Quantum Electronics **36** (5) 483 – 486 (2006)

PACS numbers: 42.55.Rz; 42.55.Xi
DOI: 10.1070/QE2006v036n05ABEH013248

Study of thermooptic distortions of a Nd : YVO_4 active element at different methods of its mounting

V.V. Kijko, E.N. Ofitserov

***Abstract*. Thermooptic distortions of the active element of an axially diode-pumped Nd : YVO_4 solid-state laser are studied at different methods of its mounting. The study was performed by the Hartmann method. A mathematical model for calculating the optical power of a thermal lens produced in the crystal upon pumping is developed and verified experimentally. It is shown that the optical power of a thermal lens produced upon axial pumping of the convectively cooled active element sealed off in a copper heat sink is half the optical power observed upon convective cooling of the active element without heat sink. The experimental and theoretical results are in good agreement.**

Keywords: *laser, diode pumping, thermal lens.*

1. Introduction

The active elements of solid-state lasers are heated during laser operation due to conversion of a part of absorbed pump energy to heat [1, 2]. At present one of the most popular active elements of compact lasers is a Nd : YVO_4 crystal. Because of a low heat conductivity of this crystal, cw pumping produces a large temperature gradient in its cross section perpendicular to the pumping direction, resulting in the appearance of large temperature stresses in the crystal. The stresses change the position of the system of optical axes and the principal values of the refractive index. The optical properties of the crystal change both due to the temperature variation in the refractive index ($\mathrm{d}n/\mathrm{d}T$) and the dependence of the refractive index on thermal stresses ($\mathrm{d}n/\mathrm{d}\sigma_{x,y}$).

Thermal distortions produce a thermal lens and induced birefringence. In addition, thermal deformations of the element surface appear in the pump region, which lead to the distortion of the wave front of radiation, excitation of high-order angular oscillations, and the increase in the divergence, and also affect the polarisation characteristics of laser radiation. Thermal stresses can result in the crystal damage [1]. Thus, the heating of the active element strongly affects the properties of laser radiation and is one of the factors determining laser operation.

Diode pumping provides an increase in the lasing efficiency but it does not eliminate the problem of thermooptic influence of pumping on the parameters of laser radiation. Upon diode-end pumping, the beam cross section in the active element is a few hundreds of micrometers for a typical average power of several watts. Because the energy density in the beam waist is high, a thermal lens is produced in the crystal, which is comparable with thermooptic distortions produced in active elements upon flashlamp pumping.

In this paper, we present the results of theoretical and experimental studies of a thermal lens in the Nd : YVO_4 active element in the form of a 3 mm × 3 mm × 1 mm plate fixed by two different methods: by two end faces, which makes the cooling conditions close to convective ones (Fig. 1), and inside copper plates through an indium layer covering the entire surface of the crystal (except its central part of diameter 1.2 mm), which improves the cooling conditions (Fig. 2).

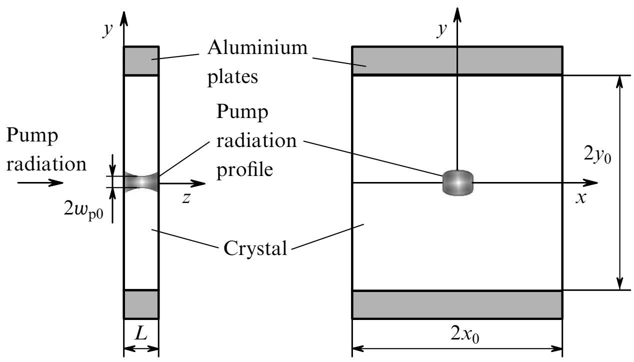

Figure 1. Mounting of the Nd : YVO_4 active element providing convective cooling of the crystal without a heat sink.

2. Theoretical model

The problem of heating the active element by pump radiation is reduced in the general case to the solution of the heat conduction equation describing the spatiotemporal variation of temperature in the active medium. The differential heat conduction equation for isotropic bodies has the form [1, 3]

V.V. Kiiko, E.N. Ofitserov A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia; e-mail: hkww@ran.gpi.ru, oficerov@ran.gpi.ru

Received 6 March 2006
Kvantovaya Elektronika **36** (5) 483 – 486 (2006)
Translated by M.N. Sapozhnikov

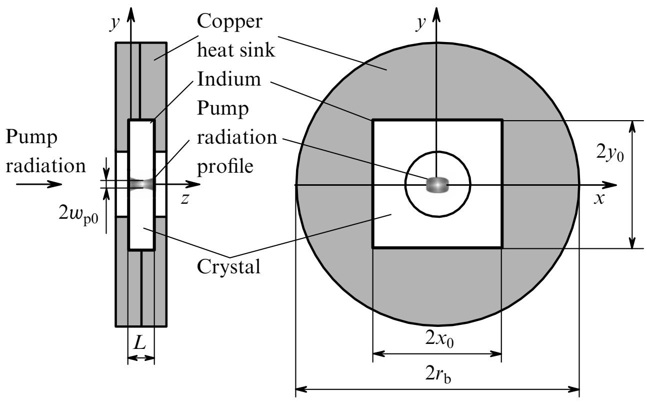

Figure 2. Mounting of the Nd : YVO_4 active element providing convective cooling of the crystal – heat sink system.

$$c(T)\rho_m(T)\,\frac{\partial T}{\partial t}\,(x,y,z,t) = \nabla[K(T)\nabla T(x,y,z,t)] + Q(x,y,z,t), \tag{1}$$

where $T(x,y,z,t)$ is the spatiotemporal temperature distribution; $Q(x,y,z,t)$ is the thermal flow, which is a part of the pump power; $c(T)$ is the specific heat; $\rho_m(T)$ is the material density; and $K(T)$ is the heat conductivity. Consider the solution of equation (1) under the following conditions:

(i) The time interval between the current instant of time and the pumping onset is sufficiently large, i.e., the initial conditions have no effect on the temperature distribution in a medium;

(ii) the heat conduction and specific heat, as well as density are independent of temperature: $K(T) = K = \text{const}$, $c(T) = c = \text{const}$, $\rho_m(T) = \rho = \text{const}$.

We will solve the heat conduction equation by using several simplifications:

(i) Due to the smallness of the cross section of the pump region (0.3 – 0.5 mm) (with respect to the transverse size of the crystal), we can use cylindrical coordinates in Eqn (1) in which the reduced radius of the crystal is $r_0 = (x_0^2 + y_0^2)^{1/2}$ [where $2x_0$ and $2y_0$ are crystal dimensions in the Cartesian coordinates (Figs 1 and 2)].

(ii) The pump region in the active medium is close to cylindrical.

(iii) The Nd : YVO_4 active element is isotropic in the heat conduction K. We assume that $K = 5.23$ W m^{-1}K^{-1} [4 – 6].

After passing to the cylindrical coordinates, Eqn (1) takes the form [2]

$$\frac{\partial^2 T}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T}{\partial \rho} + \frac{\partial^2 T}{\partial z^2} = -\frac{Q(\rho,z)}{K}. \tag{2}$$

We will consider the stationary temperature distribution in the active element by representing a thermal flow in the form of the reduced Gaussian spatial function [4, 5]

$$Q(\rho,z) = P_{\text{abs}}\,p(\rho,z)\,\frac{\alpha_{p0}}{1-\exp(-\alpha_{p0}L)}\exp(-\alpha_{p0}L), \tag{3}$$

$$p(\rho,z) = \frac{2}{\pi w_p^2(z)}\exp\left[-\frac{2\rho^2}{w_p^2(z)}\right], \tag{4}$$

where α_{p0} is the absorption coefficient of the medium at the pump wavelength; $P_{\text{abs}} = P_p[1-\exp(-\alpha_{p0}L)]$ is the absorbed pump power; P_p is pump radiation power; L is the crystal length; $w_p(z)$ is the transverse size of the pump region in the active medium, which in our case is determined by the expression [5, 6]

$$w_p^2(z) = w_{p0}^2\left[1 + \left(\frac{M^2\lambda_p z}{\pi n_0 w_{p0}^2}\right)^2\right] \approx w_{p0}^2, \tag{5}$$

where w_{p0} is the minimal radius of the pump region in the active element; λ_p is the pump wavelength; n_0 is the refractive index of the active medium; and M^2 is the quality parameter of pump radiation [7].

The solution of the differential heat conduction equation (2) was obtained for the following boundary conditions:

(i) Convective cooling of the active element (Fig. 1). In this case, the interaction between the crystal surface and environment can be written in the form of the boundary condition of the third kind

$$-\left.\frac{\partial T(\rho,z)}{\partial \rho}\right|_{\rho=r_0} = \frac{h}{K}\,[T(r_0,z) - T_m], \tag{6}$$

where T_m is the environment temperature and h is the coefficient of heat exchange with the environment.

(ii) Convective cooling of the active element with a copper heat sink (Fig. 2). Because the crystal – heat sink system has a perfect thermal contact in this case, the temperatures of the crystal contact and heat sink are equal:

$$T_{\text{cr}}(\rho,z)|_{\rho=r_0} = T_{\text{Cu}}(\rho,z)|_{\rho=r_0}, \tag{7}$$

and the specific thermal flows at each point are also equal (in the absence of heat release at these points) [8]:

$$-K\left.\left[\frac{\mathrm{d}T_{\text{cr}}(\rho,z)}{\mathrm{d}\boldsymbol{n}}\right]\right|_{\rho=r_0} = -K_{\text{Cu}}\left.\left[\frac{\mathrm{d}T_{\text{Cu}}(\rho,z)}{\mathrm{d}\boldsymbol{n}}\right]\right|_{\rho=r_0}, \tag{8}$$

where $T_{\text{cr}}(\rho,z)$ and $T_{\text{Cu}}(\rho,z)$ are the temperatures of the crystal and copper heat sink on the contact surface, respectively; and K_{Cu} is the heat conduction of copper.

The solution of the heat conduction equation in the first case has the form

$$\Delta T_1(\rho,z) = T(\rho,z) - T_m = \frac{P_{\text{abs}}}{4\pi K}\,\frac{\alpha_{p0}\exp(-\alpha_{p0}z)}{1-\exp(-\alpha_{p0}L)}$$
$$\times\left[2\exp\left(-\frac{2r_0}{w_{p0}^2}\right)\ln\left(\frac{r_0}{\rho}\right) + E_1\left(\frac{2r_0^2}{w_{p0}^2}\right) - E_1\left(\frac{2\rho^2}{w_{p0}^2}\right)\right],$$

where $E_1(2r_0^2/w_{p0}^2)$ [9] is the second order of smallness, which can be neglected. Then, we obtain

$$\Delta T_1(\rho,z) = \frac{P_{\text{abs}}}{4\pi K}\,\frac{\alpha_{p0}\exp(-\alpha_{p0}z)}{1-\exp(-\alpha_{p0}L)}$$
$$\times\left[2\exp\left(-\frac{2r_0^2}{w_{p0}^2}\right)\ln\left(\frac{r_0}{\rho}\right) - E_1\left(\frac{2\rho^2}{w_{p0}^2}\right)\right]. \tag{9}$$

The solution of the differential equation in the second case has the form

$$\Delta T_2(\rho,z) = T(\rho,z) - T_{\rm cr} = \frac{P_{\rm abs}\alpha_{\rm p0}}{4\pi K h r_{\rm b}\ln(r_{\rm b}/r_0)}$$
$$\times\frac{\exp(-\alpha_{\rm p0}z)}{1-\exp(-\alpha_{\rm p0}L)}\left\{2Khr_{\rm b}\exp\left(-\frac{2r_{\rm b}^2}{w_{\rm p0}^2}\right)\ln\left(\frac{r_0}{r_{\rm b}}\right)\right.$$
$$+hr_{\rm b}E_1\left(\frac{2r_{\rm b}^2}{w_{\rm p0}^2}\right)\ln\left(\frac{r_{\rm b}}{r_0}\right)+hr_{\rm b}\left[E_1\left(\frac{2r_0^2}{w_{\rm p0}^2}\right)-E_1\left(\frac{2\rho^2}{w_{\rm p0}^2}\right)\right]$$
$$\left.+(hr_{\rm b}\ln\rho+KK_{\rm Cu})\left[E_1\left(\frac{2r_{\rm b}^2}{w_{\rm p0}^2}\right)-E_1\left(\frac{2r_0^2}{w_{\rm p0}^2}\right)\right]\right\},$$

where $r_{\rm b}$ is the radius of the copper heat sink. By excluding the functions of the second order of smallness $E_1(2r_0^2/w_{\rm p0}^2)$ and $E_1(2r_{\rm b}^2/w_{\rm p0}^2)$, we obtain

$$\Delta T_2(\rho,z) = \frac{P_{\rm abs}\alpha_{\rm p0}}{4\pi K\ln(r_{\rm b}/r_0)}\frac{\exp(-\alpha_{\rm p0}z)}{1-\exp(-\alpha_{\rm p0}L)}$$
$$\times\left[2K\exp\left(-\frac{2r_{\rm b}}{w_{\rm p0}^2}\right)\ln\left(\frac{r_0}{\rho}\right)-E_1\left(\frac{2\rho^2}{w_{\rm p0}^2}\right)\right]. \quad (10)$$

The term $E_1(2\rho^2/w_{\rm p0}^2)$ in (9) and (10) can be represented in the form of a quadratic power series in ρ and written as $2\rho^2/w_{\rm p0}^2$ [9]. In this case, Eqns (9) and (10) can be written in the form

$$\Delta T_1(\rho,z) \approx \frac{P_{\rm abs}}{4\pi K}\frac{\alpha_{\rm p0}\exp(-\alpha_{\rm p0}z)}{1-\exp(-\alpha_{\rm p0}L)}\frac{2\rho^2}{w_{\rm p0}^2}, \quad (11)$$

$$\Delta T_2(\rho,z) \approx \frac{P_{\rm abs}}{4\pi K\ln(r_{\rm b}/r_0)}\frac{\alpha_{\rm p0}\exp(-\alpha_{\rm p0}z)}{1-\exp(-\alpha_{\rm p0}L)}\frac{2\rho^2}{w_{\rm p0}^2}. \quad (12)$$

The reduced focal distance f of a thermal lens produced in the active medium upon pumping depends on the phase shift $\Delta\Phi$ of a wave in the crystal and is described by the expression [3]

$$f = \frac{k\rho^2}{2\Delta\Phi}, \quad (13)$$

where k is the wave number. The total phase shift $\Delta\Phi$ depending on the refractive index n, which in turn depends on the gradient dn/dT produced upon pumping, is described by the equation

$$\Delta\Phi = \int_0^L k\Delta n(\rho,z)dz, \quad (14)$$

where $\Delta n(\rho,z) = \Delta T(\rho,z)(dn/dT)$. The average distribution of the temperature field over the crystal cross section can be found by the root-mean-square approximation of the temperature field distribution $\Delta T(\rho,z)$ by a parabola over the entire cross section of the active element [1]. In the first approximation, we can rest ourselves to the local component of the curvature of the temperature field at the central point.

Therefore, the reduced focal distance of the thermal lens produced in the crystal upon pumping is described by the equation

$$f = \frac{\rho^2}{2\int_0^L \Delta T(\rho,z)(dn/dT)dz}. \quad (15)$$

By substituting (11) and (12) into (15), we obtain the dependences of the reduced focal distance of the thermal lens for the two methods of crystal mounting shown in Figs 1 and 2, respectively:

$$f(P_{\rm p}) = \frac{\pi K w_{\rm p0}^2}{P_{\rm p}[1-\exp(-\alpha_{\rm p0}L)](dn/dT)}, \quad (16)$$

$$f(P_{\rm p}) = \frac{\pi K w_{\rm p0}^2\ln(r_{\rm b}/r_0)}{P_{\rm p}[1-\exp(-\alpha_{\rm p0}L)](dn/dT)}. \quad (17)$$

We will calculate the focal distance of a thermal lens produced in the active element by using the parameters of a Nd : YVO$_4$ crystal presented in [10]: $K = 5.23$ W m^{-1} K^{-1}, $\alpha_{\rm p0} = 19.3$ cm^{-1}, $P_{\rm p} = 0.5 - 2.5$ W; $w_{\rm p0} = 0.2\times10^{-3}$ m, $dn/dT = 3\times10^{-6}$ K^{-1}, $x_0 = y_0 = 1.5\times10^{-3}$ m, $2r_{\rm b} = 24\times10^{-3}$ m.

The dependences of the focal distance of the thermal lens on the pump power calculated for the two cases of active element mounting are presented in Fig. 3 (solid curves). One can see that upon convective cooling the crystal with the heat sink, the optical power of the thermal lens is half that upon convective cooling without the heat sink.

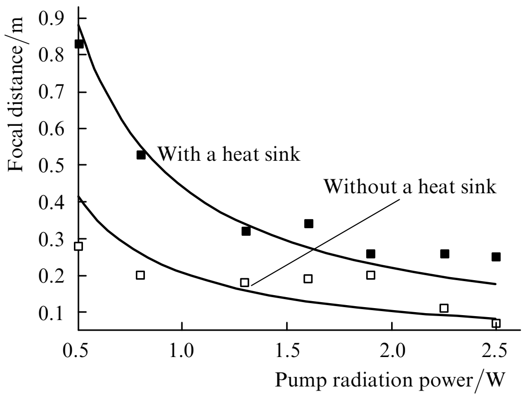

Figure 3. Theoretical (curves) and experimental (points) dependences of the focal distance of the thermal lens on the pump power for two different mounting (cooling) methods.

3. Experimental results

Thermooptic distortions were studied on the experimental setup shown in Fig. 4 by the Hartmann method [11, 12]. The setup contains an ATC-2550 laser diode with a focusing optics, its LDD-9A power supply, a Nd : YVO$_4$ active element under study, a collimating objective, a set of IKS1, 5, 6 and NS1, 8 optical filters, a Hartmann diaphragm, an LCL 902HS video camera with a power supply, and a PC. The 808-nm laser diode was used both for pumping and as a reference radiation source. The Nd : YVO active element was placed in the pump-beam waist. The Hartmann diaphragm represented a set of holes located in the nodes of the fifth-order hexagonal grid. The diameter of holes was 0.2 mm and the hexagonal grid diameter was 3.5 mm. The diaphragm was placed at the distance $d = 15.8$ mm from the video camera at the exit pupil of the optical system.

Measurements were performed in the following way. For each value of the pump radiation power, the video camera

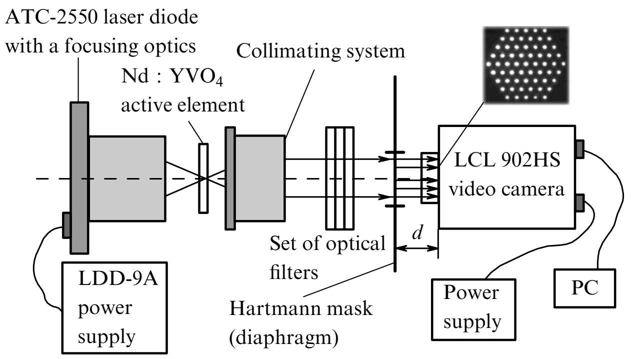

Figure 4. Scheme of the experimental setup.

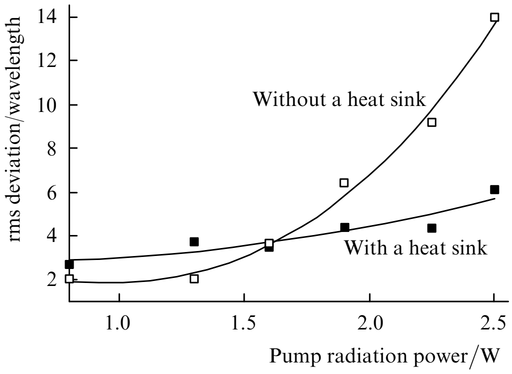

Figure 5. Theoretical (curves) and experimental (points) dependences of the root-mean-square (rms) deviation of the wave front distorted by the thermal lens from the reference wave front on the pump power.

recorded two images of the Hartmann diaphragm (Fig. 4): one for the laser diode wavelength corresponding to the maximum of absorption of the pump radiation in the active element, and the other for the radiation wavelength corresponding to the absorption minimum. The radiation wavelength of the laser diode was temperature tunable. A preliminary calibration showed that the shape of the wave front of laser diode radiation was almost independent of the diode temperature. The root-mean-square variations in this shape in the absence of the active element did not exceed 3 % upon the change in temperature by 15 °C.

Then, we determined the coordinates of the 'centres of gravity' of light spots on the images of the Hartmann diaphragm and the displacements of the 'centres of gravity' of the wave front distorted by the thermal lens (corresponding to the absorption maximum for pump radiation) with respect to the reference wave front (corresponding to the absorption minimum). The local slopes of the wave front calculated from the displacements of the 'centres of gravity' of light spots were used to reconstruct the wave-front shape described by the weight coefficients of the expansion in the first ten Zernicke polynomials. The reduced focal distance was determined from the radius of curvature of the reference sphere closest to the reconstructed wave front, which was calculated by the method of least squares.

The experimental values of the focal distance of the thermal lens obtained at different pump powers are shown by points in Fig. 3. One can see that the experimental data and theoretical curves are in good agreement.

Because the Hartmann method allows one to reconstruct the wave-front shape almost completely, we studied experimentally higher-order aberrations, which substantially affect the diffraction-limited divergence of the output radiation of a solid-state laser. The quality of the induced thermal lens was numerically estimated by calculating the root-mean-square deviation of the wave front distorted by the thermal lens from the reference wave front obtained for a minimal thermal lens in the crystal.

The experimental dependences of root-mean-square deviations (in units of the laser wavelength) on the pump power are presented in Fig. 5. One can see that for the pump power above 1.7 W, the root-mean-square deviations of the wave front from the reference wave front in the case of convective cooling the crystal without the heat sink are twice as large as those upon convective cooling the crystal with the copper heat sink.

4. Conclusions

Our study of thermooptic distortions in the Nd : YVO_4 active element at two different methods of its mounting providing two types of its cooling have shown that the mounting of the crystal inside a convectively cooled copper heat sink reduces the optical power of the induced thermal lens by half. In this case, not only the increase in the focal distance of the thermal lens is observed but also the fraction of higher-order aberrations decreases. Therefore, by cooling convectively the active element – copper heat sink system, one can expect an increase in the lasing efficiency and an improvement in the output radiation quality compared to the convective cooling of the crystal without the heat sink, all other factors being the same.

References

1. Koechner W., Bass M. *Solid-State Lasers: a Graduate Text* (New York: Springer-Verlrag, 2003).
2. Schmid M., Graf Th., Weber H.P. *J. Opt. Soc. Am. B*, **17** (8), 1398 (2000).
3. Innocenzi M.E., Yura H.T., Fincher C.L., Fields R.A. *Appl. Phys. Lett.*, **56** (19), 1831 (1990).
4. Chen Y.F. *IEEE J. Quantum Electron.*, **35** (2), 234 (1999).
5. Peng X., Xu L., Asundi A. *IEEE J. Quantum Electron.*, **38** (9), 1291 (2002).
6. Chen Y.F., Liao T.S., Huang C.F., Lin K.H., Wang S.C. *IEEE J. Quantum Electron.*, **32** (11), 2010 (1996).
7. *Optics and Optical Instruments. Lasers and Laser Related Equipment. Test Methods for Beam Widths, Divergence Angle and Beam Propagation Factor* (Geneva: ISO 11146, 2005).
8. Gröber H., Erk S. *Die Grundgesetze der Warmeubertragung* (Berlin-Gottingen-Heidelberg: Springer-Verlag, 1955).
9. Abramawitz M., Stegun I.A. *Handbook of Mathematical Functions* (New York: Dover, 1965).
10. Sennaroglu A. *Opt. Quantum Electron.*, **32**, 1307 (2000).
11. Malacara D. *Appl. Opt.*, **11** (1), 99 (1972).
12. Morales A., Malacara D. *Appl. Opt.*, **22** (4), 3957 (1983).