

RADIATION DETECTORS

PACS numbers: 07.60.Dq; 03.75.– b
DOI: 10.1070/QE2006v036n12ABEH013272

Magnetic calorimeter with a SQUID for detecting weak radiations and recording the ultralow energy release

A.I. Golovashkin, G.N. Izmailov, L.N. Zherikhina, G.V. Kuleshova, A.M. Tskhovrebov

***Abstract*. The scheme of a magnetic calorimeter for recording extremely low energy releases is developed. The calorimeter is activated by the method of adiabatic demagnetisation and its response to the energy release is measured with a superconducting quantum interference device (SQUID). The estimate of the ultimate sensitivity of the calorimeter with the SQUID demonstrates the possibilities of its application for detecting ultralow radiation intensity, recording single X-ray quanta in the proportional regime and other events with ultralow energy releases. The scheme of the calorimeter with the SQUID on matter waves in superfluid ^{4}He is proposed.**

Keywords: *calorimeters, matter waves, SQUID.*

1. Introduction

It was demonstrated more than a decade ago that the resolution of measuring instruments based on superconducting quantum interference devices (SQUIDs) can be comparable to the quantum limit $[\langle\delta\Phi\rangle(\Delta f)^{-1/2}]^2/(2L) \approx \hbar$ [1, 2] (where Δf is the frequency pass band, L is the effective SQUID inductance, and $\hbar$ is Planck's constant), the resolution over the magnetic flux being $\langle\delta\Phi\rangle = 5 \times 10^{-7}\Phi_0$ in the pass band $\Delta f = 1$ Hz [3–6] [where $\Phi_0 = 2\pi\hbar/(2e) = 2.07 \times 10^{-15}$ Wb is the magnetic flux quantum and e is the electron charge]. Such a high resolution has been achieved due to the development of microwave-biased ac SQUIDs [7] and two-stage dc SQUIDs, in which the second SQUID plays the role of an integrated low-noise amplifier of electric signals arriving from the first SQUID [8, 9]. However, the potential possibilities of such unique instruments are poorly used in practice so far.

We developed the principal scheme for recording extremely low energy releases based on an adiabatically demagnetised paramagnetic at an ultralow operating temperature ($T \simeq 100$ mK), whose magnetic moment is continuously monitored by the SQUID. Such magnetic calorimeters can be used to record ultralow intensity radiation and detect single quanta and rare events with ultralow energy releases (cosmic rays, weakly interacting massive particles, etc.). We also propose to use a calorimeter of this type with a SQUID based on matter waves in superfluid ^{4}He as a highly sensitive detector of entropy variations.

A.I. Golovashkin, L.N. Zherikhina, G.V. Kuleshova, A.M. Tskhovrebov P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia; e-mail: golov@sci.lebedev.ru;
G.N. Izmailov Moscow Aviation Institute (State Technical University), Volokolamskoe sh. 4, 125993 Moscow, Russia

Received 2 February 2006; revision received 21 July 2006
Kvantovaya Elektronika **36** (12) 1168 – 1175 (2006)
Translated by M.N. Sapozhnikov

2. Adiabatic magnetic calorimeter

2.1 Paramagnetic case (thermodynamic consideration)

In magnetic detectors [10–12] operating similarly to magnetic calorimeters, the energy released in the interaction of a particle with the paramagnetic working substance causes its heating, thereby changing the magnetic susceptibility detected by a SQUID in a constant external field B in the medium. This principle, as the operation principle of the known magnetic thermometer [13], is based on the Curie–Weiss mechanism $\chi(T) = \alpha/(T - T_C)$, where χ is the specific magnetic susceptibility; α is the Curie constant; and T_C is the Curie temperature ($T > T_C$).

We begin the thermodynamic calculation of such a detector (magnetic thermometer with a paramagnetic working substance) from the calculation of the derivative of its magnetic moment M, which has the form

$$\frac{\partial M}{\partial T} = \frac{\partial(\chi B)}{\partial T} = \frac{\partial}{\partial T}\left(\frac{\alpha B}{T - T_C}\right) = \frac{-\alpha B}{(T - T_C)^2}$$

according to the Curie–Weiss law. The equality of the mixed derivatives of the free energy F of a magnetic in an external field gives the relation, which yields, after integration, the 'paramagnetic' entropy S:

$$\mathrm{d}F = -S\mathrm{d}T - M\mathrm{d}B, \quad \frac{\partial^2 F}{\partial B \partial T} = \frac{\partial^2 F}{\partial T \partial B}, \quad \frac{\partial S}{\partial B} = \frac{\partial M}{\partial T},$$

$$S_{\mathrm{pm}} = \int \frac{\partial M}{\partial T}\,\mathrm{d}B = -\frac{\alpha\left(B^2 + B_{\mathrm{r}}^2\right)}{2(T - T_C)^2},$$

where B_{r} is the residual field of the paramagnetic. Then, the magnetic specific heat is

$$C_{\mathrm{pm}} = T\frac{\partial S_{\mathrm{pm}}}{\partial T} = \frac{\alpha T\left(B^2 + B_{\mathrm{r}}^2\right)}{2(T - T_C)^3}.$$

For $T \to T_C$, we have $C_{\mathrm{pm}} \gg C_{\mathrm{ph}} \propto T^3$ and, hence, $C = C_{\mathrm{pm}} + C_{\mathrm{ph}} + \ldots \approx C_{\mathrm{pm}}$, where C and C_{ph} are the total and phonon specific heat of the working substance, respectively. The response to the energy release ΔE inside the working substance (cylinder of height h), i.e. the increase in the magnetic flux $\Delta\Phi$ detected by the SQUID is described by the expression

$$\Delta\Phi = \frac{\mu_0 \Delta M}{h} = \frac{\mu_0}{h}\frac{\partial M}{\partial T}\Delta T = \frac{\mu_0}{h}\frac{\partial M}{\partial T}\frac{\Delta E}{C}$$

$$\approx \frac{\mu_0}{h}\frac{\partial M}{\partial T}\frac{\Delta E}{C_{\rm pm}} \approx \frac{-\mu_0 B}{h(B^2+B_{\rm r}^2)}\frac{T-T_{\rm C}}{T}\Delta E,$$

where μ_0 is the permeability of free space. The latter expression remains valid in the refined Curie-Weiss law taking into account magnetic saturation and removing the divergence of $C_{\rm pm}$ for $T \to T_{\rm C}$.

A disadvantage of the detector under study is, first, the necessity of accurate balancing of the gradient-metric system of the SQUID [14] and stabilisation of the external magnetic field B. Second, a thermal valve should be used which should satisfy the contradictory and difficultly achievable requirements such as the extremely high heat conduction in the open state (due to a high magnetic specific heat during cooling in the external magnetic field) and almost zero heat conduction in the operating regime. Third, as follows from the expressions presented above, the maximum sensitivity $\Delta\Phi/\Delta E$ (for $B \simeq B_{\rm r}$) increases as $1/B_{\rm r}$ with decreasing $B_{\rm r}$. For real paramagnetics, $B_{\rm r} \simeq 100$ G; nuclear paramagnetics have weaker residual fields, for example, $B_{\rm r} \simeq 3$ G for copper. However, to obtain the dominating magnetic specific heat, a deeper cooling (below 100 mK) is required. In addition, the cooling of the detector near $T_{\rm C}$ is inadmissible because the sensitivity rapidly decreases in this case as $\Delta\Phi/\Delta E \sim (T-T_{\rm C})/T$ according to the Curie–Weiss law.

These disadvantages can be naturally eliminated by detecting ultralow energy releases by the method of adiabatic demagnetisation employed in cryogenics to obtain ultralow temperatures [13] (Fig. 1). To actuate such a calorimeter to the operating state, a paramagnetic is first polarised by a strong magnetic field by removing simultaneously heat with the help of a heat-exchange gas. Then, the paramagnetic substance is isolated (gas is evacuated) and the external field is decreased almost to zero. In this case, the paramagnetic is cooled, while the ratio $\beta = B/T$ preserves its initial value during demagnetisation (β = const), playing the role of the adiabatic invariant, which explains formally the temperature decrease. The substance remains polarised during demagnetisation, which follows from the simplest expression for the magnetic moment M taken from the Langevin paramagnetism theory (the working substance contains N paramagnetic ions):

$$M(\beta) = N\mu_{\rm B}\frac{\exp(\mu_{\rm B}\beta/k) - \exp(-\mu_{\rm B}\beta/k)}{\exp(\mu_{\rm B}\beta/k) + \exp(-\mu_{\rm B}\beta/k)},$$

where k is the Boltzmann constant and $\mu_{\rm B}$ is the Bohr magneton. Because β = const, we have M = const.

In the absence of the external heat supply, the magnetic moment M is preserved after the end of demagnetisation. Any energy release causes the heating and disordering of the spin system, thereby reducing M, which is detected by the SQUID. It is important for its operation that measurements are performed in the absence of an external field and do not require the use of a gradient-metric system. Such a calorimeter does not require a thermal valve with unusual parameters because heat is mainly removed (before demagnetisation) at a relatively high start temperature. The possibility of 'self-cooling' of this system can be used for the development of detectors of elementary particles and submillimetre radiometers in space research, where common ultralow-temperature refrigerators based on the dilution of ^{3}He in ^{4}He cannot be employed due to weightlessness.

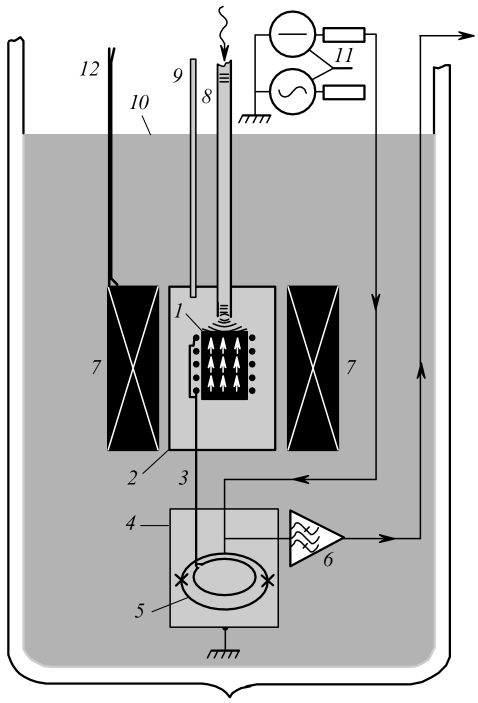

Figure 1. Scheme of the adiabatic paramagnetic calorimeter: (*1*) paramagnetic working substance of the calorimeter inside the coil of a superconducting flux transformer; (*2*) sealed shell of an adiabatic calorimeter; (*3*) low-inductance two-channel line of a superconducting flux transformer; (*4*) superconducting screen; (*5*) dc SQUID coupled with the coil of the flux transformer; (*6*) selective preamplifier of the dc SQUID; (*7*) superconducting solenoid; (*8*) sealed optical fibre or waveguide for radiation coupling to the calorimeter; (*9*) inlet and evacuation line for heat-exchange helium; (*10*) helium level in a cryostat (^{4}He or ^{3}He with pumping, $T < 4.2$ K); (*11*) current generators feeding the dc SQUID; (*12*) electrodes for current input to a superconducting solenoid.

The magnetic moment and entropy are expressed in terms of the statistical sum Z as

$$M = kT\frac{\partial}{\partial B}(\ln Z),$$

and

$$S = k\frac{\partial}{\partial T}(T\ln Z) = k\ln Z + kT\frac{\partial}{\partial T}(\ln Z),$$

respectively.

In the given case, the statistical sum of the spin system explicitly depends on the parameter β, which remains invariable during adiabatic demagnetisation (i.e. before the calorimeter operation). By using the relations

$$\frac{\partial}{\partial B} = \frac{1}{T}\frac{\partial}{\partial\beta}, \quad \frac{\partial}{\partial T} = -\frac{\beta}{T}\frac{\partial}{\partial\beta},$$

we pass from the differentiation of $\ln Z$ with respect to B and T to differentiation with respect to β:

$$M = k\,\frac{\partial}{\partial\beta}(\ln Z), \quad S = k\ln Z - k\beta\,\frac{\partial}{\beta}\,(\ln Z).$$

By substituting M from the first equality to the second one, we obtain $S = k\ln Z - \beta M$.

However, to 'save' the third law of thermodynamics, it is necessary to introduce a residual magnetic field B_{r} into these relations, which formally prevents the achievement of the absolute temperature zero when the external field B vanishes. The residual field B_{r} is included into the adiabatic invariant along with the external field:

$$\beta = \frac{B_{\mathrm{t}}}{T} = \frac{1}{T}\left(B^2 + B_{\mathrm{r}}^2\right)^{1/2}.$$

By varying the expression for entropy, we obtain

$$\Delta S = k\Delta(\ln Z) - \frac{B_{\mathrm{t}}}{T}\,\Delta M.$$

Assuming that temperature is low enough, we neglect (see section 2.3) the first term in the variation sum, and tend the external field in the second term to zero to obtain

$$\Delta S \simeq -\left[\frac{1}{T}\left(B^2 + B_{\mathrm{r}}^2\right)^{1/2}\right]\Delta M \simeq -\frac{B_{\mathrm{r}}}{T}\,\Delta M.$$

The interaction of an elementary particle with substance provides an increase in entropy due to heating, thereby reducing the magnetic moment of the system: $\Delta S = \Delta E/T \simeq -(B_{\mathrm{r}}/T)\Delta M$, which gives $\Delta M = -\Delta E/B_{\mathrm{r}}$. The change ΔM in the cross section of the working substance in the form of a cylinder of height h corresponds to the magnetic moment change $\Delta\Phi = \mu_0\Delta M/h$. The minimal detectable increments of the moment δM and the corresponding spin anisotropy δS are restricted by the SQUID resolution over the flux $\delta\Phi$:

$$|\delta S| \simeq \left|\frac{B_{\mathrm{r}}\delta M}{T}\right| = \left|\frac{hB_{\mathrm{r}}\delta\Phi}{\mu_0 T}\right|.$$

This yields the energy resolution of the calorimeter:

$$|\delta E| \simeq |T\delta S| \simeq \left|\frac{hB_{\mathrm{r}}\delta\Phi}{\mu_0}\right|.$$

One can see that the energy resolution is independent of the paramagnetic base area. This free parameter can be used for calculation of the flux transformer. Taking into account the transfer coefficient of the transformer, we can assume that the admissible effective resolution $\delta\Phi$ of the SQUID is $10^{-5}\Phi_0$ within the pass band $\Delta f = 1$ Hz. Then, for $h = 1$ cm and $B \simeq B_{\mathrm{r}} \simeq 100$ G, the calorimeter sensitivity is $\delta E = 3\times10^{-18}$ J Hz$^{-1/2}$.

Of certain interest is also a microcalorimeter in the form a micron paramagnetic film prepared in the unit planar technology with a dc SQUID. According to the expression presented above, the energy resolution δE can be 3×10^{-22} J Hz$^{-1/2}$ = 0.002 eV Hz$^{-1/2}$. However, the operation time of such a device in the adiabatic regime is considerably limited by uncontrollable heat inflows due to a low heat capacity of the film.

2.2 Ferromagnetic case

According to the third law of thermodynamics, the spin system will necessarily undergo the phase transition at low enough temperature. Let us analyse the appearance of magnetisation upon transition from the paramagnetic to ferromagnetic state under adiabatic conditions when the external field B completely vanishes. By comparing magnetic susceptibilities for $T > T_{\mathrm{C}}$ within the framework of the Ginzburg–Landau theory [15, 16] and Langevin paramagnetism theory [17], the first coefficient of the free energy expansion in the order parameter (magnetic moment) $F = F_0 + a(T - T_{\mathrm{C}})M^2 + bM^4$: can be obtained for paramagnetic ions:

$$\chi_{\mathrm{G-L}}(T > T_{\mathrm{C}}) = \frac{1}{2a(T - T_{\mathrm{C}})},$$

$$\chi_{\mathrm{L}} = \frac{\alpha}{T - T_{\mathrm{C}}} = \left.\frac{\partial M}{\partial B}\right|_{B\to0} \simeq \frac{N\mu_{\mathrm{B}}^2}{k(T - T_{\mathrm{C}})},$$

$$a = \frac{1}{2\alpha} = \frac{k}{2N\mu_{\mathrm{B}}^2}.$$

According to the Ginzburg–Landau theory, the entropy $S = -\partial F/\partial T = S_0 - aM^2 = S_0 + S_{\mathrm{fm}}$ (where $S_0 = -\partial F_0/\partial T$). This dependence is confirmed by calorimetric measurements of paramagnetic salts with the residual magnetic moment at $T \approx T_{\mathrm{C}}$ [18, 19]. The direct relation of entropy with the moment M provides the operation of the detector in the region of ferromagnetic transition. Let us choose the initial conditions of the adiabatic demagnetisation in the form $B_{\mathrm{i}} \gg B_{\mathrm{r}}$ and $T_{\mathrm{i}} \gg T_{\mathrm{K}}$. Then, the entropy preserved due to adiabaticity can be written in the form

$$S = \mathrm{const} = S_0 + S_{\mathrm{pm}} \approx S_0 - \frac{\alpha B_{\mathrm{i}}^2}{2T_{\mathrm{i}}^2}.$$

The phase transition corresponds to the condition $S_{\mathrm{pm}} = S_{\mathrm{fm}}$, which yields the expression for the spontaneous moment induced by the ferromagnetic transition under conditions of adiabatic demagnetisation:

$$\frac{\alpha B_{\mathrm{i}}^2}{2T_{\mathrm{i}}^2} = aM^2, \text{ i.e. } M = \frac{B_{\mathrm{i}}}{2aT_{\mathrm{i}}}.$$

By varying the 'ferromagnetic' entropy in the presence of the spontaneous moment M, we obtain

$$\Delta S_{\mathrm{fm}} = -2aM\Delta M = -2a\,\frac{B_{\mathrm{i}}}{2aT_{\mathrm{i}}}\,\Delta M = -\frac{B_{\mathrm{i}}}{T_{\mathrm{i}}}\,\Delta M$$

(external field $B = 0$). We determine the energy sensitivity of the calorimeter with the working substance of height h by recalculating the variation of M to the flux increment $\delta\Phi$ measured with the SQUID:

$$|\delta E| \simeq |T_{\mathrm{C}}\delta S_{\mathrm{fm}}| \simeq \left|T_{\mathrm{C}}\,\frac{B_{\mathrm{i}}}{T_{\mathrm{i}}}\,\delta M\right| \simeq \left|\frac{T_{\mathrm{C}}}{T_{\mathrm{i}}}\,\frac{hB_{\mathrm{i}}\delta\Phi}{\mu_0}\right|.$$

The initial conditions of adiabatic demagnetisation can be chosen so that $B_{\mathrm{i}}T_{\mathrm{C}}/T_{\mathrm{i}} < B_{\mathrm{r}}$. Then, it follows from the last expression that the energy resolution of a detector operating on the phase transition can be considerably higher than that of a detector operating on a 'pure' paramagnetism.

A similar approach can be applied to the development of a detector in which a superconductor is used as a working substance instead of a ferromagnetic, while the Cooper condensate density ψ^2 plays the role of the order parameter, i.e. we have the similar relation $F = F_0 + \alpha_C(T - T_c)\psi^2 + \beta_C\psi^4$ for the free energy expansion. When $T < T_c$ (where T_c is the critical temperature), the condensate density, as follows from the condition $\partial F/\partial\psi^2 = 0$, is $\psi_0^2 = [\alpha_C/(2\beta)](T_c - T)$, and the entropy $S = -\partial F/\partial T = S_0 - \alpha_C\psi^2$. Then, $\delta Q = T\delta S = -\alpha_C T\delta\psi^2$ and, hence, $\delta\psi^2 = -\delta Q/(\alpha_C T)$. In this case, the relative variation of the condensate density

$$\frac{\delta\psi^2}{\psi_0^2} = -\frac{2\beta_C}{\alpha_C^2 T(T_c - T)}\,\delta Q$$

proves to be very sensitive to the absorbed heat δQ (i.e. to the detected energy) because the coefficient at δQ grows infinitely at $T \to T_c$. The resulting variation $\delta\psi^2/\psi_0^2$ can be detected without contact by microwave absorption in the superconductor. It is obvious that in these measurements the extremely high energy amplification will take place, which is proportional to the amplitude of the probe microwave.

2.3 Paramagnetic case (statistical consideration of the ideal spin gas)

Completing the analysis of the ultimate sensitivity of the magnetic adiabatic calorimeter based on statistical physics, we return to the case of noninteracting spins. Let us determine to what extent the neglect of the first term $\Delta S_1 = k\Delta(\ln Z)$ in the sum of entropy variations obtained earlier for a paramagnetic is justified. The expression $S_1 = k(\ln Z)$ has the form close to the expression $S = k(\ln W)$ for entropy in the known statistical interpretation following from the Boltzmann H theorem, where W is the statistical weight of a state. We will use below this Boltzmann definition of entropy. Let us assume that all N spins were polarised by an external field at sufficiently low temperature (paramagnetic saturation), then the field slowly vanishes under adiabatic conditions, and temperature achieves some finite value T. If we assume that $B = 0$ and $B_r = 0$ ('ideal paramagnetism'), then the up and down spin states have the same probability: $P_\uparrow = P_\downarrow = 1/2$. The probability $W_N(m)$ that m spins from the total number N are directed upward and the rest $N - m$ spins – downward is described by the binominal distribution function $W_N(m) = C_N^m P_\uparrow^m P_\downarrow^{N-m}$ (scheme of the independent Bernoulli tests [20]). Let us calculate the change in the system entropy when one spin turns downward:

$$\delta S = S(m = N - 1) - S(m = N)$$

$$= k\ln\frac{W_N(m = N - 1)}{W_N(m = N)} = k\ln\frac{C_N^{N-1}}{C_N^N} = k\ln N.$$

Such a variation δS requires the energy release $\delta E = T\delta S = kT\ln N$ and will be accompanied by a change in the magnetic moment only by two Bohr magnetons $\delta M = 2\mu_B$. Therefore, the effective magnetic field $B_{\rm eff} \approx \delta E/\delta M = [kT/(2\mu_B)]\ln N$ for $T = 1$ K will have a very high strength equal to $(\ln N)/2$, which corresponds to a few tens of tesla. Thus, in the case of complete polarisation, the field $B_{\rm eff}$ exceeds usual values of B_r by three orders of magnitude. This means that the magnetic response proportional to $1/B_{\rm eff}$ and corresponding to the first variation term $\Delta S_1 = k\Delta(\ln Z)$ is small and can be neglected. The above considerations are close to the known method of the proof of the instability of the ferromagnetic state in the one-dimensional Ising model [17].

However, all will change drastically if the system can be demagnetised to obtain the ultimately low polarisation at the end of the process. Let us assume that in the zero external field the number of electrons in the up spin state is greater by one than $K = N/2$ (i.e. $m = N/2 + 1$), while in the down spin state their number is smaller by one than K (i.e. $N - m = N/2 - 1$). The change in entropy after the turn of a pair of spins downward is

$$\delta S = S(m = K) - S(m = K + 1) = k\ln\frac{W_N(m = K)}{W_N(m = K + 1)}$$

$$= k\ln\frac{(K + 1)!(K - 1)!}{K!K!} = k\ln\frac{K + 1}{K} \simeq \frac{k}{K}.$$

In this case, the effective magnetic field $B_{\rm eff} = (kT\mu_B^{-1})\times N^{-1}$ at $T = 1$ K, and $2K = N = 10^{15}$ is 10^{-15} T, and the calorimeter sensitivity δE is estimated as 10^{-31} J Hz$^{-1/2}$ $\approx 10^{-12}$ eV Hz$^{-1/2}$.

3. Features of the establishment of thermal equilibrium and the time selection of events

Consider now the features of the establishment of thermal equilibrium between the spin system and crystal lattice of the working substance of the calorimeter. It is known that the spin-relaxation time $\tau_{\rm sp}$, which mainly determines the longitudinal relaxation of magnetisation, considerably increases with decreasing temperature. The inelastic Raman scattering of a phonon accompanied by an electron spin flip – flop makes a contribution to $\tau_{\rm sp}$, which is proportional to $1/T^7$ [21]. The Orbach scattering involving an additional electronic level gives the dependence $\tau_{\rm sp} \sim \exp[\Delta E/(kT)]$, where ΔE is determined by the electronic level position. However, for $T < 1$ K, when $\tau_{\rm sp}$ is of the order of a few seconds, the increase in the longitudinal relaxation time drastically slows down due to direct energy exchange between the spin system and lattice, and $\tau_{\rm sp}$ becomes proportional to $1/T$. It is small values of $\tau_{\rm sp}$ that allow the use of a magnetic thermometer at ultralow temperatures. The thermal equilibrium in the spin – lattice system in metals at such temperatures is established due to electron – phonon interaction even faster than in dielectrics. In this case, the relaxation time is $\tau_{\rm sp} = K/T_e$, where T_e is the electron temperature and K is the Korringa constant. This allows the use of metal paramagnetism in the calorimeter [12]. A heavy metal adsorber doped with a paramagnetic impurity with a short thermal equilibrium time (10^{-3} s) can be promising for the development of a proportional thermal counter for X-ray quanta with the spectral resolution better than 1 – 10 eV. Such a counter can be used in many applied problems of chemical analysis by the method of X-ray fluorescence (proximate analysis, search for valuable metals in geological rocks, etc.).

In principle, nuclear magnetic moments in metals can be also used. The Bohr magneton does not enter the expression for the energy sensitivity, but because of its smallness, more stringent initial conditions $B_{\rm i} \approx 10$ T and $T_{\rm i} \approx 10$ mK will be required for the initial polarisation of nuclei. Due to conduction electrons, the relaxation times at ultralow temperatures are also small in this case. The use of a copper working substance with $B_{\rm r} \approx 3$ G provides the energy sensitivity $\delta E = 10^{-19}$ J Hz$^{-1/2}$ $= 0.6$ eV Hz$^{-1/2}$ for $h = 1$ cm and $\langle \delta\Phi \rangle = 10^{-5}\Phi_0$ in the pass band $\Delta f =$ 1 Hz. It can be expected that a 'nuclear' calorimeter will find applications in experiments on detecting weakly interacting massive particles, from which, as assumed, the dark matter of the Universe consists, comprising up to 90 % of the Universe mass according to estimates. Due to the non-baryon nature of these massive particles and the absence of electric and other charges, they can be detected only owing to their nonzero mass in head-on collisions with the nuclei of usual matter. Therefore, by observing the magnetic moment of a nuclear system, one can 'directly' detect such collisions.

As an alternative, a paramagnetic with long spin–lattice relaxation times can be used in the calorimeter. A weak interaction of elementary particles with matter is also determined by the absence of the electric charge (neutrino of three generations, supersymmetric companions of a photon, a neutron, etc.; weakly interacting massive particles and other exotic particles). Such particles can be detected with the calorimeter due to the creation of secondary charged particles or recoil nuclei in the working substance. Similarly to the detection of weakly interacting particles, we can consider the problem of observation of rare events such as the double beta decay (measurement of the neutrino mass by analysing the non-neutrino decay channel, the search for the Majorana neutrino).

The energy losses of secondary charged particles in a medium cannot be smaller than 2 MeV cm^2 g^{-1}. Indeed, the ionisation losses, i.e. the energy $\mathrm{d}E_I/\mathrm{d}x$ of a charged particle (charge $Q_{\rm p} = Z_{\rm p}e$) spent to ionise atoms of the medium along the unit track are proportional to the concentration $n_{\rm a}$ of atoms according to the Bethe formula [22]:

$$-\frac{\mathrm{d}E_I}{\mathrm{d}x} = \frac{1}{4\pi\varepsilon_0^2}\frac{Z_{\rm p}^2 e^4 n_{\rm a}}{m_{\rm e} V_{\rm p}^2} F\left(\frac{V_{\rm p}}{c}, \nu_{\rm e}\right),$$

where $F(V_{\rm p}/c, \nu_{\rm e})$ is the relativistic factor in the form of a logarithmic function whose arguments are the particle velocity $V_{\rm p}$ divided by the speed of light c and the characteristic rotation frequency $\nu_{\rm e}$ of electrons on the Bohr orbit of an atom in the medium; ε_0 is the permittivity of free space; and $m_{\rm e}$ is the electron mass. Correspondingly, the ratio of $|\mathrm{d}E_I/\mathrm{d}x|$ to the material density ρ is independent of ρ and is approximately equal to 2 MeV cm^2 g^{-1}. This value specifies the lower limit of the ratio of total losses to the material density when the total losses of the particle energy per unit track length also include other channels, for example, bremsstrahlung, radiative losses, photoeffect, etc.

A secondary single-charged particle propagating in a paramagnetic of density ~ 5 g cm^{-3} loses at least 10 MeV per 1 cm of its track. On each lattice period along the track, a particle loses the energy more than 1 eV, which results in the instant local heating up to 10^4 K. In this case, $\tau_{\rm sp}$ decreases at once by a few orders of magnitude down to microseconds, and a rapid and strongly nonequilibrium heat exchange occurs between the lattice and the spin system. After the end of this exchange, a greater part of energy is transferred to electrons nearest to the track. The magnetic system is thermalised during the spin–spin relaxation time, which is usually much shorter than the equilibrium time $\tau_{\rm sp}$. An advantage of a detector with large $\tau_{\rm sp}$ is the possibility of the natural temporal selection of events by sampling abrupt jumps of the magnetic moment against the background of a slow drift caused by uncontrollable heat inflows to which long establishment times of thermal equilibrium correspond.

The adiabatic magnetic calorimeter with a short relaxation time can be treated as the ideal proportional detector of particles. The thermal response of such a detector is proportional to the particle energy losses in an adsorber, and if the total cross section for interaction with the working substance of the calorimeter is sufficiently high, the response amplitude corresponds to the initial energy of the particle. The estimates of the energy sensitivity presented above demonstrate that the energy resolution can achieve fractions of electron volts for fluxes of the order of 10^4 particles per second. Such a resolution would considerably increase, for example, the efficiency of X-ray fluorescence measurements of the content of valuable metals in rocks, which is now determined by using silicon–lithium semiconductor proportional detectors with a resolution of hundreds electron volts.

The direct transformation of the energy of an incident particle to a thermal response with a high amplitude resolution would solve the known problem of the development of a proportional neutron detector. To measure the neutron energy, it is necessary first to transfer this energy to a charged particle. Thus, energy in organic scintillators is transferred to a proton and then is 'emitted' and measured from the intensity of the total scintillation. The proton energy is transformed to emission quite deterministically. However, energy transfer from a neutron to a proton depends on an uncertain parameter – the rotation angle of the neutron trajectory in the centre-of-mass system after the collision. This uncertainty can be demonstrated by solving the system of equations following from the laws of conservation of energy and momentum for the elastic collision of two particles. As a result, the instrumental function of such a detector becomes too wide. By measuring with a SQUID the thermal response of a large enough crystal of heavy water (to slow down a neutron completely, only a few centimetres are required) containing a diluted paramagnetic salt, it would be possible to solve the problem of the development of a proportional neutron detector.

The theoretical estimates of the ultimate resolution of detectors on overheated superconducting granules [23] in the proportional detection regime yield the value at the level of fractions of electron volts. However, the realisation of such a high resolution in practice is out of the question, and the real energy resolution of these detectors is hundreds of electron volts, which probably excludes their use in accurate investigations of the beta-decay spectra and applied problems of X-ray fluorescence. A considerable disadvantage of these detectors is a low efficiency of the use of the semiconductor mass (only ~ 1 % of the total number of granules), which substantially reduces the detection probability of a particle incident on a detector. The expected frequency of events such as the detection of weakly interacting massive particles, reduced to the detector mass, is estimated as $\sim 10^{-4}$ event (day kg)$^{-1}$. In principle, the mass of the magnetic adiabatic calorimeter considered above remains a free parameter, which also makes promising its use for detecting such particles. Moreover, magnetic thermal detectors (including calorimeters proposed here) have a significant advantage over thermal detectors of elementary particles of other types because they admit an increase in the working substance mass without the sensitivity deterioration.

The total interaction cross section and the detection probability of a particle increase with increasing mass. In 'usual' non-magnetic systems (for example, an adsorber and a semiconductor thermistor), the heat capacity of the working substance increases with increasing mass, while the detected thermal response naturally decreases. In magnetic detectors, the thermal response decreases with increasing mass, but the detected change in the magnetic moment increases. In fact, the sensitivity is preserved because the functions of the working substance and temperature sensor in magnetic systems are combined. We can say that the magnetic adiabatic calorimeter measures not the thermal response (as the intense thermodynamic quantity) but the total change in the system entropy (as the extensive quantity proportional to the adsorber mass).

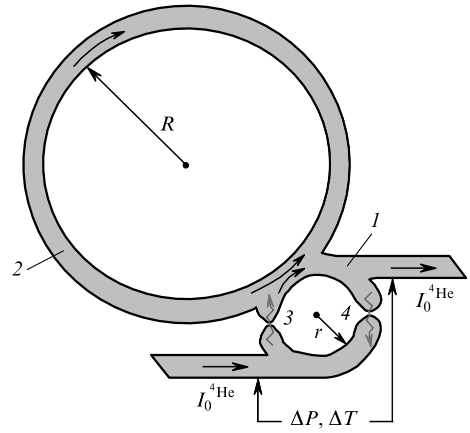

Figure 2. Scheme of a SQUID on superfluid helium: (*1*) quantum interferometer on matter waves in superfluid ^{4}He (dc$I_0^{^4\mathrm{He}}$ SQUID); (*2*) superfluid angular momentum transformer; (*3,4*) Josephson weak couplings (quantum nanochops); $I_0^{^4\mathrm{He}}$ is the fixed total helium flux.

4. Magnetic calorimeter and quantum interferometer on matter waves in superfluid ^{4}He

Consider another possible application of the adiabatic magnetic calorimeter with a SQUID for direct measuring the increment of the entropy of a paramagnetic working substance. Note that, while we have mainly considered so far pulsed measurements of ΔS, below we will deal with continuous calorimetric measurements of the deviation $S(t) - S(t = 0)$ of entropy from its initial value. It is known [24] that ^{4}He undergoes the transition to the superfluid state at $T < T_\lambda = 2.17$ K (where T_λ is the λ point temperature), and under these conditions the macroscopic coherent effects are observed in helium, to which non-dissipative processes correspond (helium flow without friction and super heat conduction). Attempts have been made to find macroscopic quantum coherent effects occurring in ^{4}He below the λ point. This search is caused by the analogy between superfluidity and superconductivity, and the main goal is to find the Josephson effect in liquid helium.

The observation of the nonstationary Josephson effect in ^{4}He was reported in a number of papers [24–26]. The role of the tunnel Josephson junction is played in these experiments by a quantum choke – a nanohole (or a system of such holes [26]) in a nanometre-thick membrane (direct analogy with the Diem bridge [14]). Such a small size of the Josephson weak coupling is determined by the exclusively small coherence length in superfluid ^{4}He [17]. If the analogue of the stationary Josephson effect can be realised in circular tube (*1*) (Fig. 2) filled with superfluid liquid and containing two quantum chokes [(*3*) and (*4*)], then the conditions required for manufacturing a superfluid dc SQUID will be fulfilled. These conditions include additionally the development of methods for recording the supercritical ^{4}He flux and the development of the operation principle of an analogue of a superfluid flux transformer.

The supercritical flux is an analogue of the quasi-particle component $I_\mathrm{q} = I_0 - I_\mathrm{c}(\Phi/\Phi_0)$ (where Φ_0 is the magnetic flux quantum) of the total flux I_0 introduced and removed through the poles of a superconducting ring in a usual dc SQUID. The resulting critical current I_c in a ring with two Josephson junctions is a periodic function of the external magnetic flux threading the ring and measured with the SQUID: $I_\mathrm{c} = I_\mathrm{c}(\Phi/\Phi_0)$ [14, 24]. The periodic dependence appears here due to the interference of a superconducting condensate propagating through two interfering trajectories – through the first and second Josephson junctions [17], while the phase difference in the junctions is determined by the Bohm–Aronov effect [27]. As a result, by measuring the quasi-particle component I_q for fixed I_0, we can determine I_c, and then the flux Φ itself with an accuracy to an integer number Φ_0 of quanta, which a usual SQUID makes [14]. By recording the supercritical component $I_\mathrm{q}^{^4\mathrm{He}}$ of the helium flux for a fixed total flux $I_0^{^4\mathrm{He}}$ in a ring with two weak couplings, we can determine the superfluid component $I_\mathrm{c}^{^4\mathrm{He}}$ of the helium flux and its phase, which contains information on parameters measured with a dc SQUID with superfluid ^{4}He (dc$^{^4\mathrm{He}}$ SQUID).

Such a method of determining $I_\mathrm{q}^{^4\mathrm{He}}$ can be based (Fig. 2) either on the measurement of the pressure difference appearing during the flow of supercritical helium through a weak coupling (the superfluid component passes through a weak coupling without friction) or on the measurement of heat transfer by quasi-particles of the supercritical component (the supercritical component has no entropy). In the first case, the third weak coupling can be used to measure the difference ΔP of pressures in front of and behind the ring. If the pressure difference is applied to the third junction (nanochoke), acoustic vibrations should be generated on it due to the nonstationary Josephson effect, the frequency Ω of these vibrations being proportional to ΔP. However, according to the elementary theory of the nonstationary Josephson effect [24], the amplitude P_0 of these vibrations should be much smaller than ΔP. The realistic estimates of P_0 show that it is necessary to fabricate a supersensitive microphone capable of detecting acoustic vibrations with the amplitude on the level of fractions of picopascal [28]. In the second case, the entropy increment carried by the supercritical component $I_\mathrm{q}^{^4\mathrm{He}}$ to the output of the ring of a superfluid interferometer can be measured if the output will be thermally loaded on the working substance of the magnetic calorimeter.

What parameters and signals can be measured with a dc$^{^4\mathrm{He}}$ SQUID? The phase difference at which the resulting critical current $I_\mathrm{c} = I_\mathrm{c}(\varphi)$ is established is determined from the expression

$$\varphi - \varphi_0 = \frac{1}{\hbar}\oint \mathscr{P} \mathrm{d}\boldsymbol{r} = \frac{1}{\hbar}\oint (\boldsymbol{p} - q\boldsymbol{A}) \mathrm{d}\boldsymbol{r}.$$

A usual SQUID responds to the second term in the integrand ($q = 2e$ is the Cooper pair charge). By using the Stokes theorem, we can pass from the vector potential $\boldsymbol{A}$ in the integrand to the magnetic field flux $\boldsymbol{B}$ threading the ring and related to the flux quantum:

$$\frac{q}{\hbar}\oint \boldsymbol{A} \mathrm{d}\boldsymbol{r} = \frac{2e}{\hbar}\int\int \boldsymbol{B} \mathrm{d}\boldsymbol{s} = \frac{2\pi\Phi}{\phi_0}.$$

In this case, the contribution of the first term is compensated by the calibration transformation of the second one. The first term for superfluid ^{4}He proves to be the only one at all because $q = 0$. Then, under the conditions of rotation of ^{4}He with the angular frequency ω in a toroidal tube of radius r (including two weak couplings), we have the momentum $p = m\omega r$ (where m is the helium mass) and $|\mathrm{d}\boldsymbol{r}| = r\mathrm{d}\theta$ $(0 < \theta < 2\pi)$. In this case, the first term (circulation related to the action quantum $\hbar$) can be transformed to the ratio of the angular momentum Λ of superfluid helium to Planck's constant $\hbar$:

$$\frac{1}{\hbar}\oint \mathscr{P} \mathrm{d}\boldsymbol{r} = \frac{1}{\hbar}\oint m\omega r^2 \mathrm{d}\theta = \frac{2\pi\Lambda}{\hbar}.$$

Similarly to superconductivity, $I_{\mathrm{c}}^{^4\mathrm{He}}$ proves to be a periodic function of the phase $2\pi\Lambda/\hbar$: $I_{\mathrm{c}}^{^4\mathrm{He}} = I_{\mathrm{c}}^{^4\mathrm{He}}(2\pi\Lambda/\hbar)$, which can be also obtained within the framework of the Feynman quantum electrodynamics [17]. The phase $2\pi\Lambda/\hbar$ detected with a dc$^{^4\mathrm{He}}$ SQUID during the measurement of $I_{\mathrm{c}}^{^4\mathrm{He}}(2\pi\Lambda/\hbar) = I_0^{^4\mathrm{He}} - I_{\mathrm{q}}^{^4\mathrm{He}}$ allows one to determine the angular momentum of superfluid ^{4}He in fractions of Plank's constant, similarly to the possibility of measuring the flux with a dc$^{^4\mathrm{He}}$ SQUID in fractions of Φ_0 (the sensitivity of modern commercial SQUIDs is no worse than $10^{-5}\Phi_0$ within the pass band $\Delta f = 1$ Hz [14]).

For the high sensitivity of a SQUID (usual non-superfluid) not to remain a pure abstraction in real experiments, a special matching device was developed – a superconducting flux transformer for introducing the magnetic flux being measured from the region of size determined by specific experimental conditions to the working ring of the SQUID containing Josephson junctions. The size of this ring both for dc SQUIDs and rf-biased SQUIDs is strictly bounded above. The unique dependence of the output signal on the measured magnetic flux (within one quantum) requires the fulfilment of the condition $LI_{\mathrm{c}}(\Phi = 0) \lesssim \Phi_0$ in systems of both types, where L is the ring inductance and $I_{\mathrm{c}}(\Phi = 0)$ is the amplitude of the Josephson critical current. If the flux is measured over a large area, it is introduced to a large (signal) ring of the system of two rings forming a closed superconducting circuit in which the total flux is preserved. A small ring is coupled with the SQUID with the maximum possible mutual inductance coefficient. The design of this superconducting flux transformer is described in detail in [29].

In the case of superfluidity, it is difficult to create a similar matching device – an angular momentum transformer, for two reasons. First, it is difficult to find a direct analogue of the magnetic field that would provide the coupling between the transformer and SQUID. Second, the theory of superfluidity has only one dimensional parameter, namely, the coherence length, whereas in the case of superconductivity, another parameter also exists – the London penetration depth λ_{L} of the field. At scales smaller than λ_{L}, the momentum of non-dissipative motion of particles can be transferred from one closed contour to another, to which the mutual inductance coefficient $\mathscr{M} = \mu_0\lambda_{\mathrm{L}}$ corresponds. In the case of superfluidity, it is necessary to find other approaches to develop the transformer, and based on the relativistic nature of a magnetic field, a non-extensive macroscopic parameter should be found by analogy, which would depend explicitly on the flow velocity. Such a parameter (pressure P) is the Bernoulli integral – the known solution of the Euler equation in hydrodynamics in the absence of viscous friction which corresponds to the superfluidity condition. To understand how the coupling of two liquid motions appears in two closed circuits, i.e. how the angular momentum is transferred from the first circuit to the second one, we assume that the superfluid liquid moves at the velocity V_1 in the first circuit and was initially at rest in the second circuit, i.e. $V_2 = 0$. Let us write the Bernoulli integral for the direct contact region where both circuits merge in fact:

$$P_1 + \frac{\rho V_1'^2}{2} = P_2 + \frac{\rho V_2'^2}{2} = \mathrm{const}.$$

After pressure equalisation ($P_1 = P_2$), the same velocity of the liquid motion should be established in both circuits, and thus the angular momentum will be transferred from the first circuit to the second one (Fig. 2).

The combination of a dc$^{^4\mathrm{He}}$ SQUID and magnetic calorimeter within one system (i.e. the combination of a working paramagnetic with a usual SQUID) is somewhat similar to the fabrication of two-stage dc SQUIDs mentioned above. In this case, the role of the first stage is played by the dc$^{^4\mathrm{He}}$ SQUID and the role of the second stage – by the magnetic calorimeter. Both these stages have extremely high gains: the first one – due to the Josephson nonlinearity in ^{4}He, and the second one – as a magnetic transformer with a usual SQUID at the output. The difficulties encountered in the development of such an interferometer on matter waves in superfluid ^{4}He are quite comparable with problems appearing in the practical realisation of a quantum computer. However, unlike the computer case, where some principle problems concerning its development have not been solved so far, the conceptual development of a helium interferometer can be considered completed at present.

A quantum interferometer on superfluid ^{4}He combined in a unified system with the angular momentum transformer proposed above [(*1*), (*2*) in Fig. 1] is a supersensitive sensor of rotations with low angular velocities. The sensor is a universal detector of ultralow mechanical vibrations reduced to rotations in its own reference system. Such an instrument can be used to verify the nontrivial predictions of the general relativity theory such as the existence of the Lenze–Thirring effect [30], gravitational waves, etc.

Let us estimate the angular momentum imparted by a gravitational wave to a superfluid transformer, when the wave vector lies in the plane of its signal ring:

$$\Lambda = pR = \left(m\omega_{\mathrm{g}}R|\delta g_{ij}|\,\frac{R}{\lambda_{\mathrm{g}}}\right)R = \frac{m\omega^2 R^3}{2\pi c}\,|\delta g_{ij}|,$$

where m is the total mass of ^{4}He that was initially at rest in the transformer; R is the ring radius; ω_g, λ_g, and $|\delta g_{ij}|$ are the frequency, length, and amplitude of the gravitational wave, respectively; R/λ_g is the quasi-stationarity factor of the ring used as an antenna. For $m = 1$ kg, $R = 1$ m, and $\omega_g = 4500$ rad s^{-1}, the imparted angular momentum proves to be of the order of an action quantum ($\Lambda = \hbar$) when the amplitude of oscillations of the metric tensor in the gravitational wave is $|\delta g_{ij}| \approx 10^{-32}$. Such a sensitivity of the detector exceeds by ten (!) orders of magnitude the sensitivity of the best modern gravitational detectors. In this case, the dc ^{4}He SQUID can probably measure Λ in fractions of Planck's constant, similarly to a usual modern dc SQUID detecting the value $10^{-5}\Phi_0$ within the pass band $\Delta f = 1$ Hz. Although, of course, the transfer coefficient of the flux transformer of the SQUID can be also much lower than unity. Thus, this coefficient for modern superconducting flux transformers is $\sim$0.01.

References

1. Clarke J. *Phys. Today*, **39** (3), 36 (1986).
2. Voss R.F. et al. *Proc. Second Int. Conf. on Supercond. Quantum Devices* (Berlin, 1980) p. 94.
3. Voss R.F., Laibowitz R.B., Raider S.I., Clarke J. *J. Appl. Phys.*, **51**, 2306 (1980).
4. Ketchen M.B., Voss R.F. *Appl. Phys. Lett.*, **35**, 812 (1979).
5. Clarke J. *IEEE Trans. Electron Dev.*, **27**, 1896 (1980).
6. Ketchen M.B., Jaycox J.M. *Appl. Phys. Lett.*, **40**, 736 (1982).
7. Pierce J.M., Opfer J.E., Rorden L.H. *IEEE Trans. Magn.*, **10**, 599 (1974).
8. Falferi P. *Class. Quantum Grav.*, **21**, S973 (2004).
9. Gottardi L., Podt M., Bassan M., et al. *Class. Quantum Grav.*, **21**, S1191 (2004).
10. Buhler M., Umlauf E. *J. Low Temp. Phys.*, **93**, 697 (1993).
11. Fausch T., Buhler M., Umlauf E. *J. Low Temp. Phys.*, **93**, 703 (1993).
12. Bandler R. et al. *Low Temp. Phys.*, **93**, 709 (1993).
13. Lounasmaa O.V. *Experimental Principles and Methods Below 1 K.* (London, New York: Acad. Press, 1974).
14. Schwartzand Br.B., Foner S. (Eds) *Superconductor Applications: SQUIDs and Machines* (New York: Plenum Press, 1977).
15. Ginzburg V.L., Landau L.D. *Zh. Eksp. Teor. Fiz.*, **20**, 1064 (1950).
16. Landau L.D., Lifshits E.M. *Course of Theoretical Physics* (New York: Pergamon Press, 1960; Moscow: Izd. Fiz.-Mat. Lit., 1976) Vol. 5, Part. 1.
17. Feynman R.P. *Statistical Mechanics: a Set of Lectures* (Reading, Mass.: Benjamin, 1972; Moscow: Mir, 1975).
18. Shal'nikov A.I. (Ed.) *Fizika nizkikh temperatur* (Low-temperature Physics) (Moscow: Inostr. Lit., 1959).
19. Steenland M.J., de Klerk D., Gorter C.J. *Physica,* **15** (8-9), 711 (1949).
20. Kolmogorov A.N., Zhurbenko I.G., Prokhorov A.V. *Vvedenie v teoriyu veroyatnostei* (Introduction to the Theory of Probability) (Moscow: Nauka, 1982).
21. Poole Ch.P. *Electron Spin Resonance* (New York, London, Sydney: John Wiley & Sons, 1967).
22. Tsipenyuk Yu.M. *Printsypy i metody yadernoi fiziki* (Principles and Methods of Nuclear Physics) (Moscow: Energatomizdat, 1993).
23. Waysand G. et al. *J. Low Temp. Phys.*, **93**, 485 (1993).
24. Tilley D.R., Tilley J. *Superfluidity and Superconductivity* (New York, Cincinnati, Toronto, London, Melbourne: Van Nostrand Reinhold Comp., 1974).
25. Richards P.L., Anderson P.W. *Phys. Rev. Lett.*, **14**, 540 (1965).
26. Hoskinson E., Packard R.E., Haard Th.M. *Nature*, **433**, 376 (2005).
27. Golovashkin A.I., Zherikhina L.N., Tskhovrebov A.M., Kuleshova G.V. *Abstracts of Papers, I Int. Conf. on the Fundamental HTSC Problems* (Zvenigorod, 2004) p. 283.
28. Golovashkin A.I., Gudenko A.V., Zherikhina L.N., et al. *Pis'ma Zh. Eksp. Teor. Fiz.*, **60**, 595 (1994).
29. Golovashkin A.I., Kuleshova G.V., Tskhovrebov A.M., et al. *Kratk. Soobshch. Fiz. FIAN*, (11), 6 (2002).
30. Khriplovich I.B. *Obshchaya teoriya otnositel'nisti* (General Relativity Theory) (Moscow: Izd. Institute of Computer Studies, 2002).