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Reconstruction of the spatiotemporal distribution of the effective reflectance of a phase-conjugate mirror from the oscillograms of incident and reflected pulses

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***Abstract*.** **The spatiotemporal characteristics of nonlinear optical adaptive processes with the use of the effective reflectance of a phase-conjugate (PC) mirror based on the stimulated scattering of light or three-wave mixing are considered. The efficiency of the use of this physical parameter for studying the characteristics of PC mirrors in the case of inertialless nonlinearity is demonstrated by the example of phase conjugation of magnetostatic waves.**

Keywords: *PC mirror, effective reflectance.*

1. Introduction

Phase conjugation [1] resulting in the formation of a wave that is complex conjugate to the incident wave virtually has no frequency restrictions and was observed in a broad spectral range [2]. The versatility of this effect was recently confirmed by the fabrication of a phase-conjugate (PC) mirror upon parametric interaction of magnetostatic waves with nonstationary local pumping [3]. In this paper, we consider by this example the spatiotemporal distributions of the effective reflectance of a PC mirror [4], which are typical for stimulated scattering and three-wave mixing in the visible and other frequency ranges and differ only in the material parameters of a nonlinear medium.

2. Formulation of the inverse problem for a PC mirror based on stimulated scattering or three-wave mixing

Consider a plane electromagnetic wave $E_1 = a_1(z,t)\times \exp[\mathrm{i}(\omega t - k_1 z)] + \text{c.c.}$ propagating along the z axis in a layer of a nonlinear active medium bounded by the planes $z = 0$ and $z = L$ and a counterpropagating reflected wave $E_2^* = a_2^*(z,t)\exp[\mathrm{i}(\omega t - k_2 z)] + \text{c.c.}$ The time dependences of the intensities of the signal and reflected waves in the plane $z = 0$ are assumed specified (they are determined by the oscillograms of corresponding pulses). The formation of the reflected wave can be described by means of the efficient reflectance. Such an approach requires the solution of the inverse scattering problem, while the use of initial experimental data having errors makes this formulation of the problem ill-posed [5]. Such problems can be solved only by numerical methods.

We consider the inverse problem for a PC mirror by the example of three-wave parametric interaction. In most such experiments, the characteristic length L of parametric interaction of the waves considerably exceeds their wavelength λ_i ($L \gg \lambda_i$, where $i = 1, 2, 3$), which allows the use of a system of truncated equations for the amplitudes of the signal ($i = 1$) and reflected ($i = 2$) waves interacting in the pump-wave field with the amplitude $a_3(z,t)$ ($i = 3$) [3]:

$$\left(\frac{\partial}{\partial t} + \Gamma_1 + v_1\frac{\partial}{\partial z}\right)a_1(z,t) = a_3(z,t)V_{12}a_2^*(z,t), \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + \Gamma_2 - v_2\frac{\partial}{\partial z}\right)a_2^*(z,t) = a_3^*(z,t)V_{12}^*a_1(z,t), \qquad (2)$$

where $a_i(z,t)$ are the amplitudes of the corresponding waves; v_i is the group velocities of the waves; Γ_i are the decay parameters; and V_{12} is the coefficient of parametric interaction of the waves. For the degenerate case ($\omega_1 = \omega_2 \equiv \omega$), the group velocities of the waves and their decay parameters are identical ($v_1 = v_2 \equiv v$, and $\Gamma_1 = \Gamma_2 \equiv \Gamma$).

Note that the system of equations (1), (2) is not complete for the direct scattering problem. Thus, for SBS mirrors, it should be supplemented with the truncated Navier – Stokes equation, and for PC mirrors based on stimulated scattering of other types – by similar equations [6]. For the inverse scattering problem, in which the time dependences (oscillograms) of the incident and reflected pulses on the input face of the PC mirror are assumed known, the third equation is absent. The inverse problem of determining the effective reflection coefficient is solved irrespective of the specific type of scattering or three-wave mixing.

We will consider the general case of parametric interaction of the waves described by Eqns (1) and (2) taking into account phase conjugation resulting in the formation of the reflected wave PC wave:

$$a_2^*(z,t) = r(z,t)a_1(z,t), \qquad (3)$$

where $r(z,t)$ is the effective amplitude reflectance. By passing to intensities $I_i(z,t) \sim |a_i^2|$ in (1) and (2) and taking (3) into account, we can write

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$$\left(\frac{\partial}{\partial z}+\frac{2\Gamma}{\upsilon}+\frac{1}{\upsilon}\frac{\partial}{\partial t}\right)I_1(z,t)=-R(z,t)I_1(z,t), \tag{4}$$

$$\left(\frac{\partial}{\partial z}-\frac{2\Gamma}{\upsilon}-\frac{1}{\upsilon}\frac{\partial}{\partial t}\right)I_2(z,t)=R(z,t)I_1(z,t), \tag{5}$$

where

$$R(z,t)=-2\,\mathrm{Im}\left(ra_3\frac{V_{12}}{\upsilon}\right) \tag{6}$$

is the effective (taking into account the wave propagation and amplification) reflectance for the wave intensities, which is of interest to us.

Knowing the oscillograms of the signal and reflected PC wave, we can formulate the initial conditions for the intensities of these waves:

$$I_1(z=0,t)=I_1^0(t),\, I_2(z=0,t)=I_2^0(t),\, I_2(z=L,t)=0. \tag{7}$$

By integrating Eqns (4) and (5), taking (7) into account, we obtain the general equations assuming weak decay ($\Gamma\approx 0$):

$$I_1(z,t)=I_1^0\left(t-\frac{z}{\upsilon}\right)\exp\left[\int_z^0 R\left(\eta,t+\frac{\eta-z}{\upsilon}\right)\mathrm{d}\eta\right], \tag{8}$$

$$I_2(z,t)=\int_L^z R\left(\mu,t+\frac{z-\mu}{\upsilon}\right)I_1\left(\mu,t+\frac{z-\mu}{\upsilon}\right)\mathrm{d}\mu. \tag{9}$$

The nonstationarity of the local pump regime used in [3] is determined by a short action time τ_3 of the pump wave compared to the characteristic durations of the signal (τ_1) and reflected (τ_2) waves: $\tau_3\ll\tau_{1,2}$. This condition of the local interaction considerable simplifies the system of equations (8),(9). For the input plane $z=0$, we can write

$$I_2^0(t)=\int_L^0 I_1^0\left(t-\frac{2z}{\upsilon}\right)R\left(z,t-\frac{z}{\upsilon}\right)$$
$$\times\exp\left[\int_z^0 R\left(\eta,t+\frac{\eta-2z}{\upsilon}\right)\mathrm{d}\eta\right]\mathrm{d}z. \tag{10}$$

Equation (10) in N separate intervals $t_j-t_{j-1}=\Delta t\ll\tau_{1,2}$ ($N=\tau_2/\Delta t$, $L=\upsilon\tau_2$, $j=1,\dots,N$), for which $R(z,t)\approx R(z,t_j)$, is equivalent to the Fredholm equations of the first kind in the corresponding intervals:

$$I_{2j}^0(t)=\int_L^0 K(z,t)F_j(z)\mathrm{d}z, \tag{11}$$

where the kernel $K(z,t)=I_1^0(t-2z/\upsilon)$ and

$$F_j(z)=R_j(z)\exp\left[\int_z^0 R_j(2z-\eta)\mathrm{d}\eta\right]$$

are the jth components of the unknown function $F(z)$ in Eqn (11), from which the corresponding components $R_j(z)=R(z,t_j-z/\upsilon)$ of the required function are found. Their sum

$$R(z,t)=\sum_{j=1}^{N}R(z,t_j) \tag{12}$$

is the piecewise continuous approximation of the effective reflectance. Therefore, we can formulate and solve the inverse scattering problem of determining the approximate effective reflectance $R(z,t)$ (12) satisfying the Fredholm equation of the first kind (11) with boundary conditions determined from the oscillograms of signal and reflected pulses.

Thus, having only two experimental oscillograms of the signal pulse $I_1^0(t)$ incident on the PC mirror and reflected pulse $I_2^0(t)$, we can determine sufficiently accurately the spatiotemporal distribution of the effective reflectance (12) by solving ill-posed scattering problem (11) with boundary conditions (7).

3. Comparative analysis of the direct and inverse problems for a PC mirror

We solved the inverse problem by using the regulating Tikhonov algorithm. This algorithm allows one to find the unique solution of the ill-posed problem by using *a priori* known properties of real pulses such as the smoothness and positiveness of their temporal shape. The modified variant of programs realising this algorithm [5] is written in the high-level Delphi language to simplify the integration with the existing program packets for automation of scientific studies.

To verify the correctness of the method used here, we performed a comparative analysis of solutions of the direct and inverse scattering problems for a broad class of model experiments. The result of these numerical experiments confirmed the stability and uniqueness of the solution of the inverse problem in the case of a random scatter in the input parameters up to 7 % of the maximum value $I_1^0(t)$. One of the variants of the comparative analysis is shown in Fig. 1. The conditions under which it was performed are analogous to the conditions for the inversion of pulses in

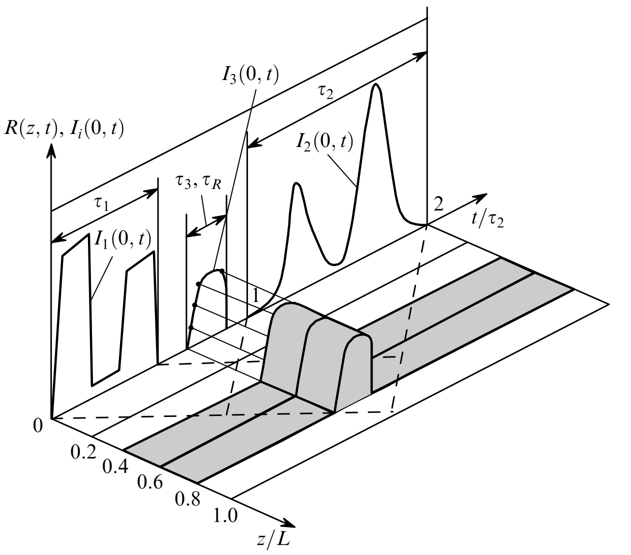

Figure 1. One of the variants of the numerical comparative analysis of the solution of the direct and inverse problems of parametric interaction of the signal I_1 and reflected I_2 waves and the pump wave $I_3(z,t)$ (hatched region) whose spatiotemporal intensity distribution in the direct problem coincides with that for $R(z,t)$ in the inverse problem (with an accuracy to the normalisation factor). The parameter τ_R is the characteristic action time of the effective reflectance $R(z,t)$.

time upon their reflection from a three-wave PC mirror [3]. A specific feature of the experimental results obtained in [3] is that they can be compared both with solutions of the direct and inverse problems for a PC mirror due to the existence of oscillograms for all three pulses and because the entire interaction region ($0 \leqslant z \leqslant L$) is uniformly filled by the pump pulse I_3 during the time τ_3 (Fig. 1). In the case of SBS, for example, the spatiotemporal intensity distribution of a hypersonic wave in the nanosecond range cannot be recorded in experiments.

In the numerical example considered here, equations (1) and (2) and the specified spatiotemporal pump-pulse intensity distribution I_3 (hatched region in Fig. 1) are initial for the direct problem, while the shape of the reflected pulse is the unknown quantity (I_2).

For the inverse problem, equations (1), (2) and oscillograms of the incident and reflected pulses are initial, while the distribution of the effective reflectance $R(z, t)$ is the unknown quantity. In the case of inertialless formation of a PC mirror, the absence of saturation or the presence of nonconjugate component in the reflected pulse, the solutions of the direct $[I_3(z, t)]$ and inverse $[R(z, t)]$ problems should coincide with an accuracy to the normalisation factor (hatched region in Fig. 1). The violation of these conditions complicates the physical interpretation of the effective reflectance.

4. Determination of $R(z, t)$ upon parametric interaction of magnetostatic waves in the case of inertialless nonlinearity

After the unique solution of the inverse model problem (see section 3), we determined the spatiotemporal distribution of the effective reflectance for real experiments on the parametric interaction of magnetostatic waves with nonstationary local pumping performed in [3]. Figure 2 presents the calculated distribution of the effective reflectance $R(z, t)$ in the section $z = \text{const}$ (solid curve), which well agrees with the experimental pump-pulse intensities I_3 taken from [3] (points). Such an agreement between the time dependences of these physical quantities means that the nonlinear parametric interaction of the waves is inertialless (within the pump pulse duration) and saturation is absent. If we tend the characteristic action time τ_R of the effective reflectance $R(z, t)$ to zero, the phase-conjugation

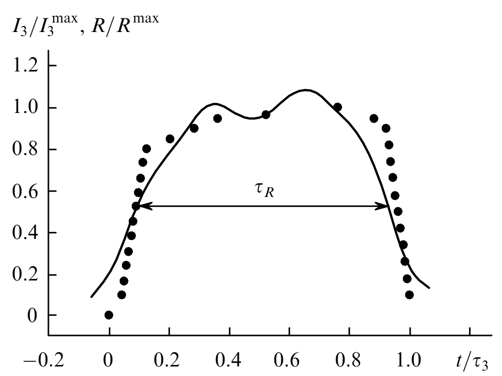

Figure 2. Distribution of the effective reflectance $R(z, t)$ in the section $z = \text{const}$ (solid curve) calculated from the experimental oscillograms of pulses presented in [3], and experimental pump-pulse intensities I_3 (points) from [3].

quality (reproduction of the time-inverted signal I_1 and reflected I_2 pulses) will be perfect. These processes can be easily analysed with the help of the distribution $R(z, t)$. The uniform distribution $R(z, t)$ along the z axis also corresponds to the uniform distribution of the pump intensity I_3 along the entire interaction region observed in [3].

5. Conclusions

We have considered an important parameter of a PC mirror – the effective reflectance, which characterises quantitatively and qualitatively the properties of PC mirrors. The use of this parameter has been demonstrated by analysing the results of the study of a PC mirror upon the parametric interaction of magnetostatic waves [3]. The effective reflectance should be taken into account upon the formation of pulses reflected from the PC mirror with the required temporal shape different from the incident pulse shape, which occurs, for example, during the compression of pulses.

Such inverse problems should be solved by giving special attention to the stability of obtained solutions and to its physical interpretation taking into account the nonstationary nature and quality of phase conjugation.

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