

# Spontaneous breaking of spectral symmetry in solid-state ring lasers

A.A. Kamysheva, N.V. Kravtsov, E.G. Lariontsev, S.N. Chekina

**Abstract.** The spontaneous breaking of the symmetry of temporal and spectral characteristics of radiation in counter-propagating waves in solid-state ring lasers (SRLs) is found upon periodic modulation of the pump power. A comparison of experimental results with numerical simulations showed that this effect is well described by a standard SRL model.

**Keywords:** solid-state ring laser, spectral nonreciprocity, nonlinear radiation dynamics, spontaneous breaking of symmetry.

## 1. Introduction

The spontaneous breaking of symmetry can occur in a variety of dynamic systems. Thus, the study of the nonlinear dynamics of lasers has revealed the spontaneous breaking of spatiotemporal [1–4], polarisation [5], and phase [6] characteristics of radiation. Most of the studies in this field concern gas and semiconductor lasers. As far as we know, these phenomena in solid-state ring lasers (SRLs) were not considered in detail.

A bidirectional SRL in the absence of optical non-reciprocities of the ring resonator in the case of a symmetric feedback between counterpropagating waves via backscattering is a symmetric oscillatory system. Such a system can naturally have symmetric stationary and transient lasing regimes in which all the parameters of counterpropagating waves (their intensity, carrier frequency, and radiation field and power spectra) are identical. Such is indeed the case in most of the lasing regimes of SRLs studied so far (see, for example, reviews [7, 8]). In particular, symmetry is observed in the stationary regime of mutual synchronisation of counterpropagating waves, in self-modulation regimes of the first and second kinds and most of the quasi-stationary lasing regimes.

Some symmetric states can become unstable in a certain region of parameters, while asymmetric states become, on the contrary, stable. In this case, a bifurcation of the spontaneous breaking of symmetry appears in a laser. The asymmetric states can be divided into two groups:

states with the asymmetric intensities of counterpropagating waves (both stationary and non-stationary) and states with different phase characteristics of counterpropagating waves. In this paper, we consider mainly states of the first type.

One of the well-known cases of spontaneous breaking of symmetry is the appearance of unidirectional lasing upon a weak coupling of counterpropagating waves via backscattering or in the absence of such coupling. In this case, two regimes of unidirectional lasing with travelling waves propagating in opposite directions can simultaneously exist in a ring laser [8]. The development of one or other of these regimes in experiments is determined by random factors.

In this paper, we show that upon periodic modulation of the pump power in a bidirectional SRL, the spontaneous breaking of symmetry occurs in the regions of parametric resonances between the modulation frequency and self-modulation frequency under certain conditions, which gives rise to specific quasi-periodic lasing regimes with different temporal structures and different emission spectra of counterpropagating waves.

## 2. Experimental setup

A ring laser under study was made of a high-quality  $\text{Nd}^{3+}$ :YAG crystal in the form of a polyhedral prism with one spherical and three plane faces providing the existence of a closed nonplanar trajectory for counterpropagating waves due to total internal reflection [8]. The geometrical perimeter of a ring resonator was 2.6 cm and the non-planarity angle was  $80^\circ$ . The laser was pumped by a semiconductor laser diode with a power supply providing a continuous variation of the pump power and its modulation in the frequency range  $\omega_p/2\pi$  between 10 and 100 kHz. The relative modulation depth  $h$  achieved 100%. The temperature of the laser was maintained with an accuracy of  $\sim 0.05^\circ$ .

The intensities of counterpropagating waves detected with a broadband photodetector (LFD-2) were displayed with a four-channel Tektronix TDS-2014 oscilloscope. The time dependences of the intensity  $I_{1,2}$  of counterpropagating waves and their spectra  $J_{1,2}$  could be recorded simultaneously.

## 3. Experimental results

The aim of our study was to find the breaking of the temporal and spectral symmetry of the radiation intensity of counterpropagating waves in the absence of the amplitude and frequency nonreciprocities of the resonator

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in the case of symmetric coefficients of coupling via back-scattering. We studied non-stationary lasing regimes appearing upon the periodic modulation of the pump power.

In the absence of pump modulation, the self-modulation regime of the first kind existed in the chip laser under study, the amplitudes of self-modulation intensity oscillations of counterpropagating waves being equal, which points to the symmetry of coupling coefficients. The self-modulation oscillation frequency  $\omega_m/2\pi$  was 180 kHz, while the main relaxation oscillation frequency  $\omega_r/2\pi$  was 60 kHz for the excess over the pump threshold  $\eta = 0.15$ . Studies were performed in the regions of nonlinear (parametric) resonances when the pump modulation frequency  $\omega_p$  and the self-modulation frequency  $\omega_m$  are related by the expressions

$$n\omega_m = k\omega_p, \quad (1)$$

where  $n, k = 1, 2, \dots$

Consider first the case of a parametric resonance for  $n = 1, k = 2$ . Such a resonance takes place when the modulation frequency is varied within a narrow frequency region near 90 kHz. Variations in the pump modulation depth in the range  $3\% < h < 15\%$  in this region gave rise to bistable asymmetric quasi-periodic regimes. Both these regimes are similar and differ only in the propagation direction of a stronger wave. Figure 1 presents typical oscillograms and spectrograms of the intensity of counterpropagating waves in these regimes. One can see that discrete equidistant spectral components separated by the interval equal to the modulation frequency dominate in the intensity spectrum in these regimes. The main specific feature of these regimes is a substantial difference between the temporal structures of radiation in the opposite directions and spectral nonreciprocity (the intensity spectra of counterpropagating waves prove to be different). Some difference between the temporal structures of radiation in bistable regimes can be explained by a small difference between the coupling coefficients of counterpropagating waves (which can appear, for example, due to parasitic reflections in the receiving channel) or by a small optical nonreciprocity due to the influence of the geomagnetic field.

Based on the results presented above, we can conclude that the spontaneous breaking of the spectral symmetry of radiation of counterpropagating waves takes place in the indicated region of parameters. The difference between the temporal and spectral structures of radiation of counterpropagating waves can be also described by the correlation coefficients of intensities  $K_I$  and intensity spectra  $K_\omega$ . For oscillograms and spectrograms presented in Fig. 1,  $K_I = -0.26$  and  $K_\omega = 0.86$ .

The width of the modulation frequency interval in which the spontaneous breaking of spectral nonreciprocity occurs depends on the pump modulation depth and the excess over the pump threshold. For  $h = 12\%$  and  $\eta = 0.15$ , quasi-periodic regimes with the spectral nonreciprocity exist in the modulation frequency range  $85 \text{ kHz} < \omega_p/2\pi < 93 \text{ kHz}$ . Outside this region, regimes of asynchronous dynamic chaos are observed with virtually identical emission spectra of counterpropagating waves, for which correlation coefficients are close to unity.

The breaking of the temporal and spectral symmetry of radiation of counterpropagating waves was also experimentally observed in some other narrow regions of parametric resonances (for example, at the pump modulation frequencies close to 60 and 120 kHz).

## 4. Results of numerical simulation

The possibility of spontaneous spectral symmetry breaking in regions of parametric resonances in SRLs with periodically modulated pumping was also demonstrated by numerical simulation. We used the standard SRL model, which in the absence of optical nonreciprocity and for symmetric coupling coefficients [equal moduli ( $m = m_1 = m_2$ ) and zero phases of complex coupling coefficients] is described by the system of differential equations for the complex amplitudes  $E_{1,2}$  of counterpropagating waves and spatial harmonics of the inverse population  $N_0, N_+,$  and  $N_- = N_+^*$  [8]

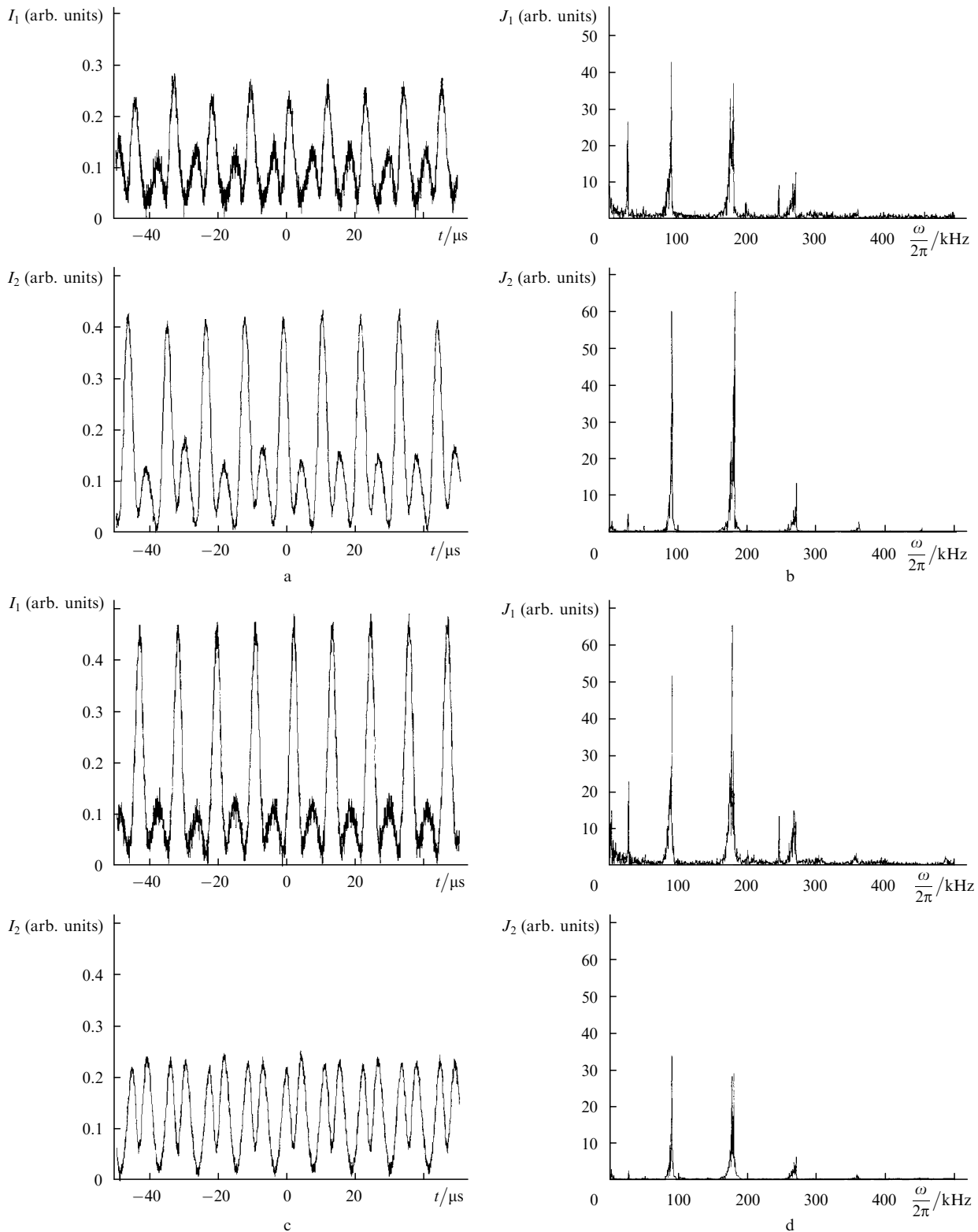
$$\begin{aligned} \frac{dE_{1,2}}{dt} &= -\frac{\omega}{2Q} E_{1,2} + \frac{i}{2} m E_{2,1} + \frac{\sigma l}{2T} [N_0 E_{1,2} + N_\mp E_{2,1}], \\ T_1 \frac{dN_0}{dt} &= N_{\text{th}}(1 + \eta) - N_0 [1 + a(|E_1|^2 + |E_2|^2)] \\ &\quad - N_+ a E_1 E_2^* + N_- a E_2 E_1^*, \end{aligned} \quad (2)$$

$$T_1 \frac{dN_\pm}{dt} = -N_\pm [1 + a(|E_1|^2 + |E_2|^2)] - N_0 a E_2 E_1^*,$$

where  $\omega/Q$  is the resonator bandwidth;  $N_{\text{th}}$  is the threshold inversion population;  $T = L/c$  is the round-trip transit time for light in the resonator;  $L$  is the perimeter length of the ring resonator;  $l$  is the active element length;  $a = T_1 c \sigma / (8 \hbar \omega \pi)$  is the saturation parameter;  $T_1$  is the inverse population relaxation time;  $\sigma$  is the laser transition cross section; and  $\eta$  is the excess of the pump power over the threshold. We assume here that the laser emits at the frequency coinciding with the centre of the luminescence line.

Parameters were selected close to the parameters of the chip laser under study:  $\omega_m/2\pi = 170 \text{ kHz}$ ,  $\eta = 0.15$ , radiation losses per round-trip transit in the resonator were set equal to 4%, which corresponds to the relaxation frequency  $\omega_r/2\pi = 61.7 \text{ kHz}$  for  $\eta = 0.15$ .

Let us present the results of numerical simulation for parametric resonance (1) for  $n = 1$  and  $k = 2$ . For the pump modulation depth  $h = 5\%$ , bistable regimes with different temporal and spectral structures of radiation of counterpropagating waves were observed in the modulation frequency interval  $86.3 \text{ kHz} < \omega_p/2\pi < 88.2 \text{ kHz}$ . Figure 2 shows the temporal and spectral characteristics of counterpropagating waves for one of these regimes. The temporal and spectral characteristics in another bistable regime are the same, but for the waves propagating in the opposite direction. Outside the region of spontaneous symmetry breaking, the regimes with symmetric intensity spectra of counterpropagating waves are established: for  $\omega_p/2\pi > 88.2 \text{ kHz}$ , the quasi-sinusoidal lasing regime QS-I appears [7], and for  $\omega_p/2\pi < 86.3 \text{ kHz}$  the regime of asynchronous dynamic chaos develops. The projections of phase portraits on the  $(I_1, I_2)$  plane for these lasing regimes are shown in Fig. 3. One can see that the radiation intensity distributions for counterpropagating waves in the QS-I regime are symmetric, whereas in the quasi-periodic regime with the spontaneous spectral symmetry breaking these distributions are asymmetric. In the asynchronous chaos regime, the intensity distribution is also asymmetric; however, the intensity spectra are almost identical.

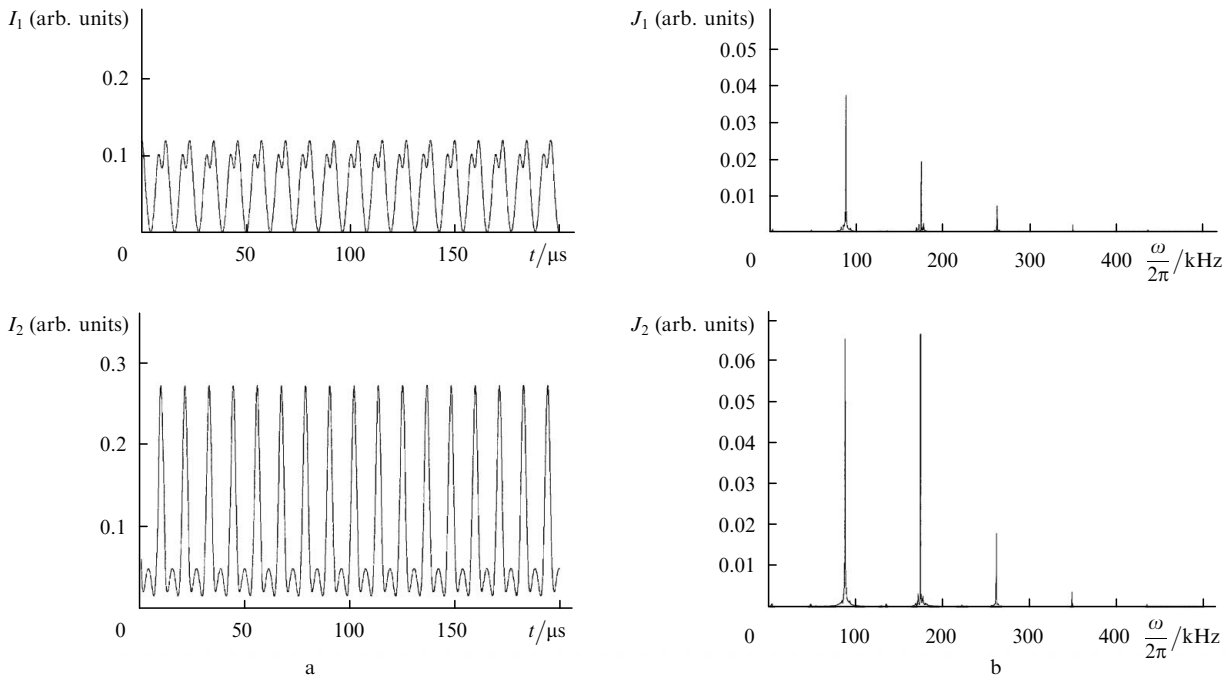


**Figure 1.** Intensity oscillograms for counterpropagating waves  $I_{1,2}$  (a, c) and intensity spectra  $J_{1,2}$  (b, d) in bistable lasing regimes with spontaneous spectral symmetry breaking for  $\eta = 0.15$ ,  $h = 8\%$ , and  $\omega_p/2\pi = 90$  kHz.

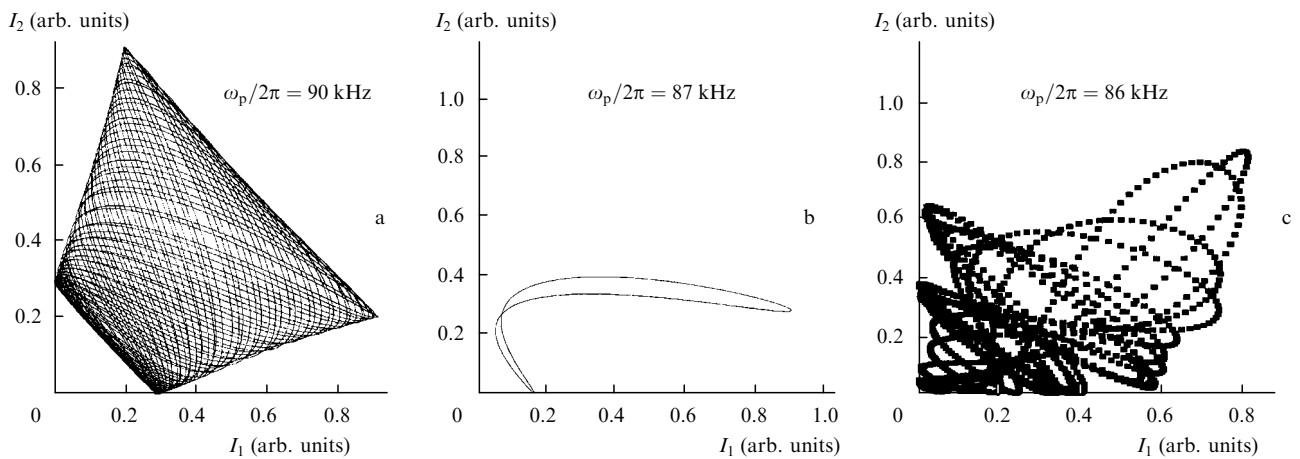
## 5. Conclusions

We have demonstrated experimentally and theoretically a new possibility of the spontaneous breaking of symmetry of temporal and spectral characteristics of radiation in a SRL upon periodic modulation of the pump power. The symmetry breaking takes place in the parametric resonance

regions between the pump modulation frequency and self-modulation oscillation frequency. Spontaneous symmetry breaking leads to the development of bistable quasi-periodic lasing regimes in the indicated frequency regions, which have different temporal and spectral structures of radiation for counterpropagating waves and also different average intensities of counterpropagating waves.



**Figure 2.** Time dependences of the intensities  $I_{1,2}$  of counterpropagating waves (a) and spectra  $J_{1,2}$  (b) in one of the bistable lasing regimes obtained by numerical simulation for  $\eta = 0.15$ ,  $h = 5\%$ , and  $\omega_p/2\pi = 87$  kHz.



**Figure 3.** Projections of phase portraits on the  $(I_1, I_2)$  plane obtained by numerical simulation for  $\eta = 0.15$ ,  $h = 5\%$ , and different values of  $\omega_p/2\pi$ .

A comparison of experimental results with numerical simulations has shown that the phenomena considered in the paper are well described by the standard SRL model.

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