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Multiphoton correlations in parametric down-conversion and their measurement in the pulsed regime

O.A. Ivanova, T.Sh. Iskhakov, A.N. Penin, M.V. Chekhova

***Abstract*. We consider normalised intensity correlation functions (CFs) of different orders for light emitted via parametric down-conversion (PDC) and their dependence on the number of photons per mode. The main problem in measuring such correlation functions is their extremely small width, which considerably reduces their contrast. It is shown that if the radiation under study is modulated by a periodic sequence of pulses that are short compared to the CF width, no decrease in the contrast occurs. A procedure is proposed for measuring normalised CFs of various orders in the pulsed regime. For nanosecond-pulsed PDC radiation, normalised second-order CF is measured experimentally as a function of the mean photon number.**

Keywords: *parametric down-conversion, biphotons, multiphoton correlations.*

1. Introduction

Spontaneous parametric down-conversion (SPDC) is one of the most efficient sources of nonclassical light, i.e., light whose properties can be described only within the framework of consistent quantum theory. In particular, the 'nonclassicality' of the SPDC radiation is related to the pair photon correlation, i.e., the extremely large value of the normalised second intensity moment. Comparison of this normalised moment with the others (the first one and the third one) shows that the SPDC radiation violates the concept of a classical probability distribution existing for photon numbers [1]. It is due to this fact that the SPDC radiation can be considered as nonclassical and called two-photon, or biphoton, light. This pair correlation disappears as one passes from small parametric gains (the so-called spontaneous regime) to large parametric gains (hereafter, we will call this regime stimulated). Light emitted through stimulated PDC also manifests nonclassical properties but they are no longer related to the anomalously large second-order intensity moment. Such light cannot be already considered as two-photon light; it can be called even-photon-number light, and it manifests squeezing effects [1].

O.A. Ivanova, T.Sh. Iskhakov, A.N. Penin, M.V. Chekhova Department of Physics, M.V. Lomonosov Moscow State University, Vorobyevy gory, 119992 Moscow, Russia; e-mail: masha@qopt.phys.msu.su

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The normalised second-order CF for the PDC radiation* can be represented in the form [2]

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2 + g_{\rm nc}^{(2)}(\tau), \quad (1)$$

where $g^{(1)}(\tau)$ is the normalised first-order CF, so that the first two terms give the 'thermal' part of $g^{(2)}(\tau)$. The last term is its nonclassical part. The maximal value of $g_{\rm nc}^{(2)}(\tau)$ is $g_{\rm nc}^{(2)}(0) = 1/N$, where N is the number of photons per mode. Thus, as the parametric gain and, hence, the number of photons per mode increases, the value of the normalised second-order CF at its maximum tends to 2, which is the typical value for fields with thermal statistics, and its nonclassical part disappears. The width of the CF 'thermal' part, $|g^{(1)}(\tau)|^2$, is equal to the coherence time τ_0 of the PDC radiation; the width of its nonclassical part, $g_{\rm nc}^{(2)}(\tau)$, is in the general case slightly different from τ_0 [3] but has the same order of magnitude.

In a number of recent works PDC was suggested as a source of three-photon or four-photon entangled states. In these works, various interference effects were observed in the measurement of the third-order and fourth-order intensity correlation functions. In particular, such experiments were interpreted as realisations of the Greenberger – Horne – Zeilinger paradox [4] or interference of three-photon and four-photon states [5 – 7] and used to test Bell's inequalities for spin-1 quantum systems [8]. At the same time, it was mentioned in [9] that the observed effects were a mere consequence of the well-known two-photon interference, and the violation of the 'four-photon' Bell's inequality registered in [8] could be explained by the simultaneous violation of two 'two-photon' Bell's inequalities.

In their turn, the authors of [9] carried out an experiment that allowed them to distinguish 'true' correlations between photon pairs from 'independently generated' pairs. Both the experiment and its further theoretical description by other authors [10] showed rather low visibility of four-photon interference (about 50 %). Note that in all such experiments, as a rule, the visibility of three- and four-photon interference does not exceed 60 % – 70 %. Exceptions are works where four-photon interference was observed together with two-photon one [5, 9].

Despite numerous experiments, it is still discussed in the literature how one can distinguish between four-photon states and accidentally overlapping photon pairs. The

*For the case where PDC is nondegenerate in at least one parameter (frequency, angle, or polarisation).

question is still open: do four-photon correlations observed in PDC indicate that entangled four-photon states are generated?

To answer this question, one can consider the generation of photon 'quadruples' directly, similarly to the generation of photon pairs via SPDC [11]. Such four-photon correlations can be called 'true'. It turns out that the difference between the four-photon correlations observed in SPDC and 'true' four-photon correlations is in the dependence of the normalised fourth-order intensity moment on the mean photon number. However, this dependence was not measured in any of the known works, although simultaneous measurement of intensity moments of different orders depending on the mean photon number was indeed carried out (see, for instance, [12]).

One of the main difficulties in the measurement of the normalised intensity CFs for PDC is their extremely small width, which prevents, for the case of relatively 'slow' detection, the measurement of the nonclassical part of the CF. As we will show below, this difficulty can be overcome by detecting PDC in the regime of sufficiently short pulses. However, no method has been developed so far for measuring normalised intensity CFs for pulsed radiation. Further, we will consider the measurement of normalised intensity CFs for cw and pulsed radiation, present the results of measuring the second-order CF for pulsed SPDC and show that the fourth-order CF for the PDC radiation can be measured only in the regime of femtosecond pulses.

2. Multimode detection and measurement of intensity correlation functions

Consider measurement of the nth-order intensity correlation function. Similarly to the way one measures $g^{(2)}(\tau)$ by means of a beamsplitter, two photodetectors, and double coincidence circuit, in order to measure the nth-order correlation function one should split the beam in n parts by means of $n-1$ beamsplitters and detect the resulting beams by n detectors. Further, coincidences of the photocounts of all detectors should be registered by means of an n-fold coincidence circuit depending on the delays introduced by electronics after the detectors. In the stationary case, the nth-order correlation function has $n-1$ time arguments. When photocount coincidences are detected, it is important that an n-fold coincidence scheme has a finite time resolution T_c. During the measurement, the correlation function $g^{(n)}(\tau_1, ..., \tau_{n-1})$ is integrated over the width of the instrumental function, which is given by T_c. If $T_c/\tau_0 \equiv m \gg 1$, then the measured value of the CF at its center is

$$g^{(n)}_{\text{meas}} \approx 1 + \frac{g^{(n)}(0, ..., 0) - 1}{m^{n-1}}, \tag{2}$$

where m can be called the number of modes and defined, as usual, as the ratio of the detection volume to the coherence volume*. In this case, however, the detection time should be understood as the coincidence circuit resolution time T_c if it is larger than the time constant of the detectors T_{det}. In the opposite case, if $T_{\text{det}} > T_c$ then $m \equiv T_{\text{det}}/\tau_0$. This result becomes very clear if one considers the second-order CF (see Fig. 1).

Let $g^{(2)}(\tau)$ have a width τ_0 and the coincidence circuit resolution be $T_c \gg \tau_0$. Then the measured second-order CF will be a convolution of $g^{(2)}(\tau)$ with the instrumental function, which, in the simplest case, has the form of a rectangle with the width T_c. As a result, the measured value of the CF at its center will be

$$T_c + \int \left[g^{(2)}(\tau) - 1\right] d\tau \approx T_c + \left[g^{(2)}(0) - 1\right]\tau_0,$$

and its 'background' value will be T_c. After the normalisation to this background value, the measured CF at the maximum will become [13]

$$g^{(2)}_{\text{meas}}(0) = 1 + \frac{g^{(2)}(0) - 1}{m}. \tag{3}$$

Usually, the spectrum of PDC is rather broad ($\Delta\lambda \approx 1 - 20$ nm). Correspondingly, the coherence time is not larger than hundreds of femtoseconds. Since the typical detection time T_{det} is about 1 ns, the number of detected modes in PDC exceeds 10^3. One can mention works where the PDC spectrum is specially narrowed by placing the nonlinear crystal into a cavity [14]. In such cases, m can be close to unity. However, it is still large in most of experiments, and when the CF is measured, only the first and the third terms 'survive' in formula (1). The 'thermal' part of the CF can be detected only at $m \approx 1$.

The value of the measured CF at the centre at $m \gg 1$ is [see (3)]

$$g^{(2)}_{\text{meas}}(0) = 1 + \frac{1}{mN}. \tag{4}$$

One can see that if the parametric gain is small enough, so that $N \lesssim m^{-1}$, then, despite the large number of detected modes, the 'noncassical' part of $g^{(2)}(\tau)$ is still revealed. However, if it is a higher-order CF that is measured for the PDC radiation, the 'multimodeness' of detection is more crucial and does not allow one to detect the values of the normalised CF above unity. This happens namely because of the fact that SPDC does not have 'true' correlations of the third and higher orders. This question will be discussed in more detail in Section 5.

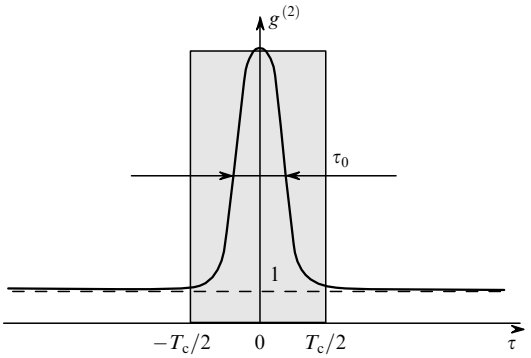

Figure 1. Typical shape of the intensity CF $g^{(2)}(\tau)$ of cw radiation. The rectangle shows the instrumental function of the coincidence circuit with the resolution T_c. The dashed line shows the background.

*The total number of modes also includes the number of transverse modes, which is defined as the ratio of the detection area to the coherence area. However, in practice, the number of transverse modes can be made close to unity, so that further on m will be considered as the total number of modes.

However, it turns out that the CF intensity can be still measured in the single-mode regime if PDC is excited by a sequence of short (femtosecond) pulses. In order to show this, consider CFs for pulsed radiation.

3. Intensity correlation functions of pulsed radiation

Consider the case where radiation is 'modulated' by a periodic sequence of pulses whose duration T_p is much larger or much smaller than the coherence time τ_0. In both cases, the intensity fluctuations of PDC and the modulation of its envelope occur at essentially different time scales, and can therefore be considered as independent.

Let the radiation under study be modulated by a sequence of pulses with the shape $Y(t)$ ($\int Y(t)\mathrm{d}t = 1$) and the period Δt. Then the second-order intensity correlation function has the form [15]

$$g^{(2)}_{\mathrm{mod}}(\tau) = g^{(2)}(\tau) g^{(2)}_{\mathrm{p}}(\tau), \tag{5}$$

where $g^{(2)}(\tau)$ is the CF of the nonmodulated radiation and

$$g^{(2)}_{\mathrm{p}}(\tau) = \Delta t \sum_{j=-\infty}^{\infty} \int Y(t + j\Delta t + \tau)\, Y(t)\mathrm{d}t \tag{6}$$

is the intensity CF for a periodic sequence of pulses with the shape $Y(t)$. Note that the value of $g^{(2)}_{\mathrm{p}}(\tau)$ in all its maxima,

$$g^{(2)}_{\mathrm{p}}(\tau = j\Delta t) \approx \frac{\Delta t}{T_{\mathrm{p}}},$$

is determined by the relative pulse duration of the sequence. It is this well-known fact that provides high efficiency of pulsed radiation for the excitation of two-photon (or multiphoton) processes compared to cw radiation with the same mean intensity.

In order to measure the CF of cw radiation at its maximum [$g^{(2)}(0)$], one can measure the corresponding CF of the modulated radiation [$g^{(2)}_{\mathrm{mod}}(0)$] and then calculate $g^{(2)}(0)$ as [15]

$$g^{(2)}(0) = \frac{\langle N_{\mathrm{c}} \rangle}{\langle N_1 \rangle \langle N_2 \rangle K}, \tag{7}$$

where $\langle N_1 \rangle$, $\langle N_2 \rangle$ are the mean photocount numbers per pulse for two detectors detecting pulsed radiation and $\langle N_{\mathrm{c}} \rangle$ the mean number of their coincidences per pulse. The factor K is given by the expression

$$K \equiv \int_{-T_{\mathrm{c}}/2}^{T_{\mathrm{c}}/2} \mathrm{d}\tau \int Y(t + \tau)\, Y(t)\mathrm{d}t. \tag{8}$$

Consider the two limiting cases: when the fluctuations of the radiation under study are 'very fast' or 'very slow' compared to the pulse duration.

(i) The most interesting situation is realised when $T_{\mathrm{p}} \ll \tau_0$, so that intensity fluctuations occur at times that are considerably larger than the pulse duration. Hence, they only lead to the fluctuations of the pulse amplitude, while the pulse shape remains constant.

In this case, the CF (5) consists of a sequence of peaks with the same shape. Figure 2 shows the shape of the correlation functions: the dashed line corresponds to $g^{(2)}(\tau)$ and the solid line, to $g^{(2)}_{\mathrm{mod}}(\tau)$ normalised by $\Delta t/T_{\mathrm{p}}$. The height of the central peak of $g^{(2)}_{\mathrm{mod}}(\tau)$ is

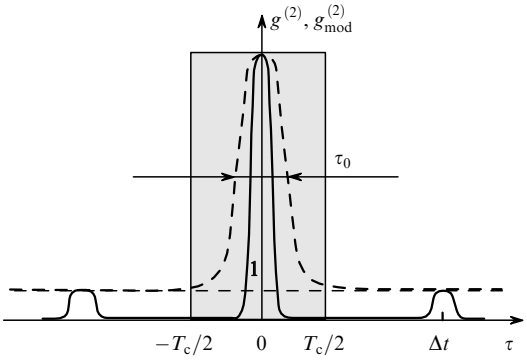

Figure 2. Typical shape of the intensity CF $g^{(2)}(\tau)$ of the radiation modulated by a sequence of pulses that are short compared to the CF width. The dashed line shows the background.

$$g^{(2)}_{\mathrm{mod}}(0) = g^{(2)}(0)\Delta t \int Y^2(t)\mathrm{d}t \approx g^{(2)}(0)\frac{\Delta t}{T_{\mathrm{p}}}, \tag{9}$$

and the heights of the other peaks are

$$g^{(2)}_{\mathrm{mod}}(j\Delta t) = \Delta t \int Y^2(t)\mathrm{d}t \approx \frac{\Delta t}{T_{\mathrm{p}}}. \tag{10}$$

When photocount coincidences are detected, the CF is integrated over the time interval T_{c} given by the coincidence circuit resolution. For the ratio of the central peak height to the height of the side peaks not to be changed by this integration, it is sufficient that the condition $T_{\mathrm{c}} < \Delta t$ is satisfied. Then the height of the central peak becomes $g^{(2)}(0)\Delta t K$ and the height of the side peaks, $\Delta t K$.

One can see that the value of the CF for the nonmodulated radiation at its maximum can be found as the height ratio of the central and side peaks of the CF for the modulated process. If it is impossible in experiment but the shape of the pulse is known, one can use relation (7) instead.

This situation is typical when PDC is excited by femtosecond pump pulses, as in many experiments with two-photon light. The pump is usually the second-harmonic radiation of a Ti : sapphire laser with a repetition rate of about 100 MHz (or 10 kHz, if a regenerative amplifier is used). Then, the condition $T_{\mathrm{c}} < \Delta t$ always holds. Therefore, if PDC is excited by femtosecond pulses, CF intensities can be measured in the single-mode regime. It is due to this fact that the authors of Ref. [16] managed to observe the bunching effect for signal and idler PDC radiation separately.

Note that in the general case, the shape of the down-converted light for pulsed PDC is different from the pump pulse shape. However, in the linear (SPDC) regime, the pulse shape for the pump and the down-converted light is the same. Further, measurement of the correlation functions will be always considered in the linear PDC regime.

A similar situation was experimentally realised in [15] for 'quasi-thermal' radiation with the typical times of intensity fluctuations on the order of milliseconds, modulated by nanosecond pulses with a repetition rate of 50 Hz.

(ii) If the pulse duration is much larger than the CF width for the 'nonmodulated' radiation, the CF of the

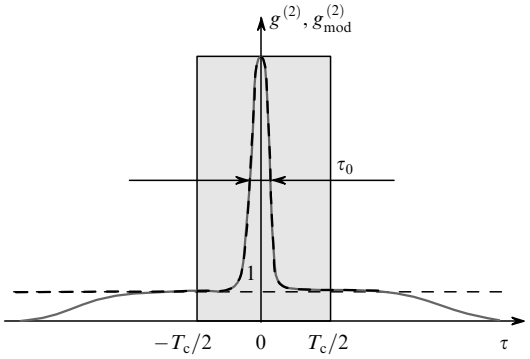

Figure 3. Typical shape of the intensity CF $g^{(2)}(\tau)$ of the radiation modulated by a sequence of pulses that are long compared to the CF width. The dashed line shows the background.

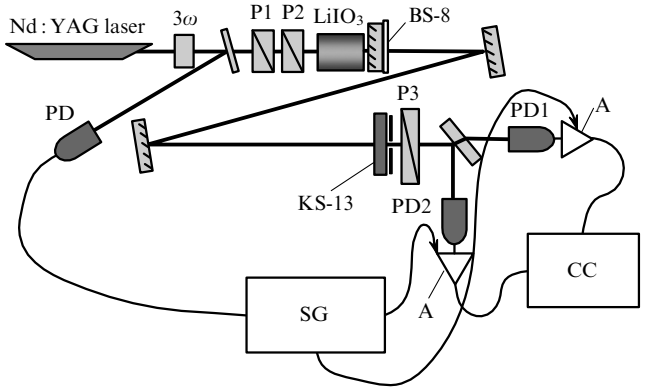

Figure 4. Experimental setup for the study of the intensity CF of pulsed SPDC as a function of the pump power: (3ω) the module generating the third harmonic of a Nd : YAG laser (with the pulse envelope duration 5.6 ns and the repetition rate, 50 Hz); (PD) the photodiode whose signal triggers the gate of the detection system; (P1) and (P2) polarising prisms; ($LiIO_3$) the nonlinear crystal; (P3) a polarisation filter; (PD1) and (PD2) avalanche photodiodes; (A) amplifiers; (SG) the strobe generator; (CC) pair coincidence circuit; (BS-8) a glass filter; (KS-13) a glass filter selecting the PDC radiation.

modulated radiation can be also represented as a product of the 'nonmodulated' radiation CF and the CF of the pulse sequence. The result (normalised by $\Delta t/T_p$) is shown in Fig. 3 by the solid line. The dashed line again shows the CF of the nonmodulated radiation. For simplicity, the side maxima are not shown here but their height is equal to unity.

From the viewpoint of the number of detected modes, the situation is qualitatively the same as for cw radiation: if time resolution is not good enough, a background will be inevitably added to the 'useful' part of the CF. However, the number of modes can be reduced if the pulse duration is smaller than the coincidence time resolution.

It is interesting that when the CF is measured in the regime of relatively short pulses (of nanosecond duration or shorter), there is a simple relation between the measured CF at its maximum and the mean photon number per pulse. Indeed, according to relation (4), the 'contrast' of the CF, i.e., the ratio of the CF maximum to its background, is

$$g_2 \equiv \frac{g^{(2)}(0)}{g^{(2)}(j\Delta t)} = \frac{1}{mN}, \tag{11}$$

where N is the number of photons per radiation mode, i.e., per coherence volume. This relation is valid for both the case $T_p \ll \tau_0$ and the case $T_p \gg \tau_0$. In the first case, $m = 1$. The value in the denominator of (11) is the mean number of photons in the detection volume. For pulses with the duration smaller than $\max\{T_{\mathrm{det}}, T_c\}$, the mean number of photons per pulse is $N_p = mN$, hence,

$$N_p = g_2^{-1}. \tag{12}$$

Relation (12) shows that to achieve high contrast in the measurement of the intensity CF for pulsed radiation, the mean number of photons per pulse must be quite small. This, especially for the case of low repetition rates, is a considerable difficulty.

4. Measurement of normalised second-order CF for nanosecond-pulsed PDC

We have measured the normalised second-order CF for nanosecond-pulsed PDC. The experimental setup is shown in Fig. 4.

PDC was excited in the lithium iodate crystal under collinear frequency-degenerate type-I phase-matching. As a pump, the third-harmonic radiation of a Nd : YAG laser was used, with a repetition rate of 50 Hz and a pulse envelope duration of 5.6 ns. The pump power was varied from 1 microwatt to 10 milliwatt by means of two Glan prisms placed in front of the crystal. After the crystal, the pump radiation was rejected by a UV mirror and a BS-8 glass filter, and the PDC radiation was directed to the measurement part of the setup. Selection of a single spatial mode was provided by a 1-mm aperture placed in the measurement part of the setup at the distance 2.64 m from the crystal, where the pump diameter was 1.2 mm. A KS-13 filter placed in front of the measurement part transmitted the PDC radiation and rejected the most part of background light. The PDC radiation was detected with two avalanche photodiodes placed behind the beamsplitter, and the photocount pulses of the photodiodes were fed to two gated amplifiers forming TTL pulses. The gate signal, with the duration 40 ns, was triggered by the photodiode PD detecting the pump pulse. Photocount coincidences were detected by a coincidence circuit with the resolution $T_c = 4.2$ ns. The measured values were mean photocount numbers per pulse in both detectors, $\langle N_1\rangle$, $\langle N_2\rangle$, and the mean coincidence number per pulse, $\langle N_c\rangle$. In order to make the mean photocount numbers per pulse optimal for the measurement (about 0.1) at any pump intensity, the PDC radiation intensity was attenuated at the input of the measurement part using a polarisation filter.

First of all, we measured the dependence of $\langle N_1\rangle$, $\langle N_2\rangle$ on the average pump power $\bar{P}$ (Fig. 5). The value of $\langle N_1\rangle_{\mathrm{eff}}$ plotted along the vertical axis should be understood as the effective photocount number, since the measured average number of photocounts per pulse never exceeded 0.1 and $\langle N_1\rangle_{\mathrm{eff}}$ was calculated with an account for the polarisation filter transmission. The dependence of $\langle N_1\rangle_{\mathrm{eff}}$ on the average pump power has a nonlinear form at $\bar{P} = 1 - 7$ mW. This indicates that the stimulated regime of PDC was reached.

By using the obtained dependence, one can find the peak value of the parametric gain $F_{\max}$ within the whole range of interest. Indeed, the instantaneous pump power P and the instantaneous parametric amplification coefficient F are related linearly,

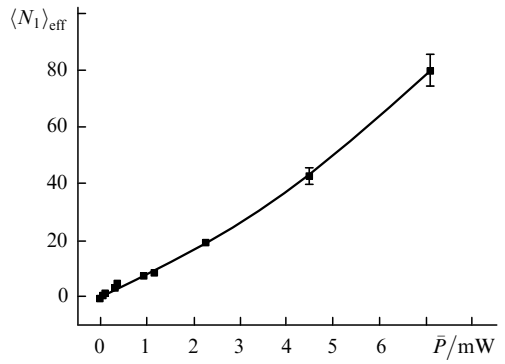

Figure 5. Average PDC signal (effective mean number of photocounts per pulse) for one of the detectors as a function of the pump average power. At $\langle N_1\rangle_{\rm eff} > 0.1$, the PDC radiation was attenuated in a known way by means of a polarisation filter.

$$F = \varkappa P, \tag{13}$$

where the constant $\varkappa$ is determined by the parameters of the crystal (quadratic nonlinearity, length, refractive index) and the pump (the beam diameter and the wavelength). At the same time, for a given pulse shape, the mean photocount number per pulse $\langle N_1\rangle$ is linearly related to the number of photons per mode N at the pulse maximum. In its turn, the peak number of photons per mode depends on the peak parametric gain $F_{\max}$ as

$$N = \sinh^2\sqrt{F_{\max}}\,. \tag{14}$$

Fitting by this formula the dependence shown in Fig. 5, we obtained the relation between the peak parametric gain $F_{\max}$ and the average power of the pump $\bar{P}$ (in mW): $F_{\max} = (0.27 \pm 0.03)\bar{P}$. Theoretical calculation of $F_{\max}$ from the reference data and the parameters of the experimental setup give the value $\varkappa = 0.24$. However, because of the large spread of the reference data on the quadratic susceptibility, the accuracy of this theoretical estimate is not better than 30 %.

It is important that in experiment we measured the integral number of photocounts per pulse (averaged over all pulses), while the nonlinear dependence (14) contains the peak photon number per mode and the peak parametric gain. Therefore, when fitting the experimental dependence with Eqn (14), the value of $\langle N_1\rangle_{\rm eff}$ was normalised by the correction coefficient

$$\kappa(F) \equiv \frac{\int_{-\infty}^{\infty} \sinh^2\sqrt{FY(x)}\,{\rm d}x}{\sinh^2\sqrt{F}},$$

which was calculated numerically (for $\bar{P} = 0 - 7$ mW, the values of κ were between 1 and 1.1).

Further, the normalised second-order intensity CF was measured as a function of the average pump power. Because of a large number of longitudinal modes, the measurements could be performed only at very small values of the power (in the spontaneous PDC regime). The normalised intensity CF was calculated using relations (7), (8), the shape of the pump pulse being measured by means of a coaxial PhEC-15 photoelement. The value of K was measured by two independent methods: from the measured pulse shape (yielding $K = 0.62$) and from the number of photocount coincidences in the regime where the 'nonclassical' correlation is absent, since both detectors register only the signal radiation of frequency-nondegenerate SPDC (yielding $K = 0.64 \pm 0.03$).

By passing, with the help of relation (13), from the average pump power to the parametric gain, which is equal, for the case of SPDC, to the number of photons per mode, one can find the dependence $g^{(2)}(N)$ for SPDC. The obtained dependence is presented in Fig. 6. Fitting it by equation (4) with the only variable parameter being the number of modes m, we found that $m = 43000 \pm 3000$. This value is close to the number of modes calculated from the known parameters of the crystal (and, hence, the known width of the spectrum) and the value $T_{\rm c}$: $m = 46500$.

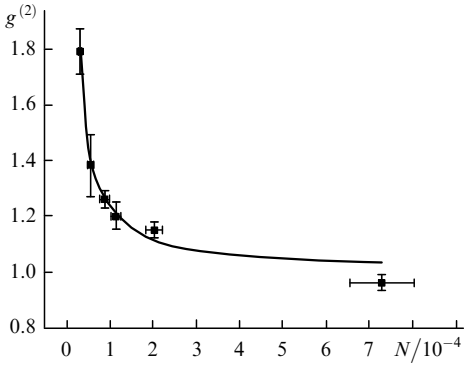

Figure 6. Dependence of the SPDC correlation function $g^{(2)}$ on the number of photons per mode N at the pulse maximum (points). Solid line is the theoretical dependence (10), the number of modes m being the fitting parameter.

Thus, the experimental dependence of the second-order normalised CF on the number of photons per mode is well described by relation (4). The obtained agreement between the experimental and theoretical dependencies confirms the validity of the proposed method of measuring the CF intensity in the pulsed regime.

5. Fourth-order CF for PDC and the problem of its measurement

In order to find out whether the four-photon correlations in SPDC are 'true' or not, it is necessary to measure the normalised fourth-order CF. It follows from Eqn (3) that

$$g^{(4)}_{\rm meas} \approx 1 + \frac{g^{(4)}(0,\ldots,0) - 1}{m^3}. \tag{15}$$

In Ref. [11], the normalised fourth-order CF was calculated for PDC with an arbitrary parametric gain. Various types of phase matching were considered: single-mode PDC, two-mode PDC, as well as PDC leading to the generation of two-photon entangled states. In all these cases, the dependence of $g^{(4)}(0)$ on the parametric gain F is similar and differs only in the coefficients standing by the powers of the hyperbolic cotangent:

$$g^{(4)}(F) = k_1 + k_2\coth^2\sqrt{F} + k_3\coth^4\sqrt{F}. \tag{16}$$

The normalised fourth-order CF takes maximal values in the single-mode case, where

$$k_1 = 24, \quad k_2 = 72, \quad k_3 = 9.$$

It follows from Eqns (15) and (16) that in the case of multimode detection, the measured fourth-order CF will be noticeably different from unity at $F \ll 1$:

$$g^{(4)}_{\mathrm{meas}}(F \ll 1) \approx 1 + \frac{9}{m^3 N^2}. \tag{17}$$

One can see that in the measurement of the fourth-order CF, to 'compensate' for the increase in the number of modes it is necessary to reduce the number of photons per mode not to $N \sim m^{-1}$, as in the measurement of the second-order CF, but to $N \sim m^{-3/2}$, which is much smaller. The relation between the mean number of photons per pulse and the 'contrast' of the fourth-order CF has the form

$$g_4 = 9 N_{\mathrm{p}}^{-2} m^{-1}. \tag{18}$$

Thus, it is almost impossible to distinguish $g^{(4)}$ from unity in the case of multimode detection. For instance, at $m = 10^4$, the 'contrast' $g_4 = 10$ is provided when the number of photons per pulse is 10^{-2}.

Note that the difficulty of measuring the fourth-order CF is caused by the fact that, according to Eqns (16), (17), the asymptotic of the normalised fourth-order CF at small N is not N^{-3}, as would be the case for four-photon light, but N^{-2} [11].

Normalised fourth-order CF can still be measured using femtosecond-pulsed SPDC, where $m = 1$ and a high 'contrast' can be achieved at mean numbers of photon per pulse about 1 or less, which, with the repetition rate being on the order of hundred MHz, is not difficult.

6. Conclusions

Thus, we have analysed the effect of multimode detection on the measured intensity CFs of higher orders. It was shown that single-mode measurement of the intensity CF is indeed possible for PDC if it is excited by sufficiently short (femtosecond) pulses.

For the measurement of normalised intensity CFs in the pulsed regime, formulas [15] were obtained, relating the value of the normalised CF with the measurable values – mean numbers of photons and coincidences per pulse – as well as the parameters of the measurement setup, the pulse shape and the coincidence resolution. These formulas, which were previously used only for the study of classical radiation (thermal and coherent), are now used for the measurement of the second-order CF for SPDC. The obtained dependence of the normalised CF on the number of photons per mode is in good agreement with the theoretical dependence, which confirms the validity of the presented method.

For the measurement of the second-order CF in the pulsed regime and single transverse-mode detection, a simple relation has been obtained: the product of the measured CF at its maximum and the mean number of photons per pulse is equal to unity.

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