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# Optimisation of a high-bit-rate optical communication link with a nonideal quasi-rectangular filter

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Abstract. The problem of data communication over a wavelength division multiplexing (WDM) high-data-rate communication link where wave division multiplexing is performed by using quasi-rectangular Bragg élters is studied numerically. It is shown that the optimisation of the mean dispersion of the link, the width and shape of the transfer function of filters provides the increase in the data communication range up to 1550 km.

Keywords: optical communication link, optical éltration, Bragg filter

# 1. Introduction

Control of dispersion and wavelength division multiplexing (WDM) considerably increase the transmission capacity of optical communication links [\[1\].](#page-3-0) The main obstacle to a further increase in the transmission capacity of communication links is the crosstalk between signals in neighbouring channels and the interbit crosstalk within one channel. To reduce the crosstalk between channels for the given transmission bandwidth, it is necessary to reduce the channel width. However, this leads to the increase in the pulse duration. As a result, pulses propagating at a given repetition rate begin to overlap and the interbit crosstalk increases.

The relation between the channel width and the bit interval can be determined by solving the problem of generating a train of pulses by using an ideal rectangular filter. Consider a short Gaussian pulse passing through a narrow rectangular filter. If the width of the pulse spectrum is much greater than the transmission bandwidth  $B$  of the filter, the output pulse will have a rectangular spectrum described by the function  $\operatorname{sinc} (\pi Bt) = [\sin (\pi Bt)]/(\pi Bt)$ . The bandwidth  $B$  should be selected so that the adjacent channels would not overlap and the frequency bandwidth would be filled completely. The optimal pulse repetition rate is the rate at which the maximum of the amplitude of each next pulse coincides with the zero of the previous pulse [\[2\].](#page-3-0)

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A filter of this type was realised in the form of a fibre Bragg grating (FBG) reflector in [\[3\].](#page-3-0)

In long communication links, additional factors appear which distort signals: the Kerr nonlinearity, noise of amplifiers, chromatic dispersion of the fibre, and nonideal characteristics of the filter. The group velocity dispersion is compensated by periodically located ébre pieces with opposite dispersion signs. The average dispersion is

$$
\langle D \rangle = \frac{D_1 L_1 + D_2 L_2}{L_1 + L_2},
$$

where  $L_i$  and  $D_i$  are the fibre piece length and its dispersion; the subscripts 1 and 2 refer to a standard single-mode and compensating fibres, respectively. In the absence of nonlinear interaction, the average dispersion does not affect the quality of the received signal if the dispersion is compensated at the end of the communication link. However, due to nonlinearity, the incomplete compensation of dispersion becomes optimal. The optimal average dispersion is determined by the parameters of the communication link and signal, it is unknown beforehand and remains one of the optimisation parameters. Another control parameter is the filter bandwidth.

The aim of this paper is to optimise the optical system for increasing the range over which bit sequences can be communicated with the acceptable bit error rate.

## 2. Spectrally flat format

To simulate the optical data communication channel, a bit sequence composed of short Gaussian pulses was preliminary transmitted through an FBG-based optical filter. Because not any spectral characteristic of the filter can be realised, it is necessary, by specifying preliminary the required characteristic, to solve the inverse scattering problem and to find the profile of the FBG with the reflection spectrum close to the required spectrum. Then, the direct scattering problem should be solved and the `real' spectral characteristic of the filter determined. In the general case the real characteristic for a finite FBG will differ from the required one not only by the reflectance amplitude but also by the group delay.

The grating profile was synthesised from the specified complex reflectance by the known method of discrete piling [\[4\].](#page-3-0) The reflectance  $|R|_{\text{max}}$  at the centre was set equal to  $1 - 5 \times 10^{-5}$ . The reflectance amplitude was described by a quasi-rectangular function with the required width and front steepness:

$$
|R(\xi)| = \frac{\cosh(2\pi \times 1.8) - 1}{\cosh(2\pi \times 1.8) + \cosh(2\pi \times 12.4\xi)},
$$
 (1)

where  $\xi = (\omega - \omega_0)/\omega_0$  is the dimensionless frequency detuning from the reflection band centre  $\omega_0$ . The phase of the reflectance was selected to provide the absence of the group delay because the frequency-dependent group delay distorts the signal. The family of quasi-rectangular func-tions was taken from paper [\[5\]](#page-3-0) where the profile of the FBG with the reflection spectrum expressed in terms of hyperbolic functions (1) was found analytically.

The 'real' transfer function of the filter is presented together with the group delay in Fig. 1a, and the proposed FBG profile is shown in Fig. 1b. One can see from Fig. 1a that the inhomogeneity of the group delay upon the decrease of the reflectivity by an order of magnitude compared to its maximum value is about 5 ps. Therefore, the delay within the reflection band is much smaller than the distance between pulses and the duration of a pulse.



**Figure 1.** Reflection spectrum  $|R|^2$  of the FBG (solid curve) and group delay  $\tau$  (dashed curve) (a) and the grating profile  $\delta n/n$  reconstructed from the reflection spectrum by the method of layer-to-layer recovery (b); z is the coordinate along the grating,  $\delta n(z)$  is the envelope of the oscillating addition to the average refractive index n.

After propagation through such a filter, a narrow Gaussian pulse acquires the sinc-like shape. The bandwidth  $B$  of the optical filter was chosen so that the zeroes of the function are located at the middles of 25-ps bit intervals. As a result, the crosstalk between adjacent bits decreases. At the receiving end of the link, channels are separated with the help of filters that are similar to those used at the beginning of the link. Pulses propagate through the élter again, which results in the additional change in their shape. Figure 2 shows pulse profiles after one and two passages through the



Figure 2. Optical pulse shape  $|A|^2$  normalised to the maximum power after one (dashed curve) and two (solid curve) passages through the optical filter.

optical filter. The first zero of the function after the second filter virtually coincides with the first zero of the function sinc ( $\pi Bt$ ). The long tails of the function decrease, of course, much faster than those of the function sinc  $(\pi Bt)$  because the spectral profile of the filter reflectance was described by smooth function (1).

#### 3. Generalised nonlinear Schrödinger equation

The propagation of optical pulses in a fibreoptic communication link with the distributed dispersion is described by the generalised nonlinear Schrodinger equation [\[1\]](#page-3-0)

$$
i\frac{\partial A}{\partial z} - \frac{\beta_2(z)}{2} \frac{\partial^2 A}{\partial t^2} + \sigma(z)|A|^2 A
$$
  
= 
$$
i \left[ -\gamma(z) + \sum_{k=1}^N r_k \delta(z - z_k) \right] A.
$$
 (2)

Here,  $z$  is the distance along the communication link;  $t$  is the time; A is the light-wave amplitude;  $\beta_2 = -\lambda_0^2 D/(2\pi c)$  is the group velocity dispersion;  $\lambda_0$  and c are the wavelength and speed of light, respectively;  $\sigma$  is the Kerr nonlinearity coefficient;  $z_k$  are the coordinates of point amplifiers with the gains  $r_k$ ; N is the number of amplifiers; and  $\gamma(z)$  is the signal attenuation coefficient. The parameters  $\sigma$  and  $\beta_2$  are written as functions of the coordinate to take into account their change on passing from one type of the fibre to another. The nonlinearity coefficient was calculated from the expression  $\sigma = 2\pi n_2/(\lambda_0 a_{\text{eff}})$ , where  $n_2$  is the nonlinear refractive index and  $a_{\text{eff}}$  is the effective cross section of the eigenmode of the fibre.

We considered an optical system consisting of pieces of a standard single-mode fibre ( $L_1 = 40$  km) and of a dispersion-compensating fibre  $(L_2 = 6.8 \text{ km})$  and also erbium-doped fibre amplifiers with the noise coefficient 4.5 dB. The parameters of ébres of both types used in calculations are presented below.

Standard single-mode

Dispersion compen-



We studied data transmission over eight 50-GHz-spaced frequency channels in the wavelength range between 1548.78 and 1551.78 nm. The channels were combined and separated by means of 45-GHz filters shifted by 4 GHz with respect to the channel centre. The shift of the optical filter with respect to the channel centre is a control parameter over which optimisation was also performed. The sinc-like sequence was produced by transmitting short 1.7-ps, 85-mW pulses through the optical fibre. The second output optical filter was located in front of the receiver for the separation of channels. Behind the receiver, a third-order 50-GHz electric Butterworth filter was placed.

## 4. Results of simulations

Figure 3 presents the dependences of the width of a single pulse on z at points where the distribution of the intensity  $|A|^2$  has the minimum width (at the so-called chirp-free points) for different values of the average dispersion  $\langle D \rangle$ . One can see that the width of optical pulses at chirp-free points decreases during their propagation due to a high negative average dispersion. Therefore, by controlling the average dispersion, we can reduce the overlap of pulses and suppress the nonlinear interaction of adjacent pulses (bits), thereby improving the signal transmission. A similar result was obtained in [\[6\]](#page-3-0) for Gaussian pulses.



Figure 3. Width w of a single optical pulse at chirp-free points as a function of the propagation length  $z$  for the average dispersion of the communication link  $\langle D \rangle = 0$ ,  $-0.03$ ,  $-0.06$ ,  $-0.145$ ,  $-0.23$ ,  $-0.29$ ,  $[-0.32, -0.35, -0.38 \text{ ps nm}^{-1} \text{ km}^{-1}$  (for curves from top to bottom). The pulse width is stabilised with increasing the absolute value of the average dispersion  $\langle D \rangle$ .

The quality of a communication system is estimated by the bit error rate (BER), which is deéned as the ratio of the number of erroneous bits to the total number of transmitted bits. In our case,  $BER = 10^{-9}$ . The direct measurement of this value would require calculations and measurements with a very great number of pulses. We assumed instead that the probability densities  $p_i$  ( $i = 0, 1$ ) for appearing zeroes and units are described by the normal distribution

$$
p_i(x) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right],
$$
 (3)

where  $\mu_i$  and  $\sigma_i$  are the average values and standard deviations, respectively. From this, the factor  $Q = (\mu_1 - \mu_2)$  $\mu_0$ /( $\sigma_1 - \sigma_0$ ) is found. It is assumed that the acceptable value of the Q factor for communication links is  $Q \ge 6$ . The BER is calculated by the expression

$$
\text{BER} = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp(-Q^2/2)}{\sqrt{2\pi}Q}, \ \ Q \gg 1. \tag{4}
$$

We calculated the  $Q$  factor by using 25 pseudorandom sequences, each of them containing 128 bits. The  $Q$  factor was found as the median average of 25 values calculated for each of the sequences [\[7\].](#page-3-0) Figure 4 presents the distances over which signals can be transmitted without any special error correction calculated for different values of the average dispersion  $\langle D \rangle$ . Because the effect of nonlinearity is smaller when pulses propagate in communication links with a high negative dispersion, such a regime provides better transmission compared to the case of the zero or small average dispersion.



Figure 4. Ranges of data transmission with the acceptable BER =  $10^{-9}$ for the average dispersion of the communication link  $\langle D \rangle = 0$  ( $\diamondsuit$ ),  $-0.145$  (o),  $-0.23$  ( $\bullet$ ),  $-0.32$  ( $\Box$ ), and  $-0.38$  ps nm<sup>-1</sup> km<sup>-1</sup> ( $\bullet$ ). The two last values of  $\langle D \rangle$  provide the best result.

In [\[2\],](#page-3-0) the separation of signals among channels was simulated by means of a super-Gaussian filter with the reflectance  $R(\omega) \propto \exp[-(\omega - \omega_0)^6/(2\Delta^6)]$ , where  $\omega_0$  and  $\Delta$ are the central frequency and half-width of the spectrum, respectively. It was assumed that filtration does not cause the additional group delay of pulses. The communication range of 1200 km was obtained for the 40 Gbit  $s^{-1}$  transmission rate in each channel. We considered here the filter that was more close to that used in experiments. Its group delay varied from the centre of the reflection band to the  $-10$  dB level approximately by 5 ps. It would seem that a pulse distorted by the inhomogeneous group delay should propagate over a smaller distance. However, the nonlinear interaction was compensated during optimisation by a rather high average negative dispersion, which allowed the increase in the transmission range up to 1550 km.

### 5. Conclusions

The improvement of the parameters of a communication link by using a flat top filter or a high average dispersion of the link was studied in papers [\[2\]](#page-3-0) and [\[6\],](#page-3-0) respectively. The application of an FBG-based filter was discussed in [\[3\].](#page-3-0) These methods were combined in our paper and a new shape of a highly reflecting FBG-based filter was used [see expression (1)].

<span id="page-3-0"></span>Our calculations have shown that, by using nonideal filters with the frequency-depended group delay, which distort the pulse shape, a long data transmission range can be achieved (1550 km) in a ébreoptic communication link at the 40-Gbit  $s^{-1}$  transmission rate in each channel. For this purpose, it is necessary to select the front steepness and width of filters providing the minimal overlap of pulses and spectral channels, and the average dispersion of the link compensating for the nonlinear interaction of pulses.

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