4}-z_0}{V(z_{\rm BEC})\tau}\right)\left[\alpha+1-\exp\left(-\alpha\,\frac{z_{3,4}-z_0}{V(z_{\rm BEC})\tau}\right)\right]$$

$$=\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]^{-1}\left[\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right]_{\rm crit}. \qquad (24)$$

It follows from this that it is reasonable to restrict the length L of the interaction region of isomers with the field in zone IV by the inequalities

$$z_3 \leqslant L + z_{\rm BEC} \leqslant z. \qquad (25)$$

The total gain G over the entire length of the interaction region in zone IV is determined from the expression

$$\ln G = \sigma n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\tau\left[1-\exp\left(-\frac{L}{V(z_{\rm BEC})\tau}\right)\right]$$

$$-\frac{\sigma n^*_{\rm BEC}(z_{\rm BEC})V(z_{\rm BEC})\tau}{1+\alpha}$$

$$\times\left[1-\exp\left(-\frac{1+\alpha}{V(z_{\rm BEC})\tau}\,L\right)\right]-\chi n(z_{\rm BEC})L. \qquad (26)$$

The requirement of the one-pass amplification with $G \geqslant 1$ determined the threshold relative concentration of the isomer at the input to zone IV:

$$\left.\frac{n^*_{\rm BEC}(z_{\rm BEC})}{n(z_{\rm BEC})}\right|_{\rm crit} \geqslant \frac{\chi L}{\sigma V(z_{\rm BEC})\tau}\left\{\left[1-\exp\left(-\frac{L}{V(z_{\rm BEC})\tau}\right)\right]\right.$$

$$\left.-\frac{1}{1+\alpha}\left[1-\exp\left(-\frac{1+\alpha}{V(z_{\rm BEC})\tau}\,L\right)\right]\right\}^{-1}. \qquad (27)$$

In this case, the ratio $L/V(z_{\rm BEC})\tau > 1$ satisfying inequality (7) is found from the condition that the total gain achieves its maximum $G_{\max}$ for $L = L_{\max}$. The value of $L_{\max}$ is determined as the largest root of the equation

$$\left[1-\exp\left(-\frac{\alpha L_{\max}}{V(z_{\rm BEC})\tau}\right)\right]\exp\left(-\frac{L_{\max}}{V(z_{\rm BEC})\tau}\right)$$

$$=\frac{\chi\, n(z_{\rm BEC})}{\sigma\, n^*_{\rm BEC}(z_{\rm BEC})}, \qquad (28)$$

which coincides numerically with the second characteristic coordinate (24) $L_{\max} = z_4 - z_{\rm BEC}$.

For silver and cesium for $n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC}) = 0.315$ (20), we have $L_{\max}/V(z_{\rm BEC})\tau = 5.3$ and 4.1, i.e. $L_{\max} = 6.48\times 10^2$ and 3.9×10^4, respectively. For these values of $L_{\max}[V(z_{\rm BEC})\tau]^{-1}$ satisfying inequality (7), the relative threshold concentrations (27) of the isomer $[n^*_{\rm BEC}(z_{\rm BEC})/n(z_{\rm BEC})]_{\rm thr} = 0.016$ and 0.0425 for silver and cesium, respectively, prove to be considerably lower than estimate (20).

Thus, for $n(z_{\rm BEC}) = 1.8\times 10^{14}$ cm^{-3}, the maximum total gain (26) for silver is $G_{\max} = 1.0023$ and for $n(z_{\rm BEC}) = 1.3\times 10^{13}$ cm^{-3}, $G_{\max} = 1.0013$ for cesium.

Zone V. Here, waste atoms are removed into a collector.

5. Conclusions

Thus, the answer to the question raised in Introduction about the possibility of amplification of gamma-ray flux due to stimulated emission from metastable states of isomeric nuclei proves to be positive despite the counteraction of strong negative factors and regardless of apprehension that these factors can result in the absolute prohibition of amplification. The obtained estimates of the total gain G only slightly exceed unity, in accordance with the value of the artificially introduced test parameter $G_{\rm A}$ (8) determining the upper bound of the maximum achievable G.

The considered experimental configuration based on the so-called quantum conveyer for atoms [3] solves in a natural way the problem of atomic beam compression, which is the key problem both for achieving the superthreshold concentration of isomers and for overcoming the critical condition of the formation of the atomic Bose condensate without a rapid decrease in the gas temperature.

It should be emphasised once more that the question of the fundamental value of the lifetime $\Theta_{\rm BEC}$ of atoms in a Bose condensate still remains open and requires further theoretical and experimental studies. However, the answer to this question is especially important both for the problem of stimulated gamma-ray emission of isometric nuclei and for solving the problem of observation of extremely narrow gamma lines with the natural width emitted from long-lived metastable nuclear states.

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