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Self-phase modulation of radiation in fibreoptic communication lines

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***Abstract*. Optical nonlinear effects of self-phase and cross-phase modulation and stimulated Brillouin scattering (SBS) determining the data transmission quality in fibreoptic communication lines are studied. The Stokes radiation power upon SBS is calculated taking into account the nonlinear broadening of the optical signal spectrum caused by self-phase modulation. The mechanism of signal distortion caused by the joint action of self-phase modulation and chromatic dispersion is considered. The main theoretical results are verified experimentally.**

Keywords: *fibreoptic communication line, self-phase modulation, cross-phase relaxation, four-wave mixing, stimulated Brillouin scattering.*

1. Introduction

To increase the data transmission range in modern fibreoptic communication lines (FOCLs), erbium-doped fibre amplifiers operating at a wavelength of about 1.5 μm are used. However, at high radiation powers various optical nonlinear phenomena appear in a fibre such as stimulated Raman scattering (SRS), self-phase modulation (SPM) and cross-phase modulation (CPM), stimulated Brillouin scattering (SBS) and four-wave mixing [1, 2]. SPM and CPM are the most important optical nonlinear effects in FOCLs with a small number of sections without optical amplifiers. These types of modulation appear due to the dependence of the refractive index of a nonlinear medium in which radiation propagates on the radiation intensity (the so-called Kerr effect) and can cause, under certain conditions, the broadening of optical pulses.

The Kerr effect should be considered by taking into account the spectrum of an optical signal transmitted in the communication line and the method of data coding. Usually, semiconductor laser diodes with a very narrow emission line ($\sim$ 10 MHz) are employed as radiation sources in FOCLs. To transfer the data signal, the diode current is modulated at a certain frequency, which results in the intensity modulation of the diode laser emission (the so-called direct modulation). In this paper, unless otherwise stated, the data signal coding is considered at which high and low power levels are represented by the unit and zero bits, respectively, two any successive unit bits being transferred without the drop of the radiation power to the value corresponding to the logic-0 level.

The Kerr effect in optical communication lines can play both a positive and negative role. The spectral broadening caused by SPM and CPM increases the SBS threshold. In this case, for bit rates 2.5 Gbit s^{-1} in the case of a directly modulated laser diode, SBS becomes negligible. On the other hand, the joint action of the Kerr effect and the group-velocity dispersion (GVD) of light pulses distorts the pulse shape, thereby increasing the error probability during data transfer. Under certain conditions, the Kerr effect can produce phase noise. The limiting linewidth of a laser diode determined by its intrinsic phase noise was obtained in [3]. The influence of spontaneous emission of optical amplifiers on the phase noise and signal broadening was studied in [4, 5].

So far the SBS threshold was calculated by neglecting the Kerr effect, while the joint action of SPM, CPM, and dispersion in FOCLs was considered only in the calculation of the optical pulse broadening [6]. Note also that experimental data on the influence of SPM and CPM on the number of errors during data transfer in FOCLs and confirming theoretical calculations were virtually absent.

Our paper is devoted to the study of restrictions imposed by SPM and CPM on the communication range in fibres. We calculated the SBS threshold taking SPM into account and confirmed it experimentally. We also measured the minimal admissible radiation power incident from the communication line on a photodetector for the specified error rate, which drastically increases due to the joint action of the Kerr effect and dispersion. The obtained results were analysed qualitatively.

2. Effect of SPM of radiation on SBS

Stimulated Brillouin scattering is backward scattering of light by acoustic vibrations of a medium. As the output radiation power exceeds a critical value, the scattering intensity increases avalanche-like, resulting in the degradation of the transmission quality. The spectrum of scattered light in the 1.5-μm region is shifted to the red by 10 GHz with respect to the incident light and its width Γ is $\sim$ 20 MHz.

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In [7], the critical power $P_{\rm cr}$ of SBS is defined as the pump radiation coupled into a fibre for which the power of scattered light measured at the fibre input is $P_{\rm cr}$. It is assumed that the pump depletion due to SBS is absent. Note that this definition is not valid when the pump depletion is taken into account because the complete conversion of pump radiation to scattered radiation is impossible due to the law of conservation of energy. The real conversion efficiency of the threshold pump power to the power of scattered radiation is of an order of per cent. For typical parameters of a single-mode fibre, $P_{\rm cr}$ is described by the expression

$$P_{\rm cr} \approx 21\frac{\alpha A}{g_0}. \qquad (1)$$

The values of parameters used in calculations for a standard single-mode fibre in the 1.5-μm region are presented below.

Fibre loss factor $\alpha/{\rm m}^{-1}$	5×10^{-5}
Nonlinear coefficient $\gamma/{\rm W}^{-1}\ {\rm m}^{-1}$	1.621×10^{-3}
Number M of transverse fibre modes	2
Temperature T/K	300
FWHM of the SBS gain band Γ/Hz	2×10^{7}
Peak SBS gain $g_0/{\rm W}^{-1}$ m	4×10^{-11}
Effective cross section of the fibre core $A/{\rm m}^2$	5×10^{-11}
Acoustic phonon frequency (Brillouin frequency) $\nu_{\rm Br}$/Hz	1.11×10^{10}
Pump radiation frequency $\nu_{\rm p}$/Hz	1.93×10^{14}

The SBS threshold depends not only on fibre parameters but also on the signal linewidth [6, 8, 9]. If the SBS gain and pump spectra have Lorentzian profiles, expression (1) is still valid for the threshold power, but the maximum of the function $g(\nu)$ is described by the expression

$$g_{\max} \approx \frac{\Gamma}{\Gamma + \Gamma_{\rm p}} g_0, \qquad (2)$$

where $\Gamma_{\rm p}$ is the pump line width. Thus, in the case $\Gamma_{\rm p} \gg \Gamma$, the SBS gain decreases by a factor of $\Gamma_{\rm p}/\Gamma$.

In the general case, the SBS gain and, hence, the threshold power is determined by the convolution of the spectra of spontaneous Brillouin scattering and pump radiation at the linear stage of the process development [8, 9]. For this reason, the type of modulation (amplitude, phase or frequency) plays an important role. In [10], various types of modulation of the transmitted signal are considered in detail. In [9, 11], the dependences of the SBS gain on the bit rate in an optical fibre were obtained for the amplitude, phase, and frequency modulations of the input signal. The threshold powers calculated for amplitude-modulated input signals and different bit rates (622, 1000, 1250, and 2500 Mbit s^{-1}) were about 15 dBm. Hereafter, power expressed in dBm is defined as the decimal logarithm of power in milliwatts.

We measured the SBS threshold in a single-mode fibre of length 200 km by using an unmodulated distributed-feedback laser diode emitting the 1550-nm line of width 15 MHz.

The solid curve in Fig. 1 shows the dependence of the signal power at the fibre output on the input power. Experimental data are in good agreement with the calculated threshold values. The threshold power calculated for unmodulated radiation was about 9.7 dBm. The dashed line in Fig. 1 shows a similar dependence upon transmission of alternating zero and unit bits at the bit rate $B =$ 2500 Mbit s^{-1}. The SBS threshold upon direct modulation of a laser diode in the power range studied (1 – 400 mW) was not achieved, which contradicts theoretical estimates (18 – 19 dBm). We explain this by the broadening of the signal spectrum caused by the SPM of radiation [10].

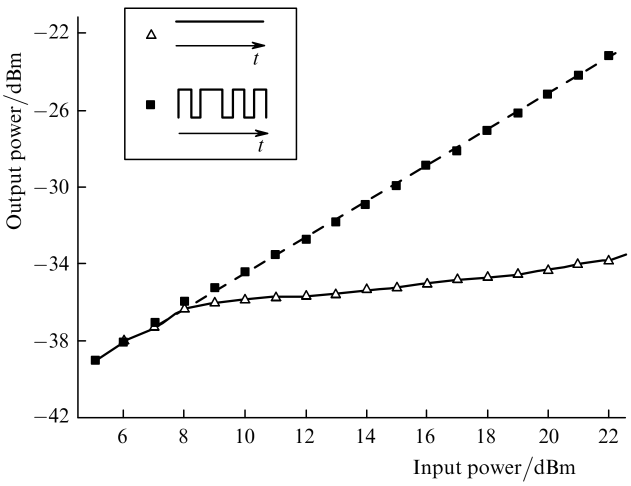

Figure 1. Dependences of the signal power at the output of a single-mode fibre of length 200 km on the input power for a continuous input signal (△) and a signal in the form of the bit sequence (■).

The shapes of the signal spectrum observed at the output of a fibre of length 200 km are shown in Fig. 2. Measurements were performed with an optical spectrum analyser with a resolution of 0.2 nm. The spectra are normalised to the maximum spectral power density and correspond to different radiation powers (from 3 to 25 dBm) coupled to the fibre. One can see from Fig. 2 that, as the input power is

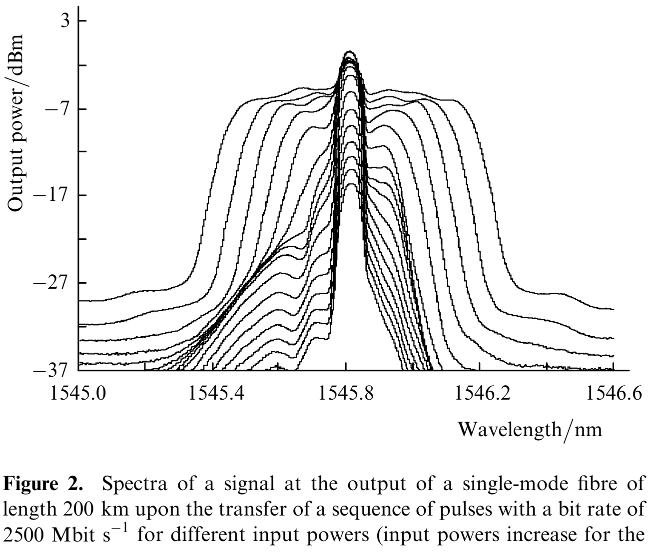

Figure 2. Spectra of a signal at the output of a single-mode fibre of length 200 km upon the transfer of a sequence of pulses with a bit rate of 2500 Mbit s^{-1} for different input powers (input powers increase for the curves from bottom to top).

increased, the signal spectrum considerably broadens at the -10 dB level. The 1545.8-nm line broadens within the resolution of the analyser (0.1 nm).

In earlier papers, the threshold SBS powers were calculated by neglecting the influence of SPM. We developed the analytic model of SBS taking into account SPM and used it to calculate the critical power [12–14]. The conversion factor of the pump to SBS radiation in long communication lines ($L \gg \alpha^{-1}$, where L is the fibre length) has the form

$$\Psi(\chi) = Mk_{\rm B}T\frac{2\pi^{3/2}}{3^{3/4}}\frac{\nu_{\rm p}}{\nu_{\rm Br}}\left(\frac{\Gamma\chi}{\tau}\right)^{1/2}\frac{\gamma}{\alpha}G(\chi)^{-3/2}\exp G(\chi), \quad (3)$$

where $k_{\rm B}$ is the Boltzmann constant; $\chi = \langle\nu^2\rangle_0\tau/\Gamma$ is the normalised root-mean-square width of the pump spectrum; $\langle\nu^2\rangle_0$ is the root-mean-square width of the pump spectrum at the fibre input; τ is the pulse repetition period; and

$$G(\chi) = \frac{3^{3/4}}{32}\left(\frac{\pi}{\ln 2}\right)^{1/2}\frac{g_0}{\gamma A}\frac{1}{\chi}.$$

Expression (3) was derived by using the following assumptions. First, it was assumed that the pump depletion caused by SBS is absent. In this case, the pump conversion factor should be always much lower than unity [$\Psi(\chi) \ll 1$]. Second, the region of admissible widths of the pump spectrum was assumed to be bounded above:

$$\chi \ll \frac{3^{3/4}}{32}\left(\frac{\pi}{\ln 2}\right)^{1/2}\frac{g_0}{\gamma A} = 75.$$

The dependence $\Psi(\chi)$ calculated by using parameters presented above is shown in Fig. 3.

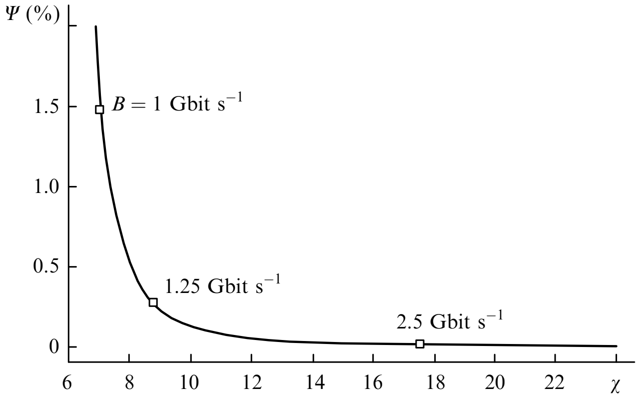

Figure 3. Conversion coefficient Ψ of the pump to SBS as a function of the normalised root-mean-square width χ of the pump spectrum at the fibre input.

Let us use the definition of the critical SBS power introduced in [3]: $P_{\rm cr} = P_{\rm p}(0) = P_{\rm s}(0)$, where $P_{\rm p}(0)$ and $P_{\rm s}(0)$ are powers of the pump and signal, respectively, at the fibre input. Taking into account the law of conservation of energy, this definition has meaning only for the solution obtained in the absence of pump depletion due to backward scattering. For $\Psi = 1$, the SBS threshold corresponds to the normalised root-mean-square width of the pump spectrum $\chi_{\rm cr} = 4.8$ and the bit rate $B_{\rm cr} = 680$ Mbit s^{-1}. As the bit rate B is increased, the pump conversion efficiency drastically decreases, and the SBS threshold cannot be achieved even at arbitrarily high powers. In this case, the nonlinear broadening of the pump spectrum 'dominates' over the SBS process. Thus, for $B > 1$ Gbit s^{-1} and high enough signal powers [expression (3) was obtained by assuming that $P_{\rm p} \gg 35$ mW] Brillouin scattering in FOCLs is negligible.

The squares in Fig. 3 correspond to the values of Ψ calculated for different bit rates in FOCLs. The root-mean-square width of the pulsed pump spectrum with a Gaussian profile was estimated by the expression $\langle\nu^2\rangle_0 \approx 2(B/\pi)^2\ln 2$ [6]. Note that the obtained SBS efficiencies are overestimated. This is explained by the following. First, pulses transmitted in fibres have steeper leading edges compared to Gaussian pulses, especially at low bit rates. Second, in the case of a directly modulated laser diode, its emission line acquires the chirped structure, resulting in its additional broadening. For this reason, the SBS effect is rather weak already for $B \sim 622$ Mbit s^{-1}.

3. Joint action of SPM and GVD

We measured the degradation of the signal-to-noise ratio at the output of a communication line of length 100 km by using a dispersion compensator at the input or output of the line. The results are presented in Fig. 4. We determined an increase in the minimal admissible power on a photodetector for the specified error rate, which was normalised to the radiation power corresponding to the limiting case when nonlinear effects were absent. When a compensating fibre was located at the line output, no degradation of the signal-to-noise ratio caused by SPM was observed. When the dispersion compensator was located at the line input, the degradation occurred because in this case only dispersion broadening of the input pulse was compensated before it was broadened due to SPM.

We measured the degradation of the signal-to-noise ratio at the input of the FOCL of length 100 km as a function of

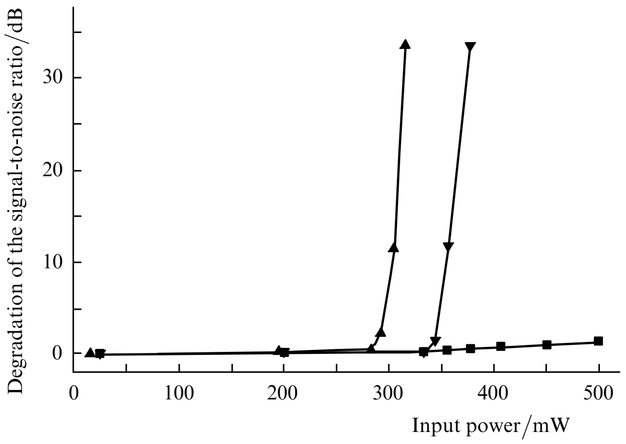

Figure 4. Dependences of the degradation of the signal-to-noise ratio caused by the joint influence of SPM and GVD at the output of a FOCL of length 100 km on the input power obtained without a dispersion compensator (▲) and with the compensator located at the input (▼) and output (■) of the FOCL.

the input optical power for several information channels. Three adjacent channels [23th ($\lambda = 1558.98$ nm), 25th (1557.36 nm), 27th (1555.75 nm)] and one remote channel [35th (1549.32 nm] were combined. The channel numbers are presented according to the standard ITU network. The results of measurements are presented in Fig. 5. Let us define the threshold power upon the joint action of SPM and GVD as the power at the line input at which the minimal admissible power on a photodetector for the specified error rate begins drastically increase. When the adjacent 25th and 23th channels are added to the 27th channel, the threshold power decreases, while upon the joint action of the 27th and 35th channels, this power is almost the same as that for one 27th channel.

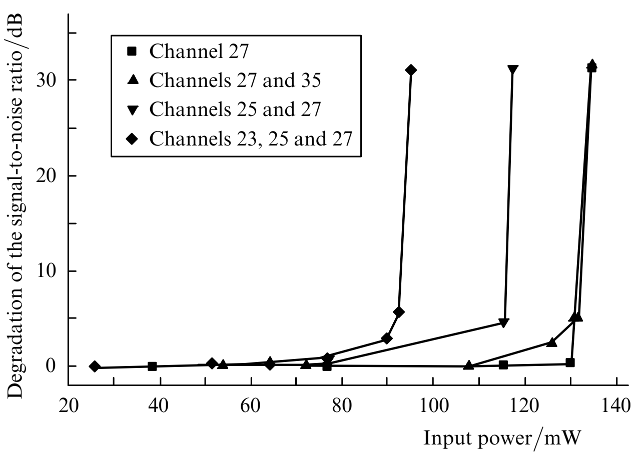

Figure 5. Dependences of the degradation of the signal-to-noise ratio due the joint influence of CPM and GVD at the output of a FOCL of length 100 km on the input power for different combinations of information channels used.

These results can be qualitatively explained by considering the system of equations describing the propagation of three optical signals along the fibre axis z [6]:

$$\mathrm{i}\frac{\partial A_1}{\partial z} = -\frac{\mathrm{i}}{2}\alpha A_1 + \frac{1}{2}\beta_2\frac{\partial^2 A_1}{\partial \tilde{t}^2} - \gamma(|A_1|^2 + 2|A_2|^2 + 2|A_3|^2)A_1,$$

$$\mathrm{i}\frac{\partial A_2}{\partial z} = -\frac{\mathrm{i}}{2}\alpha A_2 - \mathrm{i}d_2\frac{\partial A_2}{\partial \tilde{t}} + \frac{1}{2}\beta_2\frac{\partial^2 A_2}{\partial \tilde{t}^2}$$

$$-\gamma(|A_2|^2 + 2|A_1|^2 + 2|A_3|^2)A_2, \qquad (4)$$

$$\mathrm{i}\frac{\partial A_3}{\partial z} = -\frac{\mathrm{i}}{2}\alpha A_3 - \mathrm{i}d_3\frac{\partial A_3}{\partial \tilde{t}} + \frac{1}{2}\beta_2\frac{\partial^2 A_3}{\partial \tilde{t}^2}$$

$$-\gamma(|A_3|^2 + 2|A_1|^2 + 2|A_2|^2)A_3,$$

where A_i are the amplitudes of optical waves in channels ($i = 1$, 2, 3); β_2 is the GVD parameter; $\tilde{t} = t - z/v_{\mathrm{g}}$ is the time measured in the reference system moving with the group velocity v_{g} of the pulse; and $d_i = (v_{\mathrm{g}1} - v_{\mathrm{g}i})v_{\mathrm{g}1}^{-1}v_{\mathrm{g}i}^{-1}$ is the group-velocity mismatch parameter ($i = 2$, 3). This equation can be solved only numerically. The nonlinear term in equation (4) is usually much greater than the dispersion term. However, despite this, the GVD cannot be treated as perturbation. Because of a strong frequency modulation caused by SPM, even a weak influence of dispersion considerably changes the pulse shape. The spectral broadening caused by SPM is proportional to the maximum phase shift

$$\phi_{\max} = \gamma[|A_1(0)|^2 + 2|A_2(0)|^2$$

$$+ 2|A_3(0)|^2]\alpha^{-1}[1 - \exp(-\alpha L)].$$

The maximum phase shift for one channel is approximately equal to $\gamma P_0\alpha^{-1}$, for two channels – to $3\gamma P_0\alpha^{-1}$, and for three channels – to $5\gamma P_0\alpha^{-1}$ (here, P_0 is the initial pump power). This means that the threshold power should decrease by a factor of three after the addition of one channel and by a factor of five after the addition of two channels. The experimental dependence of the threshold power on the number of channels is weaker. This is explained by the fact that the dispersion separation of pulses described in (4) by terms with coefficients d_i was neglected in calculations. Two spectral channels interact over the fibre length of the order of $t_0/|d_i|$, where t_0 is the pulse duration. For the bit rate $B = 2.5$ Gbit s^{-1}, the 27th and 35th channels are 'separated' by a distance less than 1 km, which is much smaller than the lengths of nonlinear and dispersion interactions. Therefore, they have no time to interact efficiently with each other, which is well demonstrated in Fig. 5. The curves for one (27th) and two (27th and 35th) channels in fact repeat each other.

4. Conclusions

We have calculated the SBS efficiency in a FOCL taking into account the influence of SPM. The upper limit of the conversion efficiency of the pump to Stokes radiation in a standard single-mode fibre in the 1.5-μm region was estimated for different bit rates. In the case of the direct modulation of a laser diode transmitter at a bit rate of 622 Mbit s^{-1}, SBS is negligible (its efficiency is lower than 6.6 %). In this case, the nonlinear broadening caused by SPM 'dominates' over SBS. We have measured the minimum admissible radiation power incident from the line output on a photodetector for the specified error rate as a function of the input optical power and the number of channels. It has been shown that SPM and CPM considerably reduce the SBS threshold. A dispersion compensator at the communication line output compensates pulse distortions caused by the joint action of SPM, CPM, and dispersion. The results obtained in the paper can be useful for designing modern long-range fibreoptic communication systems.

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