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On the calculation of the gyro-factor in a semiconductor ring laser

P.G. Eliseev

Abstract. **The frequency splitting due to the Sagnac effect is considered in the active laser gyro with a ring cavity filled with a medium having the high refraction index and nonzero dispersion. It is shown that the medium reduces the gyro-factor by n^* times, where n^* is the group refractive index. However, in the case of dynamic anomalous dispersion, it is possible in principle to increase the gyro-factor without increasing the ring cavity size.**

Keywords: *laser gyros, semiconductor ring laser, gyro-factor*.

1. Introduction

Gas lasers in which the influence of the dispersion of a medium is negligible are successfully used in laser gyros [1 – 5]. The spectral splitting $\Delta\nu$ of a cavity mode caused by the Sagnac effect in such an active laser gyro [1] is proportional to the angular rotation velocity Ω, $\Delta\nu = K\Omega$, where K is the gyro-factor. For a ring laser with the contour area A and the perimeter length L, we have $K = 4A/(\lambda L)$. For a circular contour, $K = 2R/\lambda$, where λ is the wavelength in vacuum and R is the circle radius.

The recent studies of monolithically integrated semiconductor ring lasers (SRLs) [6] have demonstrated certain prospects for their applications in rotation sensors. For example, mode beatings were observed between independent SRLs with the cavity length (perimeter) above 1 cm [6]. However, because of a comparatively small size of SRLs, the gyro-factor K is too small for applications in navigation. And this is not the only, but substantial problem.

Because a semiconductor active medium has the high refractive index and dispersion, the influence of these quantities on the gyro-factor is important. However, the relevant theoretical results are quite ambiguous. It was shown already in [2] that the frequency splitting $\Delta\nu$ decreases with increasing the refractive index of the medium. In [7], some papers were considered in which the influence of the refractive index n on the gyro-factor was calculated. The dependence of the frequency splitting due to the Sagnac effect on n was varied in these papers from $\Delta\nu \propto n$ to $\Delta\nu \propto n^{-2}$. Therefore, the gyro-factor for the refractive index $n = 3.6$, which is typical for semiconductors, changes by 46.6 times depending on the approximations used in calculations. The influence of the refractive index dispersion on the gyro-factor is also unclear. It was shown in [3] that $\mathrm{d}n/\mathrm{d}\nu$ appears in the calculation of $\Delta\nu$; however, in [4] it was claimed that the dispersion does not affect K in the first-order approximation. In [7], the dispersion appears in the expression for K together with a factor proportional to Ω and, therefore, its influence is negligible at small velocities.

Note also that because dispersion can be changed by various dynamic methods (for example, by 'slowing down' or 'accelerating' light), the question arises of whether it is possible to increase K without further increasing the SRL size? The answer depends on whether the dispersion influences the Sagnac effect. We show how it influences this effect and present the relevant calculation for the non-relativistic case. Because the frequency splitting components caused by the Sagnac effect in a dispersion medium are characterised by different n, their phase velocities prove to be different. This leads to a decrease or increase in the splitting depending on the dispersion type (normal or anomalous).

In this paper, a SRL is considered in which a cavity is filled with an emitting medium having the refractive index $n > 1$ and dispersion $\mathrm{d}n/\mathrm{d}\nu \neq 0$. For example, in a bulk GaAs at a wavelength of ~ 880 nm, we have $n = 3.61$ and $\nu\mathrm{d}n/\mathrm{d}\nu \sim 1.08$. The Fresnel drag coefficient is not zero but 0.923. Therefore, taking into account these parameters is obligatory. Because the gyro-factor K is inversely proportional to the group refractive index, i.e. is also sensitive to n and its dispersion, it is possible in principle to change its value by producing the dynamic anomalous dispersion.

2. Calculation of the frequency splitting

Consider a homogeneous ring cavity of a circular shape of radius R formed, for example, by an integrated-optical waveguide in a laser semiconductor. Let us assume that two counterpropagating travelling waves, which are the independent modes of the cavity, exist in the laser. They have the same frequency ν_0 in the immobile cavity, which is related to the perimeter length L by the expression

$$\nu_0 = \frac{cq}{n_0 L}, \tag{1}$$

where c is the speed of light; n_0 is the refractive index at frequency ν_0;

$$q = n_0 L \nu_0 / c = \nu_0 T_0 \tag{2}$$

P.G. Eliseev P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia; present address: Center for High Technology Materials, University of New Mexico, Albuquerque, NM, USA; e-mail: eliseev@chtm.unm.edu

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is the longitudinal index of the mode; and $T_0 = n_0 L/c$ is the round-trip transit ('phase') time in the cavity. The value of the longitudinal index is not specified, it is only important that it is conserved during rotation, i.e. we are always dealing with the same modes.

Let us assume now that the cavity rotates with the constant angular velocity Ω (for definiteness, clockwise). In this case, the counterpropagating waves become non-equivalent and their frequencies become different due to a change in the perimeter length at which the phase should be reproduced, and a change in the phase velocity. Because the interval of Ω of interest for navigation measurements corresponds to comparatively small linear velocities, we are dealing with the nonrelativistic case, i.e. the quantity $(\Omega R/c)^2$ can be neglected. However, the result should not contradict the theory of relativity and, in particular, the value of K should be independent of the velocity of transient linear motion.

Consider a wave propagating in the same direction as the medium does. The length L^+ at which the wave phase is reproduced will increase by the distance which the medium passes for the round-tripe transit time T^+, i.e. by $\Omega R T^+$, and will be $2\pi R + \Omega R T^+$. In this case, the phase velocity is

$$V^+ = c/n^+ + \alpha^+ \Omega R, \tag{3}$$

where n^+ is the refractive index at the frequency ν^+ by which the frequency of the wave changes; and α^+ is the drag coefficient. Now, we obtain the equation for T^+:

$$T^+ = \frac{L^+}{V^+} = \frac{2\pi R + \Omega R T^+}{c/n^+ + \alpha^+ \Omega R}, \tag{4}$$

whose solution has the form

$$T^+ = \frac{2\pi R}{c/n^+ - \Omega R(1-\alpha^+)}. \tag{5}$$

For the counterpropagating wave, we have similarly

$$L^- = 2\pi R - \Omega R T^-, \quad V^- = c/n^- - \alpha^- \Omega R, \\ T^- = \frac{2\pi R}{c/n^- + \Omega R(1-\alpha^-)}. \tag{6}$$

The frequencies of these modes correspond to the condition of conservation of the mode index mentioned above,

$$q = \nu_0 T_0 = \nu^+ T^+ = \nu^- T^-, \tag{7}$$

which gives

$$\begin{aligned} \Delta\nu = \nu^- - \nu^+ &= \nu_0 T_0 \left(\frac{1}{T^-} - \frac{1}{T^+} \right) \\ &= \frac{\nu_0 T_0}{2\pi R} \left[\frac{c}{n^+} + \Omega R(1-\alpha^+) - \frac{c}{n^-} + \Omega R(1-\alpha^-) \right] \\ &= \frac{\nu_0 n_0}{c} \left[2\Omega R(2 - \alpha^+ - \alpha^-) + \frac{c}{n^+} - \frac{c}{n^-} \right]. \end{aligned} \tag{8}$$

By taking into account the first-order dispersion, we can obtain

$$\frac{c}{n^+} - \frac{c}{n^-} \approx -\frac{c}{n_0^2} \Delta\nu \frac{\mathrm{d}n}{\mathrm{d}\nu}, \tag{9}$$

and this substitution gives the equation for $\Delta\nu$:

$$\Delta\nu \left(1 + \frac{\nu_0}{n_0} \frac{\mathrm{d}n}{\mathrm{d}\nu} \right) = 2 \frac{\Omega R \nu_0 n_0}{c} (1-\alpha). \tag{10}$$

Here, $\alpha = (\alpha^+ + \alpha^-)/2$ and the solution has the form

$$\Delta\nu = 2 \frac{\Omega R \nu_0 n_0}{c} (1-\alpha) \left(1 + \frac{\nu_0}{n_0} \frac{\mathrm{d}n}{\mathrm{d}\nu} \right)^{-1}. \tag{11}$$

3. Results and discussion

The frequency splitting for a rotating ring laser with a rotating medium is described by a more general expression (expression (51) in [4])

$$\Delta\nu = \frac{2\upsilon}{c} \frac{\oint n^2 (1-\alpha)\, \mathrm{d}r}{\oint n\, \mathrm{d}s}, \tag{12}$$

which, however, neglects dispersion. Here, integration is performed over the ring laser perimeter; ds is the perimeter length element; dr is the radial displacement element; and υ is the linear rotation velocity. This expression is valid for uniformly filled cavities of arbitrary shapes. By integrating over a uniform circle of radius R and taking into that $\upsilon = \Omega R$, we obtain

$$\Delta\nu = 2\Omega R \nu n(1-\alpha)/c, \tag{13}$$

which is identical to expression (11) neglecting dispersion.

Consider the role of the drag coefficient α (see also discussion in [4]). It was initially obtained by Fresnel in the form

$$\alpha = 1 - \frac{1}{n^2}. \tag{14}$$

The corresponding effect was confirmed by Fizeau in experiments with a moving medium in an interferometer arm. Later, Lorentz introduced the dispersion addition and obtained the drag coefficient in the form

$$\alpha = 1 - \frac{1}{n^2} + \frac{\nu}{n} \frac{\mathrm{d}n}{\mathrm{d}\nu} \tag{15}$$

(the derivation is presented in [8]). If expression (15) is applied for rotating cavities, the problem of invariance to the uniform translational motion required by the theory of relativity appears. Expression (14) does not present such a problem. Dispersion exerts some effect if the frequency is changed. The remark of Einstein [9] on this subject (see discussion in [4]) is that the dispersion addition appears due to the Doppler effect between an immobile source and a moving interferometer in the experiments considered by Lorentz. In our case, a source moves together with the cavity, which allows the use of expression (14). By substituting (14) into (11), we obtain

$$\Delta\nu = 2 \frac{\Omega R \nu_0}{c} \left[n_0 \left(1 + \frac{\nu_0}{n_0} \frac{\mathrm{d}n}{\mathrm{d}\nu} \right) \right]^{-1} = \frac{2\Omega R}{\lambda n^*}, \tag{16}$$

where

$$n^* = n_0 \left(1 + \frac{\nu_0}{n_0} \frac{\mathrm{d}n}{\mathrm{d}\nu} \right)$$

is the group refractive index and λ is the wavelength in vacuum. The gyro-factor is

$$K = \frac{2R}{\lambda n^*}, \quad (17)$$

or in the general form,

$$K = \frac{4A}{L\lambda n^*}. \quad (18)$$

We derived expression (11) by neglecting the quantity of the order of $[(\Delta\nu/n)\mathrm{d}n/\mathrm{d}\nu]^2$ compared to unity. For $n = 3.6$, $\mathrm{d}n/\mathrm{d}\nu = 3\times10^{-14}$ Hz^{-1} (parameters of a GaAs laser), and $\Delta\nu = 100$ MHz, this quantity is 7×10^{-13}. We also excluded the dispersion of the coefficient α, which corresponds to the neglect of the quantity of the same order of magnitude. Thus, these approximations are quite acceptable. For $R = 1$ cm, $\lambda = 1$ μm, and $n^* = 3.8$ (InGaAs laser), the gyro-factor K can be as high as 5.2×10^3. The calculations of $\Delta\nu$ in a rotating coordinate system are presented in the Appendix. Expression (A3) obtained is completely equivalent to (17), which means that in this case the frequency splitting is invariant to the selection of the coordinate system.

According to these expressions, when a ring cavity is placed in a medium with the group refractive index n^*, the Sagnac effect is reduced by n^* times. This can be explained by a strong influence of the optical drag effect. In the limit of complete dragging $\alpha \to 1$, no frequency splitting occurs.

The introduction of a quartz plate into the cavity in experiments with gas ring lasers [10] resulted in a decrease of K, in qualitative agreement with (17). The dispersion introduced in these experiments was too small to exert any significant effect. The advantage can be achieved by decreasing n^*, for example, in the case of anomalous dispersion. However, the known semiconductor lasers operate in the normal dispersion region where $n^* > n$.

Consider the possibility of changing the gyro-factor by producing dynamic dispersion, for example, due to the nonlinear interaction between modes [11 – 13]. In this case, comparatively small variations in the refractive index n occur in a narrow spectral interval, resulting in a considerable induced dispersion. Therefore, the group index experiences variations, in particular, changes in sign in the vicinity of the 'strong' mode frequency. The frequencies at which n^* vanishes are called points of 'critically anomalous dispersion' where the group velocity tends to infinity. The value of K can be in principle increased near such a point according to (17) when n^* approaches zero. Thus, by using the nonlinear interaction of modes in a semiconductor ring laser, the anomalous dispersion can be obtained by selecting properly the regime and frequency detuning corresponding to the 'fast' light concept.

4. Conclusions

In this paper, the expression has been obtained for the gyro-factor K in a homogeneous ring laser taking into account dispersion and frequency dragging. Due to the discrepancy of the data in the literature, we presented the derivation of expressions for small rotation velocities in the stationary and rotating coordinate systems. The gyro-factor calculated in the nonrelativistic case is independent of the coordinate system. The normal dispersion reduced K; however, the gyro-factor can be increased in principle due to dynamic anomalous dispersion. The calculation can be applied to an integrated-optical semiconductor ring laser.

Appendix

Expression (17) can be obtained in the rotating coordinate system as follows.

In a system rotating at the velocity Ω in the same direction, the ring cavity is immobile, so that the length is the same for both waves: $L^+ = L^- = 2\pi R$. Because the medium is also immobile in this case, the drag effect is absent. However, the phase velocities of the counterpropagating waves in this noninertial system become different. In the absence of the medium, as shown in [14], the velocities are $V_{\mathrm{r0}}^{\pm} = c \pm 2\Omega A/L$, where the subscripts r and 0 correspond to the rotating system and the absence of the medium, respectively. For a cavity filled with a medium, we have

$$V_{\mathrm{r}}^{\pm} = V^{\pm} \mp \Omega R = c/n^{\pm} \mp \Omega R/(n^{\pm})^2. \quad (A1)$$

The relativistic expressions for phase velocities in a rotating medium have been derived in [15] [expressions (2.34) and (2.35)]. Expression (A1) can be obtained from these expressions by passing to nonrelativistic velocities. After the calculation similar to that presented above, we obtain

$$\Delta\nu = \nu_0 T_0 (L^-/V^- - L^+/V^+)$$
$$= \frac{n_0}{\lambda}\left\{\frac{c}{n^-} - \frac{c}{n^+} + \Omega R\left[\frac{1}{(n^+)^2} + \frac{1}{(n^-)^2}\right]\right\}. \quad (A2)$$

By using approximation (9) and assuming that $1/(n^+)^2 + 1/(n^-)^2 \approx 2/n_0^2$, we obtain

$$\Delta\nu = \frac{2\Omega R}{\lambda n^*}, \qquad K = \frac{2R}{\lambda n^*}. \quad (A3)$$

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