

# On the entanglement of electronic states of impurity atoms in nanoparticles

A.M. Basharov, V.N. Gorbachev, N.V. Znamenskii

**Abstract.** By using the derived master equations, it is shown that the decay of electronic impurities in a nanocrystal can be described as a collective relaxation of particles. A set of entangled states of impurity atoms is found, which have immunity to this relaxation. These states can be used for decoherence-free quantum processing.

**Keywords:** nanocrystal, electronic impurity states, localised phonon mode, dispersion limit, collective relaxation, entanglement of quantum states.

## 1. Introduction

Nanocrystals and glasses activated with rare-earth ions are used in scintillation devices as luminophores and in biomedical problems as fluorescence probes [1, 2]. In this paper, we discuss a new possibility of using nanocrystals doped with atoms or ions as a ‘bricks’ of the elemental base in quantum information devices. This possibility is determined by the specific features of the dynamics of electronic states of impurity atoms caused by electron–phonon interaction in the nanocrystals.

The unit of quantum information is a qubit or a two-level quantum particle [3]. The physical realisations of qubits together with main operations performed with them have been actively studied over a decade [3, 4]. The main elements of quantum information processes including quantum calculations and quantum communication are the superposition and entangled states of qubits.

Entangled quantum states are also considered as the states with quantum correlations. The methods of preparation of quantum-correlated systems and their robustness to relaxations (decoherence) are the subject of numerous theoretical and experimental studies. At present the decoherence-free entangled states have been experimentally realised only in the system of photons [5]. Thus, the search

for situations in which entangled states insensitive to decoherence can exist is of current interest. In addition, from the practical point of view the realisations of qubits and entanglement operations in solid or even semiconductor matrices are important for their integration with other semiconductor devices. One of the examples is quantum dots [6] and another is activated nanocrystals.

According to experimental data, impurity atoms or ions in nanoparticles, for example, rare-earth ions ( $\text{Pr}^{3+}$ ,  $\text{Eu}^{3+}$ ,  $\text{Ce}^{3+}$ ) in oxyorthosilicate nanocrystals ( $\text{Y}_2\text{SiO}_5$ ,  $\text{Lu}_2\text{SiO}_5$ ,  $\text{Gd}_2\text{SiO}_5$ ) exhibit a strong electron–phonon interaction with a localised phonon mode [7, 8]. Nanocrystals differ in this respect from macroscopic crystals. The frequency of a localised mode is approximately equal to the inverse propagation time of an acoustic signal over the characteristic size of the nanocrystal in a microscopic crystal. The popular model [9] consistent with experiments [7, 8] considers two impurity quantum levels interacting with a localised phonon mode similarly to a two-level atom in a resonator.

In our paper, we generalise the model [9] to the case of several impurities and the coupling between the localised mode and phonons of the matrix at the nanocrystal surface and consider then the dynamics of impurity electronic states. It is shown in the simplest case of the dispersion limit when detuning from the resonance exceeds the characteristic interaction frequency of an impurity with localised phonons that the electronic states of impurities in nanocrystals can be entangled and insensitive to decoherence. Therefore, activated nanocrystals are promising for applications in quantum information devices.

We studied the dynamics of impurities in nanocrystals by using a popular model of atoms in a microcavity with losses at mirrors [10], which has, however, some modifications.

Note first of all that the dispersion limit for atoms in a microcavity with losses at mirrors was recently considered in [11]. However, the kinetic equation was derived in this paper not correctly enough because the passage to the dispersion limit was performed in kinetic equations for the atom + microcavity photon system, whereas first the effective Hamiltonian should be obtained in the dispersion limit for the atom + microcavity photon + thermostat phonon system and then kinetic equations derived by using this Hamiltonian [12]. The difference can be both in the values of the model parameters and in the appearance of new relaxation channels. In [13], the case is described when such a correct approach results in the appearance of two-quantum relaxation mechanisms that are efficient in media with specific spectral features.

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In this paper, we obtained the kinetic equation for atoms in a nanocrystal in the dispersion limit by the method considered above and showed the correctness of the model of collective relaxation. We obtained a correct expression for parameters of the kinetic equation, which differs from the kinetic equation obtained in [11]. The kinetic equation was derived for the initial Hamiltonian without using the slowly varying amplitude approximation. Note that it was shown in [14] for the problem of interaction of ultrashort electromagnetic pulses with a two-level system in the dispersion limit (where it is called the adiabatic following approximation) how the correctly obtained dispersion limit differs from the results derived from the initial Hamiltonian in the slowly varying amplitude approximation.

By solving the kinetic equation obtained for impurities in a nanocrystal, we found that, several initial states of impurities result in the steady-state solutions with their decoherence-free entangled states. Acoustic waves in a matrix to which an impurity nanocrystal is embedded play a key role in the creation of such states.

## 2. The model

Consider a spherical nanocrystal of diameter  $L$  to which several impurity atoms are embedded. According to the Simon–Geller model [9], each atom interacts with a localised phonon mode of frequency  $\omega_0 \sim 2\pi v/L$ , where  $v$  is the sound speed in a macroscopic crystal. By assuming that the electronic states of impurities are the same, we will write the Hamiltonian for this interaction in the form

$$H = H_a + H_{\text{ph}} + V, \quad H_a = \sum_{\alpha i} \varepsilon_{\alpha} c_{\alpha i}^{\dagger} c_{\alpha i}, \quad (1)$$

$$V = \sum_{\alpha \alpha' i} g_{\alpha \alpha'} c_{\alpha i}^{\dagger} c_{\alpha' i} (a + a^{\dagger}), \quad H_{\text{ph}} = \omega_0 a^{\dagger} a.$$

Here, the subscript  $\alpha = 1, 2$  numerates a pair of electronic states with energies  $\varepsilon_1$  and  $\varepsilon_2$ , and  $c_{\alpha i}$  and  $c_{\alpha i}^{\dagger}$  are the electron annihilation and creation operators in these states of the  $i$ th impurity. The electron–phonon interaction is determined by the coupling constants  $g_{21} = g_{12} = g$ ,  $g_{11} = g_{22} = 0$ ; and  $a$  and  $a^{\dagger}$  are the annihilation and creation operators for the localised phonon mode.

The further generalisation of the Simon–Geller model involves the consideration of excitation of the localised mode due to interaction with thermostat phonons on the nanocrystal boundary. The simplest operator of this interaction has the form

$$V_{\text{loss}} = \sum_{\omega} \Gamma_{\omega} (a^{\dagger} b_{\omega} + a b_{\omega}^{\dagger}), \quad (2)$$

where  $b_{\omega}$  and  $b_{\omega}^{\dagger}$  are the thermostat phonon annihilation and creation operators. We assume that the initial state  $|\Phi_0\rangle$  of phonons in the thermostat is delta-correlated:

$$\langle \Phi_0 | b_{\omega}^{\dagger} b_{\omega'} | \Phi_0 \rangle = N(\omega) \delta(\omega - \omega'), \quad (3)$$

$$\langle \Phi_0 | b_{\omega} b_{\omega'}^{\dagger} | \Phi_0 \rangle = (1 + N(\omega)) \delta(\omega - \omega'),$$

where  $N(\omega) = (e^{\omega/kT} - 1)^{-1}$ . Because phonons with frequencies close to the localised mode frequency play a main

role, the assumption of delta-correlation is justified in the light of using the Markovian approximation to derive the kinetic equation.

The main approximation of the Simon–Geller model and proposed generalisation is the neglect of a difference between transverse and longitudinal phonons. It is also assumed that the dipole moment of the  $\varepsilon_2 \rightarrow \varepsilon_1$  electron transition is small enough to neglect the interaction between atoms and the electromagnetic field. We use the system of units in which  $\hbar = 1$ .

Therefore, the initial Hamiltonian of the problem on the dynamics of impurity electronic states in a nanocrystal taking into account the coupling of the localised phonon mode with thermostat phonons is

$$H_{\text{loss}} = H_a + H_{\text{ph}} + H_{\text{bath}} + V + V_{\text{loss}}, \quad (4)$$

$$H_{\text{bath}} = \sum_{\omega} \omega b_{\omega}^{\dagger} b_{\omega},$$

where  $H_{\text{bath}}$  is the Hamiltonian of the thermostat phonons in a matrix to which the nanocrystal is embedded.

The state vector  $|\Psi\rangle$  of the entire system satisfies the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = H_{\text{loss}} |\Psi\rangle. \quad (5)$$

One can see that Hamiltonian (4) coincides formally with the Hamiltonian describing the dynamics of atoms in a single-mode cavity with losses at mirrors [10]. To demonstrate most simply the possibility of the entanglement of impurity electronic states and the realisation of the decoherence-free entangled states, we consider the dispersion limit for Hamiltonian (4). In this case, our equations are no longer coincident with the known kinetic equations for the dispersion limit presented in [11].

## 3. Dispersion limit

Consider a nanocrystal of size at which the localised mode frequency and the electronic transition frequency for impurities satisfy the conditions

$$|\Delta| \gg g^2 \bar{n}, \quad |\Delta| \ll |\varepsilon_2 - \varepsilon_1|, \quad \Delta \equiv \omega_0 - \varepsilon_2 - \varepsilon_1, \quad (6)$$

i.e. we assume that the detuning  $\Delta$  from the resonance exceeds the characteristic energy of interaction with the localised mode ( $\bar{n}$  is the average number of phonons of the localised mode) but is much smaller than all detunings from other impurity electronic states and impurity electronic transition frequencies.

Condition (6) shows that transitions between the impurity levels  $\varepsilon_1$  and  $\varepsilon_2$  caused by the electron–phonon interaction are weak. Therefore, we will try to diagonalise the non-dissipative part of the Hamiltonian  $H_{\text{loss}}$ . For this purpose, we perform the unitary transformation of the state vector  $|\Psi\rangle$  of the whole system:

$$|\tilde{\Psi}\rangle = U |\Psi\rangle. \quad (7)$$

The transition of the vector  $|\Psi\rangle$  to a new vector (4) results in the transformation of the Hamiltonian

$$\tilde{H}_{\text{loss}} = UH_{\text{loss}}U^+ - i\hbar U \frac{\partial}{\partial t} U^+, \quad (8)$$

so that the quantum system is now described by the Schrödinger equation with the Hamiltonian (8):

$$i\hbar \frac{\partial}{\partial t} |\tilde{\Psi}\rangle = \tilde{H}_{\text{loss}} |\tilde{\Psi}\rangle. \quad (9)$$

By expressing the unitary operator in terms of the Hermitean operator

$$U = e^{-iS}, \quad S^+ = S, \quad (10)$$

and expanding the transformed Hamiltonian (5) and  $S$  into series over the coupling constants of the atoms and thermostat phonons,

$$S = S^{(10)} + S^{(01)} + S^{(11)} + \dots,$$

$$\tilde{H}_{\text{loss}} = \tilde{H}^{(00)} + \tilde{H}^{(10)} + \tilde{H}^{(01)} + \tilde{H}^{(11)} + \tilde{H}^{(20)} + \dots, \quad (11)$$

we obtain the following equalities for different expansion orders of the transformed Hamiltonian (the left digit in each pair of superscripts indicates the expansion order in the coupling constant with the localised phonon mode, while the right digit indicates this order in the coupling constant with thermostat phonons):

$$\tilde{H}^{(00)} = H_a + H_{\text{ph}} + H_{\text{bath}}, \quad (12a)$$

$$\tilde{H}^{(10)} = V - i[S^{(10)}, \tilde{H}^{(00)}] + \hbar \frac{\partial S^{(10)}}{\partial t}, \quad (12b)$$

$$\tilde{H}^{(01)} = V_{\text{loss}} - i[S^{(01)}, \tilde{H}^{(00)}] + \hbar \frac{\partial S^{(01)}}{\partial t}, \quad (12c)$$

$$\tilde{H}^{(11)} = -\frac{i}{2}[S^{(01)}, V] - \frac{i}{2}[S^{(10)}, V_{\text{loss}}] - \frac{i}{2}[S^{(01)}, \tilde{H}^{(10)}]$$

$$- \frac{i}{2}[S^{(10)}, \tilde{H}^{(01)}] - i[S^{(11)}, \tilde{H}^{(00)}] + \hbar \frac{\partial S^{(11)}}{\partial t}, \quad (12d)$$

$$\tilde{H}^{(20)} = -\frac{i}{2}[S^{(10)}, V] - \frac{i}{2}[S^{(10)}, \tilde{H}^{(10)}]$$

$$- i[S^{(20)}, \tilde{H}^{(00)}] + \hbar \frac{\partial S^{(20)}}{\partial t}, \quad (12e)$$

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To obtain the effective Hamiltonian, it is necessary to remove the term describing the linear interaction of atoms with the localised mode because in the adiabatic approximation it is this term that should be absent due to a large enough tuning from the resonance. We will require that

$$\tilde{H}^{(10)} = 0. \quad (13)$$

This can be achieved with the help of the operator

$$S^{(10)} = i \frac{g(a^+ R_- - a R_+)}{\Delta} + i \frac{g(a^+ R_+ - a R_-)}{\omega_0 + \varepsilon_2 - \varepsilon_1}. \quad (14)$$

In this case,

$$\tilde{H}^{(20)} = -\frac{i}{2}[S^{(10)}, V] = -\frac{g^2}{\Delta}(R_- R_+ + cc^+ 2R_3), \quad (15)$$

$$\tilde{H}^{(01)} = V_{\text{loss}}, \quad (16)$$

$$\tilde{H}^{(11)} = -\frac{g}{\Delta} \sum_{\omega} \Gamma_{\omega}(R_+ b_{\omega} + R_- b_{\omega}^+), \quad (17)$$

$$S^{(20)} = 0, \quad S^{(01)} = 0, \quad S^{(11)} = 0. \quad (18)$$

The collective operators for impurity electronic states used in these relations are

$$R_3 = \frac{1}{2} \sum_i (c_{2i}^+ c_{2i} - c_{1i}^+ c_{1i}), \quad R_+ = \sum_i c_{2i}^+ c_{1i}, \quad R_- = \sum_i c_{1i}^+ c_{2i},$$

and

$$[R_3, R_{\pm}] = \pm R_{\pm}, \quad [R_+, R_-] = 2R_3, \quad R_- R_+ = R^2 - R_3^2 - R_3.$$

In addition, we excluded from the Hamiltonian the term proportional to the population of impurity levels under study,

$$\frac{1}{2}(\varepsilon_2 + \varepsilon_1) \sum_i (c_{2i}^+ c_{2i} + c_{1i}^+ c_{1i}),$$

because it commutes with all other terms.

Thus, the effective Hamiltonian

$$H^{\text{eff}} = \tilde{H}^{(00)} + \tilde{H}^{(01)} + \tilde{H}^{(11)} + \tilde{H}^{(20)}$$

of the impurity nanocrystal takes the form

$$\begin{aligned} H^{\text{eff}} &= (\varepsilon_2 - \varepsilon_1)R_3 + \omega_0 a^+ a \\ &+ \sum_{\omega} \omega b_{\omega}^+ b_{\omega} - \frac{g^2}{\Delta}(R_- R_+ + cc^+ 2R_3) \\ &+ \sum_{\omega} \Gamma_{\omega}(a^+ b_{\omega} + a b_{\omega}^+) - \frac{g}{\Delta} \sum_{\omega} \Gamma_{\omega}(R_+ b_{\omega} + R_- b_{\omega}^+) \end{aligned} \quad (19)$$

in the dispersion limit, in which the effective Hamiltonian  $H_a^{\text{eff}}$  of impurities in the nanocrystal and the impurity-thermostat field coupling operator  $V_{a\text{loss}}^{\text{eff}}$ :

$$H_a^{\text{eff}} = (\varepsilon_2 - \varepsilon_1)R_3 - \frac{g^2}{\Delta}(R_- R_+ + cc^+ 2R_3),$$

$$V_{a\text{loss}}^{\text{eff}} = -\frac{g}{\Delta} \sum_{\omega} \Gamma_{\omega}(R_+ b_{\omega} + R_- b_{\omega}^+)$$

play a key role.

By using Hamiltonian (19) and initial state (3) for matrix phonons, we derive in the Markovian approximation [15, 16] the master equation for the density matrix of impurities in the nanocrystal:

$$\frac{d\rho_a}{dt} = \frac{i}{\hbar} [\rho_a, \bar{H}_a^{\text{eff}}] - \hat{\Gamma}\rho_a, \quad (20)$$

where

$$\begin{aligned} \hat{\Gamma}\rho_a = & |\chi|^2(N+1)(\rho_a R_+ R_- + R_+ R_- \rho_a - 2R_- \rho_a R_+) \\ & + |\chi|^2 N(\rho_a R_- R_+ + R_- R_+ \rho_a - 2R_+ \rho_a R_-) \end{aligned} \quad (21)$$

is the relaxation operator;  $\chi = g\Gamma_{\varepsilon_2-\varepsilon_1} \Delta^{-1} \sqrt{2\pi}$ ; and  $N = N(\varepsilon_2 - \varepsilon_1)$ .

We assume also that the dynamics of the phonon mode of the nanocrystal in the dispersion limit is mainly determined by the interaction with thermostat phonons on the nanocrystal boundary, so that the Hamiltonian  $\bar{H}_a^{\text{eff}}$  contains instead of phonon operators the average number  $\overline{a^+a}$  of phonons in the nanocrystal:

$$\bar{H}_a^{\text{eff}} = (\varepsilon_2 - \varepsilon_1)R_3 - \frac{g^2}{\Delta} [R_- R_+ + (1 + \overline{a^+a})2R_3]. \quad (22)$$

Equations (20) and (21) describe the relaxation of impurity electronic states in the nanocrystal in the dispersion limit (6). Unlike paper [11], the coefficients of the relaxation operator are independent of the boson annihilation and creation operators in the nanocrystal. The term containing operators  $R_3$  is also absent {see Eqn (5.8) in [11]}.

In conclusion, we write the equation for the density matrix of impurities in the nanocrystal in the interaction representation  $\tilde{\rho}_a$ :

$$\tilde{\rho}_a = \exp(i\bar{H}_a^{\text{eff}}t/\hbar)\rho_a \exp(-i\bar{H}_a^{\text{eff}}t/\hbar), \quad (23)$$

$$\dot{\tilde{\rho}}_a = -\hat{\Gamma}\tilde{\rho}_a,$$

where the relaxation operator is now determined by Eqn (21) [by neglecting the difference of  $\bar{H}_a^{\text{eff}}$  from  $(\varepsilon_2 - \varepsilon_1)R_3$ ].

#### 4. Consequences of the collective dynamics of impurities

It is important to note that relaxation operator (21) coincides accurately to the values of parameters with this operator obtained in the model of collective relaxation of atoms located in one spatial point in a three-dimensional electromagnetic field and in the model of a unidirectional thermostat [17]. Moreover, when the components of the density matrix change slowly, kinetic equations (23) also correspond to the model of collective relaxation. Therefore, we can use the results of our papers [17–19] to describe the dynamics of impurities in nanocrystals. The main features of this dynamics are as follows.

In the case of two impurities, of most interest for relaxation processes is the pure state with one excited particle

$$\Psi = \alpha|01\rangle + \beta|10\rangle, \quad (24)$$

where  $|\alpha|^2 + |\beta|^2 = 1$ ;  $|0\rangle$  and  $|1\rangle$  are the wave functions of impurities with energies  $\varepsilon_1$  and  $\varepsilon_2$ ;  $|01\rangle$  ( $|10\rangle$ ) are the states of two impurities in which the first (second) impurity occupies the electronic state with energy  $\varepsilon_1$  and the second (first) impurity occupies the electronic state with energy  $\varepsilon_2$ . Because the wave function  $\Psi$  is not factorised, the atoms can be entangled. In this case, the entanglement degree is determined by the entropy of one of the impurities, say, the first one:  $S = -\text{Tr}(\rho \ln \rho)$ , where  $\rho = \text{Tr}_2|\Psi\rangle\langle\Psi|$  and the trace is taken over the states of the second impurity. In our case,  $S = -p \ln p - (1-p) \ln(1-p)$ , where  $p = |\alpha|^2$ . For  $p = 0$  or  $1$ , the entropy is  $S = 0$ , which corresponds to the case of the non-entangled state for which the wave function is factorised. For  $p = 1/\sqrt{2}$ , the maximally entangled state appears, which can be antisymmetric with respect to the permutation of particles:

$$\Psi^- = (|01\rangle - |10\rangle)/\sqrt{2}. \quad (25)$$

Due to the symmetry, this state is annihilated by all two-particle collective operators and, thereby being preserved in all collective processes. In particular, if the two impurities were initially prepared in the state  $\Psi^-$ , then under the conditions of a collective decay determined by Eqn (23) their state will be preserved because  $\Gamma|\Psi^-\rangle\langle\Psi^-| = 0$ . This means that maximally entangled antisymmetric states of a pair of atoms prove to be insensitive to collective relaxation. If we consider now arbitrary states of two impurities, we will see that collective relaxation separates antisymmetric states, resulting in the formation of stationary entangled states [17].

Numerous multiparticle entangled states are of interest for applications. By using the above example, we can construct a trivial generalisation for the even number  $2n$  of impurities by considering the state  $|\Psi^-\rangle \otimes |\Psi^-\rangle \dots$ . This state describes the case of  $n$  excited impurities. However, we can also indicate the states with one excited impurity, which will be also insensitive to collective relaxation. Such states prove to be natural for multiparticle systems and can be prepared by using simple schemes. Consider the wave function describing an ensemble of  $n$  impurities inside one nanocrystal when only one impurity is excited. We will use the result

$$\eta_n(1) = q_1|10\dots 0\rangle + q_2|01\dots 0\rangle + \dots + q_n|00\dots 1\rangle \quad (26)$$

obtained in [18], where  $\sum_k |q_k|^2 = 1$ . If all the coefficients  $q_k$  are the same, the symmetric state appears, which is called the  $W$  state in the quantum theory of information:

$$W_n = (|10\dots 0\rangle + |01\dots 0\rangle + \dots + |00\dots 1\rangle)/\sqrt{n}, \quad (27)$$

States of the type  $\eta_n(1)$  can appear during the additional resonance interaction of impurities with a high-power electromagnetic wave. Because the dipole moments of transitions between the electronic levels with energies  $\varepsilon_1$  and  $\varepsilon_2$  are small, the intensity of the electromagnetic wave can be selected so that only one of  $n$  impurity atoms can absorb a photon. As a result, one excitation is distributed between all impurities in the nanocrystal and superposition (26) appears.

Consider the dynamics of  $\eta_n(1)$  in a vacuum thermostat by assuming that  $N = 0$  in (21). In this case, Eqn (23) takes the form

$$\dot{\tilde{\rho}}_a = -|\chi|^2 \mathcal{L} \tilde{\rho}_a, \quad (28)$$

$$\mathcal{L} \tilde{\rho}_a = \tilde{\rho}_a R_+ R_- - R_- \tilde{\rho}_a R_+ + \text{H. c.}$$

By using this equation, we find that the evolution of the density matrix  $\tilde{\rho}(0) \rightarrow \tilde{\rho}(t)$  for the  $\eta_n(1)$  state is determined by the expression

$$\begin{aligned} |\eta_n\rangle\langle\eta_n| &\rightarrow (Q/\sqrt{n})[\exp(-n|\chi|^2 t) - 1]|W_n\rangle\langle\eta_n| \\ &+ \text{H. c.} + (|Q|^2/n)[\exp(-n|\chi|^2 t) - 1]^2 \\ &\times |W_n\rangle\langle W_n| + (|Q|^2/n)[1 - \exp(-2n|\chi|^2 t)] \\ &\times |0\rangle\langle 0| + |\eta_n\rangle\langle\eta_n|, \end{aligned}$$

where  $Q = \sum_k q_k$ . This means that, under the condition

$$\sum_k q_k = 0 \quad (29)$$

the  $\eta_n(1)$  states do not decay unlike the decaying  $W_n$ :  $W_n \rightarrow |00\dots 0\rangle$ . This behaviour can be explained based on the symmetry considerations. The matter is that vectors  $\eta_n(1)$  under conditions (7) are reduced to the Dicke states, and therefore their space can be divided into irreducible subspaces with different types of symmetry. In this case,  $\eta_n(1)$  and  $W_n$  belong to different subspaces. For collective relaxation processes, which are described by Eqn (23), the permutation operator of particles commutes with  $\Gamma$ . Therefore, the symmetry of the state with respect to particle permutation is conserved and different subspaces are not mixed. This means that the decay  $W_n \rightarrow |00\dots 0\rangle$  is possible because both states are totally symmetric and belong to the same subspace; at the same time, the decay of  $\eta_n(1)$  to the  $|00\dots 0\rangle$  state is impossible. The insensitivity of  $\eta_n(1)$  to the collective decay is of interest for problems in which the integrity of a quantum state is important. Thus, by coding information with the help of  $\eta_n(1)$  states, these states can be stored without decay in a collective thermostat. This can be used in a quantum memory based on impurity ions or atoms in nanocrystals.

Let us discuss the factors neglected in our model, which can affect the lifetime of state (26) and lead to its decay with time. We derived the kinetic equation by assuming that the electronic states of impurity atoms are characterised by the same transition frequency. This requires the symmetric arrangement of impurity atoms in a nanocrystal because in this case the impurities will interact with the environment in the same way, which will provide the needed equality of transition frequencies. Otherwise the transition frequencies of impurity atoms will be slightly different, which will break the symmetry properties of state (26) and will result in its decay. The decay rate of state (26) can be probably estimated by using the results obtained in paper [20] where the temporal dynamics of the entanglement of excited and unexcited atoms in a collective thermostat was studied. It was shown that a steady state in the case of small detunings decays slowly compared to the stationary entanglement time of the initially factorised states. A similar picture can be also expected for many impurity atoms in a nanocrystal.

Another factor is the dipole–dipole interaction of impurity atoms. On the one hand, we neglected it by assuming that the dipole moments of transitions between Stark sublevels are very small. On the other hand, because the insensitivity of states (26) to decoherence is caused by the different quantum dynamics of the subspaces of states of the atomic system, which are symmetric or antisymmetric with respect to the permutation of particles, and the dipole–dipole interaction operator is symmetric with respect to the permutation of particles (as the main collective operators in our model), the stationary entangled states appearing in this case should be also stable (in the sense defined in [20]) with respect to the dipole–dipole interaction of atoms. However, if the dipole–dipole interaction of impurity atoms and atoms of a nanocrystal or a matrix is considerable, this will give rise to an independent single-frequency relaxation channel, which will destroy the properties of a collective thermostat and will determine the lifetime of quantum correlations of (26). Then, the insensitivity to decoherence can exist only within the time interval shorter than the decay time in this independent channel.

## 5. Conclusions

Thus, our study has shown that the collective relaxation of the electronic states of impurity atoms can be realised in nanoparticles. The constructed entangled states of impurity atoms, which are insensitive to decoherence, can be used in various problems of quantum information, making these nanocrystals promising for applications in quantum-information devices.

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