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Amplification of ultrashort laser pulses upon stimulated Compton scattering in plasma

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***Abstract*. Stimulated Compton scattering of counterpropagating laser beams in a moving plasma is studied theoretically. It is shown that, by using Compton scattering, picosecond or femtosecond laser pulses can be amplified under certain conditions by two-three orders of magnitude.**

Keywords: *laser radiation, stimulated Compton scattering, amplification.*

1. Introduction

Stimulated Compton scattering of two arbitrarily oriented photon beams with frequencies ω_1 and ω_2 in a moving plasma was considered for the first time in theoretical paper [1].

In this paper, we obtained equations describing the propagation of counterpropagating beams in a moving plasma and solved them in different approximations. It is shown that under certain conditions, picosecond or femtosecond laser pulses can be amplified by two-three orders of magnitude. The results are compared with experimental data [2] in which the amplification of one of the laser pulses was observed during stimulated Compton scattering of counterpropagating laser beams in a moving plasma. This effect can considerably increase the reflection of intense laser radiation from the counterpropagating laser plasma [3].

Stimulated Compton scattering was earlier considered as one of the possible mechanisms of laser plasma heating [4 – 9] (see also review [10] and monograph [11]). The first experimental observations of absorption of laser radiation upon stimulated Compton scattering in a laser plasma were reported in [12 – 14].

2. General relations

We will describe the interaction of laser radiation with a free electron gas caused by stimulated Compton scattering by using the kinetic equation for photons, which is invariant with respect to the choice of the coordinate system [1, 15]:

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$$\frac{\mathrm{d}N}{\mathrm{d}t} = -c\iint\left[N(1+N')f(\boldsymbol{p})\right]\mathrm{d}\sigma\mathrm{d}\boldsymbol{p}$$
$$+c\iint\left[N'(1+N)f(\boldsymbol{p}')\right]\mathrm{d}\sigma\mathrm{d}\boldsymbol{p}. \qquad (1)$$

Here, $f(\boldsymbol{p})$ is the distribution function of electrons over the vectors of their momenta

$$\boldsymbol{p} = m\boldsymbol{v}\left(1-\frac{v^2}{c^2}\right)^{-1/2};$$

m is the electron mass; $\boldsymbol{v}$ is the electron velocity; $\boldsymbol{p}'$ is the electron momentum after its interaction with a photon; c is the speed of light; $N \equiv N(\omega, \boldsymbol{q})$ is the distribution function of photons over frequencies ω with the unit wave vectors $\boldsymbol{q} = \boldsymbol{k}/k$; $k = \omega/c$ is the wave number; $N' \equiv N(\omega', \boldsymbol{q}')$; $\mathrm{d}\sigma = (1 - \boldsymbol{vq}/c)\mathrm{d}\sigma_{\mathrm{sp}}$ ($\mathrm{d}\sigma_{\mathrm{sp}}$ is the differential cross section of spontaneous Compton scattering, and the factor in parentheses describes a change in the photon flux with respect to a moving electron [16]).

The first term in the right-hand side of expression (1) determines the number of photons with frequency ω that decrease upon scattering from an electron with the momentum $\boldsymbol{p}$, while the second term determines the number of photons coming to this state.

The relation of N and N' with the spectral densities J and J' of the energy flux of nonpolarised radiations to unit solid angles with axes directed along the unit vectors $\boldsymbol{q}$ and $\boldsymbol{q}'$, respectively, is described by the expressions

$$N_{\boldsymbol{k}} = \frac{4\pi^3c^2}{\hbar\omega^3}J_{\boldsymbol{k}}, \quad N'_{\boldsymbol{k}} = \frac{4\pi^3c^2}{\hbar\omega'^3}J'_{\boldsymbol{k}}. \qquad (2)$$

In particular, if the spectral densities $J_{\boldsymbol{k}}$ and $J'_{\boldsymbol{k}}$ are constant in the frequency intervals $\Delta\omega$ and $\Delta\omega'$ and within the elements of the solid angle $\Delta\Omega$, $\Delta\Omega' \ll 1$ (and are zero outside), we have

$$N_{\boldsymbol{k}} = \frac{4\pi^3c^2}{\hbar\omega^3}\frac{I}{\Delta\omega\Delta\Omega}, \quad N'_{\boldsymbol{k}} = \frac{4\pi^3c^3}{\hbar\omega'^3}\frac{I'}{\Delta\omega'\Delta\Omega'}, \qquad (3)$$

where I and I' are the total (integrated) intensities of the corresponding light fluxes.

The differential cross section for spontaneous Compton scattering $\mathrm{d}\sigma_{\mathrm{sp}}$ is described by the Klein – Nishina – Tamm formula, which has the form [16]

$$\mathrm{d}\sigma_{\mathrm{sp}} = \sigma_0\mathrm{d}q' \qquad (4)$$

in the laboratory coordinate system, where

$$\sigma_0 = 2r_0^2\left(\frac{\hbar\omega'}{mc^2}\right)^2\frac{u_0}{\chi^2}; \qquad (5)$$

$$u_0 = 4\left(\frac{1}{\chi}+\frac{1}{\chi'}\right)^2 - 4\left(\frac{1}{\chi}+\frac{1}{\chi'}\right) - \left(\frac{\chi}{\chi'}+\frac{\chi'}{\chi}\right);$$

$$\chi = \frac{2pk}{(mc^2)^2}; \qquad \chi' = -\frac{2pk'}{(mc^2)^2};$$

$r_0 = e^2/(mc^2) = 2.8\times10^{-13}$ cm is the classical radius of an electron. Parentheses in the numerators of expressions for χ and χ' denote scalar products of the electron four-momentum

$$\left\{\boldsymbol{p};\mathrm{i}\frac{\varepsilon}{c}\right\}$$

and the four-momenta of the incident and scattering photons

$$\left\{\hbar\boldsymbol{k};\mathrm{i}\frac{\hbar\omega}{c}\right\} \text{ and } \left\{\hbar\boldsymbol{k}';\mathrm{i}\frac{\hbar\omega'}{c}\right\},$$

respectively. According to the laws of conservation of energy and momentum, the relation between the frequencies ω and ω' of the incident and scattered photons has the form

$$\omega\left(1-\frac{v}{c}\cos\theta\right) = \omega'\left[\left(1-\frac{v}{c}\cos\theta'\right)+\frac{\hbar\omega}{mc^2}(1-\cos\vartheta)\right]. \quad (6)$$

Here, θ and θ' are the angles between the electron momentum vector $\boldsymbol{p}$ and the wave vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$ of the incident and scattered photons, respectively; and ϑ is the angle between $\boldsymbol{k}$ and $\boldsymbol{k}'$ (Fig. 1).

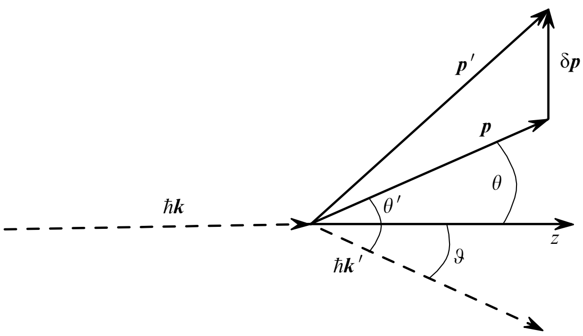

Figure 1. Scheme of stimulated Compton scattering.

By substituting expressions (2) for $N(\omega,\boldsymbol{q})$ and $N(\omega',\boldsymbol{q}')$ into (1) and taking into account only the stimulated effect, we obtain

$$\frac{1}{c}\frac{\partial J}{\partial t}+\boldsymbol{q}\frac{\partial J}{\partial \boldsymbol{r}} = \iint\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}JJ'\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}\right\}\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{q}'. \quad (7)$$

Here, $J\equiv J(\omega,\boldsymbol{q})$; $J'\equiv J(\omega',\boldsymbol{q}')$; $\tilde{\sigma}=(1-\boldsymbol{vq}/c)\sigma_0$. Consider now two counterpropagating plane electromagnetic waves along the z axis under stationary conditions ($\partial J/\partial t = 0$) (Fig. 2)

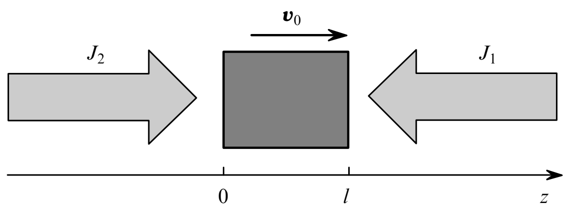

Figure 2. Scheme of the interaction of laser beams with a moving plasma. (l) is the plasma length.

$$J = J_1(z,\omega)\frac{1}{2\pi}\delta(1+\cos\theta)+J_2(z,\omega)\frac{1}{2\pi}\delta(1-\cos\theta). \quad (8)$$

Here, $\delta(\theta)$ is the Dirac delta function. In this case, only the components of the electron momentum directed along the z axis change, so that we will use below the one-dimensional distribution function of electrons over momenta $f(\boldsymbol{p})$ as the function $f(p_z)$ in (7). The form of this function is presented below.

After substituting (8) into (7) and integrating over $\mathrm{d}q'$, Eqn (7) decomposes into the system of two equations

$$\frac{\partial J_1(z,\omega)}{\partial z} = -\int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1J_1'\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{1\to1'}\right\}\mathrm{d}\boldsymbol{p} - \int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1J_2'\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{1\to2'}\right\}\mathrm{d}\boldsymbol{p},$$

$$\frac{\partial J_2(z,\omega)}{\partial z} = \int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1'J_2\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{2\to1'}\right\}\mathrm{d}\boldsymbol{p} + \int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1J_2'\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{2\to2'}\right\}\mathrm{d}\boldsymbol{p}. \quad (9)$$

The first and fourth integrals in (9) correspond to scattering within each of the beams. The second integral corresponds to the scattering of photons from the first beam to the second one, and the third integral describes scattering of photons from the second beam to the first one. Subscripts 1 and 2 refer to the states of incident photons of the corresponding beams, the same primed subscripts denote the states of photons scattered with a change in the frequency. According to (6), $\omega=\omega'$ for the $1\to1'$ and $2\to2'$ transitions and, hence, $p'=p$. As a result, the first and fourth integrals in (9) are identically zero. Therefore, we obtain finally the system of two equations

$$\frac{\partial J_1(z,\omega)}{\partial z} = -\int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1J_2'\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{1\to2'}\right\}\mathrm{d}\boldsymbol{p},$$

$$\frac{\partial J_2(z,\omega)}{\partial z} = \int\left\{\frac{4\pi^3c^2}{\hbar\omega'^3}J_1'J_2\left[f(\boldsymbol{p}')-f(\boldsymbol{p})\right]\tilde{\sigma}_{2\to1'}\right\}\mathrm{d}\boldsymbol{p}. \quad (10)$$

In the case of nonrelativistic electrons ($v\ll c$) and soft photons ($\hbar\omega\ll mc^2$), taking into account relations (4)–(6), the parameter $\tilde{\sigma}$ and the scattered photon frequency ω' can be written in the form

$$\tilde{\sigma}\simeq r_0^2\left(1\pm3\frac{v}{c}-4\frac{\hbar\omega}{mc^2}\right), \quad (11)$$

$$\omega' = \omega+\omega\left(\pm2\frac{v}{c}-2\frac{\hbar\omega}{mc^2}\right). \quad (12)$$

The expression for the momentum of an electron after its interaction with a photon has the form

$$\boldsymbol{p}' = \boldsymbol{p} + \hbar\boldsymbol{k} - \hbar\boldsymbol{k}' = \boldsymbol{p} + \frac{\hbar}{c}(\omega\boldsymbol{q} - \omega'\boldsymbol{q}'). \tag{13}$$

In the scattering geometry considered here, the unit vector $\boldsymbol{q}$ is directed along the z axis. Taking into account (12), we obtain from (13) the change in the electron momentum

$$\delta p \simeq \mp 2\frac{\hbar\omega}{c}\left(1 \pm \frac{v_z}{c} + \frac{\hbar\omega}{mc^2}\right). \tag{14}$$

The upper signs in expressions (11)–(14) correspond to the $(1 \to 2')$ photon scattering from the first beam to the second one, and the lower signs – to the $(2 \to 1')$ photon scattering from the second to first beam.

3. Stimulated Compton scattering in a plasma moving toward one of the laser beams

Because in the case of scattering of counterpropagating laser beams, only the projection of the electron momentum on the z axis changes, we will use below the one-dimensional distribution function of electrons over the momentum projections on the z axis ($p_z \equiv p$) as the function $f(\boldsymbol{p})$ in (10).

It was pointed out in [1] that under certain conditions one of the laser beams experiencing stimulated Compton scattering in a moving plasma can be amplified in principle. In this connection we will represent the distribution function $f(p)$ of electrons over their momenta in the form

$$f(p_z) = \frac{n_e}{m}\left(\frac{m}{2\pi kT_e}\right)^{1/2} \exp\left[-\frac{(p-p_0)^2}{2mkT_e}\right], \tag{15}$$

which takes into account the movement of a plasma layer as a whole at the velocity $v_0 = p_0/m$ along the z axis (see Fig. 2). Here, m is the electron mass; n_e is the electron concentration in the plasma; T_e is the electron temperature; and k is the Boltzmann constant.

Let us assume that the intensity of one of the laser beams, for example, of the first one greatly exceeds that of the second laser beam, i.e., $I_1^0 \equiv I_1(z=l) \gg I_2^0 \equiv I_2(z=0)$. Then, the influence of the second beam on the first one during Compton scattering is insignificant and, hence, the amplitude and spectrum of the function $J_1(z,\omega)$ remain invariable:

$$J_1(z,\omega) = I_1^0 f_1^0(\omega - \omega_0). \tag{16}$$

Here, ω_0 is the carrier frequency of laser radiation and $I_1^0 \equiv I_1(z=l)$ is its integrated intensity at the input into a plasma layer of thickness l. For definiteness, we will take the function $f(\omega-\omega_0)$ in the form

$$f(\omega-\omega_0) = \sqrt{\frac{2}{\pi}}\frac{1}{\Delta\omega_1}\exp\left[-2\left(\frac{\omega-\omega_0}{\Delta\omega_1}\right)^2\right], \tag{17}$$

where $\Delta\omega$ is the width of the spectral function at the 0.6 level.

Taking expressions (11), (12), (14), and (17) into account, the second equation in (10) after integration takes the form

$$\frac{\partial J_2(z,\omega)}{\partial z} = A(v_0, \Delta\omega_1, kT_e) I_1^0 J_2(z,\omega). \tag{18}$$

Figures 3–7 present the dimensionless coefficient $A(v_0, \Delta\omega_1, kT_e)/A_0$ calculated for different values of its parameters. Here, A_0 is the value of A for $kT_e = 100$ eV and $\Delta\omega_0 = 2\omega_0 w_0/c$, where $w_0 = (kT_e/m)^{1/2} = 4.2 \times 10^8$ cm s^{-1} and $\omega_0 = 1.78 \times 10^{15}$ s^{-1}.

Figure 3 shows the dependence of the coefficient A/A_0 on the relative plasma velocity v_0/w_0. It follows from our calculations that for $kT_e = 100$ eV, the maximum value $A/A_0 = 0.792$ is achieved for $v_0/w_0 = 1.12$. For $v_0/w_0 = 0$, the coefficient A/A_0 is -0.013 and for $v_0/w_0 = 0.011$, it is zero.

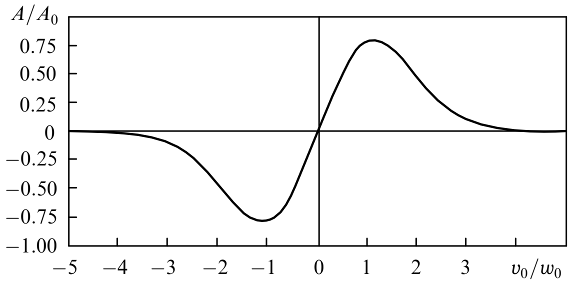

Figure 3. Dependence of the coefficient A/A_0 on the relative plasma velocity v_0/w_0 for $\omega = \omega_0$, $\Delta\omega_1/\Delta\omega_0 = 1$, and $kT_e = 100$ eV.

Figure 4 presents the dependence of the coefficient A/A_0 on the relative width $\Delta\omega_1/\Delta\omega_0$ of the spectrum of the first laser beam for $v_0 = w_0$, $\omega = \omega_0$, and $kT_e = 100$ eV. One can see that the maximum value $A/A_0 = 1$ is achieved for $\Delta\omega_1 = 0$.

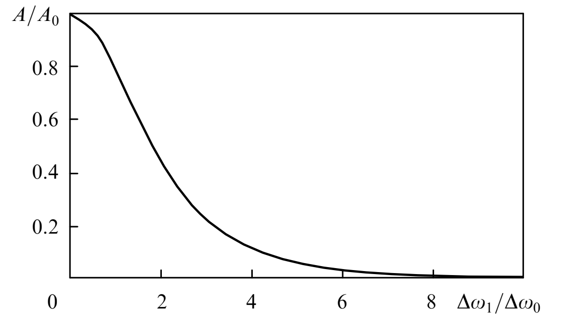

Figure 4. Dependence of the coefficient A/A_0 on the relative width $\Delta\omega_1/\Delta\omega_0$ of the spectrum of the first laser beam for $v_0 = w_0$, $\omega = \omega_0$, and $kT_e = 100$ eV.

The dependence of the coefficient A/A_0 on the laser frequency $(\omega-\omega_0)/\Delta\omega_0$ for $\Delta\omega_1 = \Delta\omega_0$ and $kT_e = 100$ eV shows that A/A_0 achieves the maximum value equal to 0.795 for $(\omega-\omega_0)/\Delta\omega_0 = -0.15$ (Fig. 5). This means that the spectral components of the second laser beam located in the low-frequency spectral region experience the maximum amplification. For $\Delta\omega_1 \to 0$, the coefficient $A/A_0 \to 1$ and the shift of the gain maximum tends to zero.

The dependence of the coefficient A/A_0 on the plasma temperature for $v_0/w_0 = 1$ and $\Delta\omega_1/\Delta\omega_0 = 0.05$ is shown in Fig. 6. One can see that as the plasma temperature is decreased from 100 to 1 eV, the coefficient A/A_0 increases almost by an order of magnitude.

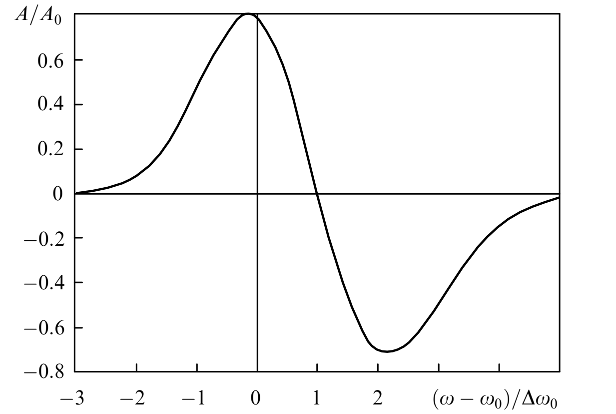

Figure 5. Dependence of the coefficient A/A_0 on the relative frequency $(\omega-\omega_0)/\Delta\omega_0$ of laser radiation for $\Delta\omega_1=\Delta\omega_0$ and $kT_e=100$ eV.

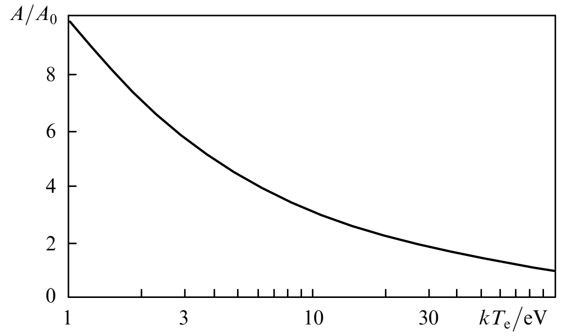

Figure 6. Dependence of the coefficient A/A_0 on the plasma temperature for $v_0/w_0=1$ and $\Delta\omega_1/\Delta\omega_0=0.05$.

Figure 7 presents the dependence of the coefficient A/A_0 on the relative width $\Delta\omega_1/\Delta\omega_0$ of the spectrum of the first laser beam for $v_0=0$, $\omega=\omega_0$, and $kT_e=100$ eV. This case is interesting from the point of view of using stimulated Compton scattering for plasma heating. The coefficient A/A_0 achieves the value -0.027 when the relative width $\Delta\omega_1/\Delta\omega_0$ of the spectrum of the first laser beam is 2.8.

If the condition

$$\frac{\Delta\omega}{2\omega_0}\ll\frac{w_0}{c} \tag{19}$$

is fulfilled, where $w_0=(kT_e/m)^{1/2}$, the function $f(p')-f(p)$ in the vicinity of $p=0$ can be represented in the form

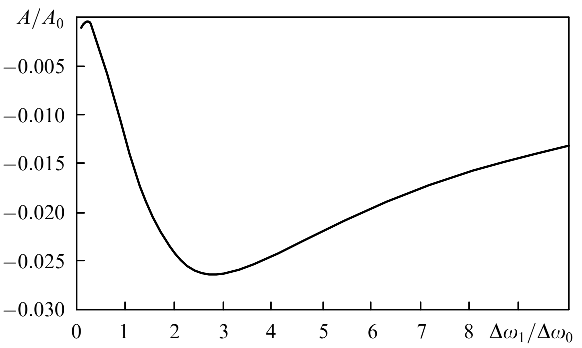

Figure 7. Dependence of the coefficient A/A_0 on the relative width $\Delta\omega_1/\Delta\omega_0$ of the spectrum of the first laser beam for $v_0=0$, $\omega=\omega_0$, and $kT_e=100$ eV.

$$f(p')-f(p)=\left.\frac{\partial f}{\partial p_z}\right|_{p=0}\delta p=\frac{n_e}{\sqrt{\pi}m^2kT_e}\left(\frac{p_0^2}{2mkT_e}\right)^{1/2}$$

$$\times\exp\left(-\frac{p_0^2}{2mkT_e}\right)\delta p, \tag{20}$$

where δp is the change in the electron momentum after scattering, which is described by expression (14).

Condition (19) determines the plasma electrons whose momenta can provide stimulated Compton scattering [the frequency of a photon scattered by an electron with the momentum $\Delta\omega$ should lie within the spectral band p of laser radiation (grey region in Fig. 8)].

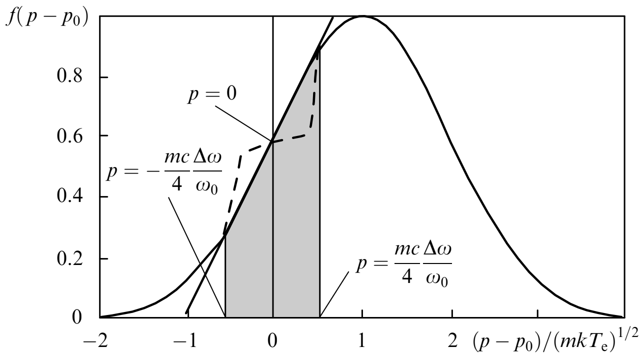

Figure 8. Normalised electron distribution function (solid curve). The solid straight line is the tangent to the distribution function at the point $p=0$, the dashed line is the result of possible saturation of the electron distribution function at a high intensity of laser radiation.

Let us assume, as before, that the intensity of one of the laser beams, for example, of the first one greatly exceeds that of the second laser beam, i.e., $I_1^0=I_1(z=l)\gg I_2^0=I_2(z=0)$. Then, the influence of the second beam on the first one during stimulated Compton scattering is insignificant and the spectrum of the function $J_1(z,\omega)$ remains invariable. In this case, the function $J_1(z,\omega)$ can be represented, as before, in the form (17). Then, taking (11), (12), (14), (16), and (17) into account, the second equation in (10) after integration in the limits determined by condition (19) can be written in the form

$$\frac{\partial J_2(z,\omega)}{\partial z}=A_0I_1^0\left(1-2\frac{\omega-\omega_0}{\omega_0}\right)J_2(z,\omega), \tag{21}$$

where

$$A_0=\frac{4\pi^3}{\sqrt{\pi}}\frac{c^2r_0^2n_e}{\omega_0^2kT_e}\left(\frac{mv_0^2}{2kT_e}\right)^{1/2}\exp\left(-\frac{mv_0^2}{2kT_e}\right). \tag{22}$$

The solution of Eqn (21) is

$$J_2(z,\omega)=J_2^0(\omega)\exp(\alpha_1z). \tag{23}$$

Expression (23) shows that in this case a weaker light pulse is amplified exponentially with the gain

$$\alpha_1(\omega)=A_0I_1^0\left(1-2\frac{\omega-\omega_0}{\omega_0}\right)=\alpha_1^0\left(1-2\frac{\omega-\omega_0}{\omega_0}\right),$$

$$\alpha_1^0=A_0I_1^0. \tag{24}$$

This result is independent of the specific form of the function $f_1(\omega)$ and its spectral width if the function is even and tends to zero for

$$|\omega - \omega_0| \geqslant \Delta\omega_{\max} = 2\omega_0 \frac{w_0}{c} = 2\omega_0 \left(\frac{kT_{\rm e}}{mc^2}\right)^{1/2}. \quad (25)$$

This means that the second laser beam with a broad spectrum restricted by condition (25) can be amplified in the interaction with a laser beam with a narrow spectrum. In other words, upon stimulated Compton scattering in counterpropagating beams, short laser pulses can be amplified with the help of laser pulses ('pump' pulses) of much longer duration. In this case, the gain is virtually independent of the 'pump' pulse duration.

When the spectrum of the function $J_2^0(\omega)$ is Gaussian,

$$J_2^0(\omega) = I_2^0 \exp\left[-2\left(\frac{\omega - \omega_0}{\Delta\omega_2}\right)^2\right], \quad (26)$$

we can show that

$$I_2^0(z,\omega) = I_2^0 \exp\left\{-2\left[\frac{\omega - \omega_0 + \alpha_0 z(\Delta\omega_2)^2/(2\omega_0)}{\Delta\omega_2}\right]^2\right\}$$
$$\times \exp\left\{\alpha_0 z\left[1 - \alpha_0 z\left(\frac{\Delta\omega_2}{2\omega_0}\right)^2\right]\right\}. \quad (27)$$

One can see from (27) that during amplification the spectrum shifts to the red by

$$\frac{\delta\omega}{\Delta\omega_2} = \frac{\Delta\omega_2}{2\omega_0}\alpha_0 z,$$

without changing its shape. This means that the temporal shape of the laser pulse does not change during amplification. In practically important cases, the spectral shift $\delta\omega$ proves to be much smaller than its spectral width $\Delta\omega_2$, i.e., we can assume that the spectra of the interacting laser beams do not change during stimulated scattering. Therefore, we can assume that

$$J_1(z,\omega) = \frac{I_1(z)}{\Delta\omega_1^{\rm eff}}, \quad J_2(z,\omega) = \frac{I_2(z)}{\Delta\omega_2^{\rm eff}}, \quad (28)$$

where $I_1(z)$ and $I_2(z)$ are the integrated intensities of the first and second laser beams; and $\Delta\omega_1^{\rm eff}$ and $\Delta\omega_2^{\rm eff}$ are the effective spectrum widths of the corresponding laser beams determined from the integral relations

$$I_1(z) = \int_{-\infty}^{\infty} J_1(z) f_1^0(\omega - \omega_0)\,{\rm d}\omega$$

and

$$I_2(z) = \int_{-\infty}^{\infty} J_2(z) f_2^0(\omega - \omega_0)\,{\rm d}\omega.$$

Then, taking expressions (11), (12), (14), (15), and (17) into account, we can represent Eqns (10) in the form

$$\frac{{\rm d}I_1(z)}{{\rm d}z} = A_0 I_1(z) I_2(z),$$
$$\frac{{\rm d}I_2(z)}{{\rm d}z} = A_0 I_1(z) I_2(z). \quad (29)$$

Note here that the form (29) of beams I_1 and I_2 is determined by their propagation in opposite directions. The system of equations (29) and expression for A_0 (without derivation) were presented earlier in [2].

Let us make some remark. If all plasma electrons are involved in scattering, the condition

$$\frac{\Delta\omega}{\omega_0} \geqslant \frac{2v}{c} = \left(\frac{kT_{\rm e}}{mc^2}\right)^{1/2}$$

is fulfilled, which is opposite to condition (19). In this case, only the two first terms can be retained in expansions of functions $J'_1(\omega')$, $J'_2(\omega')$ in a Taylor series:

$$J'_1(\omega') = J_1(\omega) + \frac{\delta\omega}{1!}\frac{\partial J_1(\omega)}{\partial\omega} + \frac{(\delta\omega)^2}{2!}\frac{\partial J_1(\omega)}{\partial\omega^2} + \ldots,$$

$$J'_2(\omega') = J_2(\omega) + \frac{\delta\omega}{1!}\frac{\partial J_2(\omega)}{\partial\omega} + \frac{(\delta\omega)^2}{2!}\frac{\partial J_2(\omega)}{\partial\omega^2} + \ldots.$$

For $v_0 = 0$, Eqns (9) can be written after integration in the form

$$\frac{\partial J_1(z,\omega)}{\partial z} = -n_{\rm e}\frac{16\pi^3 r_0^2}{m\omega}\frac{\partial J_2(z,\omega)}{\partial\omega} J_1(z,\omega),$$

$$\frac{\partial J_2(z,\omega)}{\partial z} = n_{\rm e}\frac{16\pi^3 r_0^2}{m\omega}\frac{\partial J_1(z,\omega)}{\partial\omega} J_2(z,\omega).$$

These equations were first presented and analysed in [10] (see also [11]). It was shown, in particular, that in this case the intensities of both beams in the first-order approximation at the output of a plasma layer decrease, i.e., the beams are absorbed in plasma, which coincides with the results obtained here (see Fig. 7).

4. Solution of Eqns (29) for an arbitrary relation between the intensities of interacting beams at the input to a plasma layer

The integral relation between $I_1(z)$ and $I_2(z)$ can be obtained by dividing the first equation by the second one and integrating the relation obtained:

$$I_1(z) = I_2(z) + c_1, \quad (30)$$

where c_1 is the integration constant.

By using relation (30), we can express $I_1(z)$ in the second equation in (29) in terms of $I_2(z)$ and then integrate this equation to obtain

$$I_2(z) = \frac{c_1}{\exp[-(A_0 z c_1 + c_1 c_2)] - 1}. \quad (31)$$

Let us introduce a new constant $c_0 = \exp(-c_1 c_2)$. Then, expression (31) can be written in the form

$$I_2(z) = \frac{c_1}{c_0 \exp(-A_0 z c_1) - 1}. \quad (32)$$

Integration constants c_0, c_1, and c_2 are determined from the boundary conditions

$$I_1(z=l)=I_1^0 \quad \text{и} \quad I_2(z=0)=I_2^0 \tag{33}$$

by the system of two transcendental equations

$$I_1^0=I_2(l)+c_1, \tag{34}$$

$$I_2^0=\frac{c_1}{c_0-1}, \tag{35}$$

where

$$I_2(l)=\frac{c_1}{c_0\exp(-A_0lc_1)-1}.$$

As shown above [see expression (23)], when $I_2^0/I_1^0 \ll 1$ and, hence, $c_1 \simeq I_1^0$, expression (31) can be written in the form

$$I_2(z)=I_2^0\exp(\alpha_1^0 z). \tag{36}$$

If the condition $I_2^0/I_1^0 \gg 1$ is fulfilled and, hence, $c_1 \simeq I_2^0$, expression (31) can be written in the form

$$I_1(z)=I_1^0\exp\left[\alpha_2^0(z-l)\right]. \tag{37}$$

Here, $\alpha_2^0=A_0I_2^0$. Expression (37) shows that under assumptions made above, a light beam propagating toward the plasma decays exponentially.

Therefore, the laser beam propagating in the direction of plasma movement is amplified. In this case, the inequality $\mathrm{d}f(p)/\mathrm{d}p>0$ takes place in the vicinity of $p=0$. If the direction of plasma movement is changed to the opposite, the first beam will be amplified, while the second one will decay. Then, we will have $\mathrm{d}f(p)/\mathrm{d}p<0$ in the vicinity of $p=0$.

Of special interest is the case when the relation

$$\frac{I_2^0}{I_1^0}=\frac{1}{1+A_0I_1^0l} \tag{38}$$

if fulfilled. In the given case, $c_1=0$, and expression (29) takes the form $I_1(z)=I_2(z)$. This means that, under condition (38), the intensities of interacting beams are identical in each cross section of the interaction region, and the solution of Eqns (29) takes the form

$$I_1(z)=I_1^0\frac{1}{1-A_0I_1^0(z-l)}, \quad I_2(z)=I_2^0\frac{1+A_0I_1^0l}{1-A_0I_1^0(z-l)}. \tag{39}$$

5. Comparison of the calculated laser-pulse gains with experimental data [2]

In [2], the amplification of a laser beam propagating in the direction of the plasma layer movement was experimentally observed. Experiments were performed for the initial intensity ratio of interacting beams $I_2^0/I_1^0=0.2$ and $I_2^0/I_1^0=0.7$ obtained for $I_1^0=3\times10^{14}$ W cm^{-2}. It was found that the gain in the first and second cases was 1.32 ± 0.03 and 1.12 ± 0.03, respectively.

By using relations (32)–(35), we find the dependence of $I_2(l)/I_2^0$ on I_2^0/I_1^0 in the range $[I_2^0/I_1^0=0.1; I_2^0/I_1^0=1]$. By introducing the dimensionless variables $\tilde{I}_1^0=I_1^0/I_1^0=1$, $\tilde{I}_2^0=I_2^0/I_1^0$, and $\tilde{c}_1=c_1/I_1^0$, we can represent Eqns (34) and (35), which are used for calculations of a number of values of the integration constant $\tilde{c}_1=c_1/I_1^0$ depending on $\tilde{I}_2^0=I_2^0/I_1^0$, in the form

$$1=\frac{\tilde{c}_1}{c_0\exp(-\alpha_1^0\tilde{c}_1 l)-1}+\tilde{c}_1, \quad \tilde{c}_1=(c_0-1)\tilde{I}_2^0, \tag{40}$$

where $\alpha_1^0=A_0I_1^0$. We will calculate the gain α_1^0 (in cm^{-1}) for the experimental conditions in [2]: $kT_e=100$ eV, $n_e=1.2\times10^{19}$ cm^{-3}, $v_0=10^8$ cm s^{-1}, $l=5\times10^{-2}$ cm, $\omega_0=2.73\times10^{15}$ s^{-1} (the radiation frequency of a ruby laser).

For this purpose, we will use expression (22) for A_0 written in the form convenient for calculations:

$$\alpha(\mathrm{cm}^{-1})=3\times10^5\,\frac{n_e(10^{20}\,\mathrm{cm}^{-3})I(10^{14}\,\mathrm{W\,cm}^{-2})}{\omega^3(10^{15}\,\mathrm{s}^{-1})kT_e(\mathrm{eV})}\left(\frac{mv_0^2}{2kT_e}\right)^{1/2}$$

$$\times\exp\left(-\frac{mv_0^2}{2kT_e}\right), \tag{41}$$

$$\frac{mv_0^2}{2kT_e}=2.85\,\frac{v_0^2(10^8\mathrm{cm\ s}^{-1})}{kT_e(\mathrm{eV})}.$$

By substituting the numerical values presented above into (41), we obtain $\alpha_1^0=8.75$ cm, $\alpha_1^0 l=0.44$.

Figure 9 shows the dependence $I_2(l)/I_2^0=f(I_2^0/I_1^0)$ calculated for $\alpha_1^0 l=0.44$. A discrepancy between the calculated and experimental values is explained by the fact that not all parameters required for calculations are known with the required accuracy. In addition, analytic expressions were obtained for a stationary process, whereas stimulated Compton scattering was observed under transient conditions.

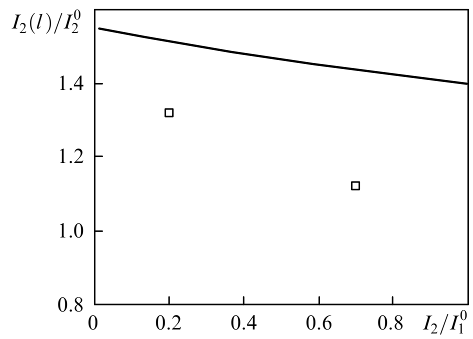

Figure 9. Dependence of the second-beam amplification $I_2(l)/I_2^0$ on the ratio I_2^0/I_1^0 (solid curve). Squares are experimental data [2].

6. Amplification of ultrashort laser pulses upon stimulated Compton scattering

It can be shown that the coefficient A_0 achieves its maximum at

$$v_0=w_0=\sqrt{\frac{\pi}{8}}\bar{v}=\sqrt{\frac{kT_e}{m}},$$

where $\bar{v}$ is the average thermal velocity of electrons in plasma, and v_0 is measures in cm s^{-1}. In this case,

$$A_0 = A_{\max} = \frac{4\pi^3}{\sqrt{2\pi}e}\frac{c^2 r_0^2 n_e}{\omega_0^3 kT_e}. \quad (42)$$

The gain $\alpha_1^{\max} = A_{\max} I_1^0$ can be calculated from the expression

$$\alpha_1^{\max}(\text{cm}^{-1}) = 1.35 \times 10^5 \frac{n_e(10^{20}\,\text{cm}^{-3}) I(10^{14}\,\text{W cm}^{-2})}{\omega_0^3(10^{15}\,\text{s}^{-1}) kT_e(\text{eV})}. \quad (43)$$

We will calculate the gain $\alpha_1^{\max}$ for the following values of parameters entering (43): $\omega_0 = 1.78 \times 10^{15}$ s^{-1}, $I_1^0 = 2 \times 10^{13}$ W cm^{-2}, $\Delta\tau_1 = 10^{-10}$ s, $\Delta\tau_2 = 10^{-12}$ s, $kT_e = 100$ eV ($v_0 = w_0 = 4.2 \times 10^8$ cm s^{-1}), $n_e = 0.4\times 10^{20}$ cm^{-3}. In this case, the gain $\alpha_1^{\max} = A_{\max} I_1^0$ is 96 cm^{-1}.

The theoretical results obtained in the stationary approximation can be used only when the interaction time of laser beams with a plasma layer is shorter than the duration of the amplified laser pulse: $l/c \leqslant \Delta\tau_2$. We assume in this case that the 'pump' pulse duration exceeds the amplified pulse duration. This condition restricts the thickness of a plasma layer by the inequality $l \leqslant c\Delta\tau_2 = 0.03$ cm. As a result, we have $\alpha_1^{\max} l = 2.88$.

By using relations (31)–(34), we determine the dependence of $I_2(l)/I_2^0$ on I_2^0/I_1^0 in the range [I_2^0/I_1^0=0.1; I_2^0/I_1^0= 1.2] for the above value of $\alpha_1^{\max} l$. This dependence is presented in Fig. 10. One can see that the gain considerably decreases when the initial intensity of the amplified laser pulse approaches the 'pump' pulse intensity. The maximum gain equal to 17.8 is achieved for $I_2^0/I_1^0 \ll 1$.

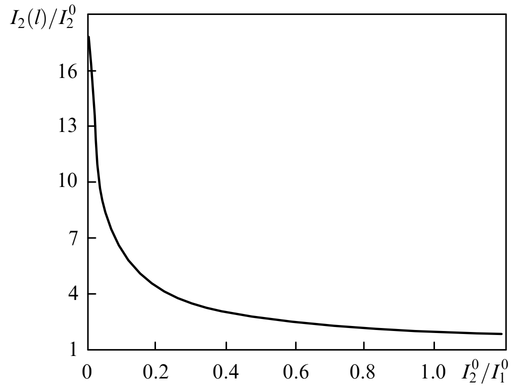

Figure 10. Dependence of the amplification of the second beam on its relative intensity.

Figure 11 presents the dependences $I_1(z)/I_1^0$ and $I_2(z)/I_1^0$ for three values of the ratio I_2^0/I_1^0.

Figure 12 shows the change in the amplitude of the second laser pulse propagating through fifteen 0.03-cm thick plasma layers (the process is stationary in each of the layers) calculated for three values of the gain $\alpha_1^{\max}$. One can see that, when the value of $\alpha_1^{\max} l$ is large, the intensity of the first beam decreases to zero after propagation through each plasma layer, while the amplitude of the second laser beam increases by I_1^0. Beginning from this instant, after propagation through remaining plasma layers, the intensity of the amplified pulse can be calculated from the expression

$$I_2^m = I_2^n + (m-n)I_1^0. \quad (44)$$

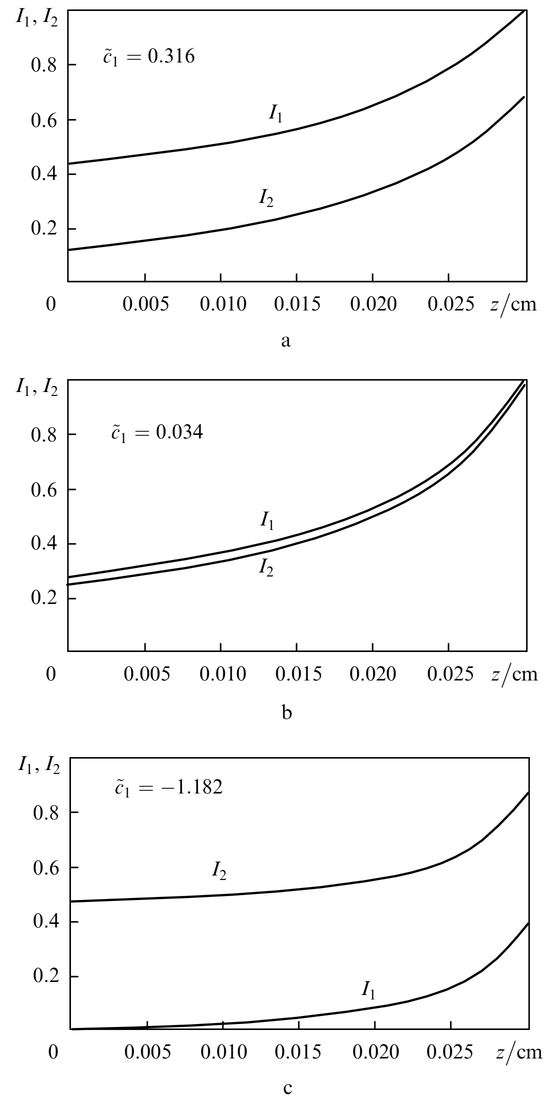

Figure 11. Changes in the intensity of amplified and 'pump' laser pulses propagating in a 0.03-cm thick plasma layer for $I_2^0/I_1^0 = 0.12$ (a), 0.24 (b), and 1.2 (c) and different values of $\tilde{c}_1$. The pulse amplitudes are normalised to $I_1^0 = 10^{14}$ W cm^{-2}.

Here, n is the number of a plasma layer, beginning from $I_1(z=0) \simeq 0$, and m is the total number of plasma layers. The correctness of expression (44) is confirmed by the data presented in Fig. 12c. In this case, already beginning from the second plasma layer, the gain can be calculated from expression (44). The corresponding value differs by 2 % from the value determined from Fig. 12c.

Note that, according to expression (30), we have $c_1 \simeq I_2^0$ and $I_2(l) = I_1^0(1 + I_2^0/I_1^0)$ for $I_2^0 \gg I_1^0$. This means that the amplification of a short laser pulse occurs at any relation between the intensities of the amplified laser pulse and 'pump' pulse.

Condition (19) restricts the duration of the amplified laser pulse:

$$\frac{\Delta\omega_{\max}}{2\omega_0} = \frac{2}{\omega_0}\frac{1}{\Delta\tilde{\tau}} \leqslant \frac{w_0}{c},$$

or

$$\Delta\tilde{\tau} \geqslant \frac{2c}{\omega_0 w_0}. \quad (45)$$

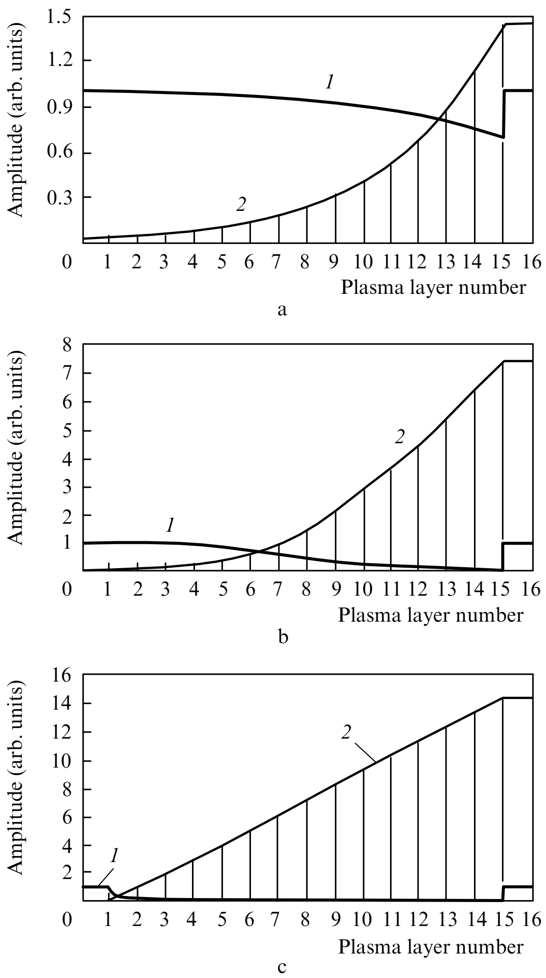

Figure 12. Amplification of the second laser pulse (of short duration) propagating through fifteen 0.03-cm thick plasma layers for $\alpha_1^{max} = 9.6$ cm^{-1}, $I_1^0 = 10^{13}$ W cm^{-2}, $I_2^m/I_2^0 = 60.4$ (a); $\alpha_1^{max} = 19.2$ cm^{-1}, $I_1^0 = 2\times 10^{13}$ W cm^{-2}, $I_2^m/I_2^0 = 310$ (b); and $\alpha_1^{max} = 96$ cm^{-1}, $I_1^0 = 10^{14}$ W cm^{-2}, $I_2^m/I_2^0 = 600$ (c). (*1*) pump laser pulse, (*2*) amplified laser pulse with the initial amplitude $I_2^0 = 0.024 I_1^0$. The pulse amplitudes are normalised to I_1^0.

For $kT_e = 100$ eV, $w = 4.2 \times 10^8$ cm s^{-1}, and $\omega_0 = 1.78 \times 10^{15}$ s^{-1}, we obtain from (42) that the duration of the amplified pulse is restricted by the value $\Delta\tau_2 \geqslant \Delta\tilde{\tau} = 3.2 \times 10^{-13}$ s. To amplify shorter pulses, it is necessary to use higher-temperature plasmas.

The main advantage of using plasma for amplifying intense short laser pulses is that plasma is a renewable medium, which almost does not change its properties during the interaction process. This circumstance was pointed out in [17]. On the contrary, the application of condensed media to amplify ultrashort laser pulses is complicated by optical damages that can be produced by laser radiation in them.

Amplification effects upon stimulated Compton scattering considered above were studied by using the kinetic equation for photons. In the case of large photon occupation numbers, these effects can be described classically. However, the calculation method, which requires the statistical approach, proves to be complicated and less illustrative than the quantum-mechanical description [18]. The interaction of radiation with plasma can be also accompanied by collective effects [8]. Under experimental conditions in [2], stimulated scattering of laser radiation by ion–sound waves could be the most probable process because spectral variations observed in experiments were small. However, as follows from analysis performed in [19], the ion–sound waves have no time to develop during the action of laser radiation. Therefore, this type of scattering can be neglected in the interpretation of experimental results [2]. This conclusion also concerns the problem considered here, in which it is assumed that plasma electrons interact with laser radiation as free particles.

Note that the possibility of amplifying ultrashort laser pulses in a plasma upon the three-wave interaction of counterpropagating electromagnetic waves with plasma oscillations is considered in paper [17], where quite complete relevant references are also presented.

There exists another circumstance that should be taken into account in the study of the interaction of intense laser radiation with electrons. The matter is that the velocity distribution function of electrons can be deformed in this case so that a 'shelf' (shown by the dashed line in Fig. 8) can be formed in the interval of velocities of electrons involved in scattering. The formation of the 'shelf' is prevented by collisions of electrons with plasma particles, so that the velocity distribution function tends to return to its initial shape. To avoid the formation of the 'shelf', the efficiency of the second of the above-mentioned process should be higher. By using the classical description of stimulated Compton scattering, it can be shown [19] that the 'shelf' will not form if the condition

$$(I_1 I_2)^{1/2} < I_{cr} \qquad (46)$$

is fulfilled, where

$$I_{cr} = 0.22 \times 10^{-33} \omega_0^2 \left(\frac{\nu_{eff}}{\omega_0}\right)^{2/3} \left(\frac{\upsilon_{T_e}}{\upsilon}\right) (\upsilon_{T_e} c)^{2/3}.$$

Here, ν_{eff} is the effective collision frequency of electrons with plasma particles; υ_{T_e} is the average thermal velocity of electrons; υ is the velocity of electrons involved in scattering; and c is the speed of light. In the examples considered above, $I_{cr} \simeq 1.3 \times 10^{16}$ W cm^{-2}, and condition (46) is fulfilled. Therefore, we can conclude that in this case the profile of the electron distribution function is not distorted during stimulated Compton scattering.

7. Conclusions

The results obtained in this paper have shown that stimulated Compton scattering of counterpropagating laser beams in a laser plasma can be used for the efficient amplification of ultrashort laser pulses. This possibility can be realised based on the theoretical results obtained in the paper.

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