Quantum Electronics **36** (1) 76–78 (2006)

LASER MODES AND BEAMS

PACS numbers: 42.15.Dp; 42.30.Va
DOI: 10.1070/QE2006v036n01ABEH003572

Tunable astigmatic π/2 converter of laser modes with a fixed distance between input and output planes

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***Abstract*. The scheme of a tunable astigmatic π/2 mode converter is proposed in which the distance between input and output planes is fixed. The converter is tuned only by rotating the cylindrical components of optical quadrupoles used in the converter around its optical axis. The Gouy phase difference in the orthogonal planes of the astigmatic π/2 converter required for mode conversion was achieved for the first time by using the scaled fractional Fourier transforms of the appropriate orders.**

Keywords: *astigmatic π/2 mode converter, optical quadrupole, tuning of parameters, scaled fractional Fourier transform.*

1. Introduction

Phase-singularity beams (PSBs), of which the Laguerre–Gaussian (LG) modes with the wave front in the form of the Archimedean spiral are typical representatives, can be most simply produced with the help of a cylindrical lens [1] or more complicated astigmatic π/2 converters [2, 3]. The latter are called so [2] because the Hermite–Gaussian (HG) eigenmodes $u_{mn}^{\rm HG}$ and $u_{nm}^{\rm HG}$ acquire the phase difference $(m-n)\pi/2$ at the output of such converters. Unlike other methods that use synthesised computer holograms [4] or special phase screens [5], when a set of LG modes with different radial (p) or angular (l) indices is obtained, in the case of astigmatic π/2 converters, only one type of the LG mode $u_{0l}^{\rm LG}$ ($l=n$) corresponds to a certain type of the HG beam $u_{0n}^{\rm HG}$ specified at the converter input. In addition, methods based on the use of holograms or phase elements are highly sensitive to the displacement of the input beam with respect to the optical axis of elements, whereas such displacements has no effect on the mode conversion quality in the case of astigmatic π/2 converters [6]. A disadvantage of the latter method for producing PSBs is the necessity of accurate matching of the input beam radius w, or more exactly of its Rayleigh length $z_{\rm R}=\pi w^2/\lambda$ with the parameters of optical elements of the converter. The scheme of a tunable π/2 converter described in [7] is based on the use of the so-called optical quadrupole (OQ) [8] and allows the conversion of beams with the Rayleigh length changing twice without the replacement of optical elements. In this case, however, the variation in the distance between components of the converter should be matched, which is very inconvenient in practice.

The main component of the astigmatic scheme for producing focused LG beams upon the π/2 conversion of HG modes [9] is an optical quadrupole as well. In this paper, it will be shown how to transform the scheme [9] to the π/2 converter which can be tuned only by rotating the cylindrical components of the optical quadrupole around the optical axis of the converter. In this case, the distance between the reference planes of the converter remains fixed irrespective of the Rayleigh length of the input beam.

2. Tunable optical quadrupole

A possible scheme of a tunable OQ consisting of two fixed cylindrical lenses with optical powers of equal moduli but opposite signs and a pair of similar lenses rotating around the optical axis of the OQ in the opposite directions was considered in [7]. An OQ can be also formed by only two cylindrical lenses, as shown in Fig. 1. The *ABCD* matrices of this device in the *xz* and *yz* planes have the form

$$T_{{\rm oq}xz,yz}=\begin{pmatrix}1 & 0\\ \mp C_2/F_0 & 1\end{pmatrix}, \quad (1)$$

where $C_2=\cos 2\varphi$; φ is the rotation angle of lenses in the xy plane; F_0 is the modulus of the focal distance of the positive and negative cylindrical lenses. Hereafter, the minus and plus signs refer to the xz and yz planes,

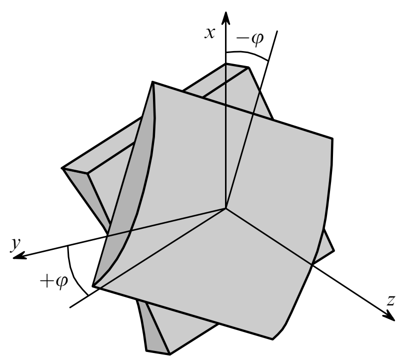

Figure 1. Optical quadrupole consisting of positive and negative cylindrical lenses with the focal distances F_0 with the same modulus.

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Received 1 February 2005; revision received 29 November 2005
Kvantovaya Elektronika **36** (1) 76–78 (2006)
Translated by M.N. Sapozhnikov

respectively. It follows from (1) that the effective optical power of the OQ changes from $\pm 1/F$ to zero when φ is changed from 0 to 45°.

3. Tunable $\pi/2$ mode converter with a fixed distance between the reference planes

The propagation of radiation to the focal plane of a spherical lens (with the focal distance f) placed directly behind an OQ is described in the coordinate planes xz and yz by the transformation matrices

$$T_{f\,xz,yz} = \begin{pmatrix} \mp C_2 f/F_0 & f \\ -1/f \mp C_2/F_0 & 1 \end{pmatrix}. \tag{2}$$

A beam with the plane wave front and the Rayleigh length $z_R = F_0/C_2$ incident on the OQ is transformed in the focal plane of the spherical lens to a beam whose radius* and curvature are described by the expressions

$$w_f = f\left(\frac{2C_2\lambda}{\pi F_0}\right)^{1/2}, \tag{3}$$

$$\rho_{xz,yz} = \frac{1}{f} \pm \frac{F_0}{2f^2 C_2}. \tag{4}$$

It follows from (4) that the wave front in the focal plane of the lens with the focal distance f has two components – spherical and saddle, whose values are determined by the relation between the Rayleigh length of the input beam and the chosen focal distance f of the lens. Both components of the curvature can be compensated by mounting the second lens with the focal distance f and adding another OQ. In this way we obtain a Fourier transformer (FT) placed between two OQs (Fig. 2). In this case, cylindrical lenses of the same type (with the focal distance $\pm F_0$) can be used in OQ1 and OQ2. Only the rotation angles of cylindrical components of the OQs with respect to the optical axis of the device will be different, as shown below. A similar scheme, but with identical OQs and an FT of different type (which is, of course, insignificant) was considered earlier in [9].

By introducing the notation $\cos 2\varphi_{1,2}/F_0 = 1/F^*_{1,2}$ in matrices (1) for OQ1 and OQ2, we obtain

$$T_{xz,yz} = \begin{pmatrix} \mp f/F_1^* & f \\ f/F_1^* F_2^* - 1/f & \mp f/F_2^* \end{pmatrix} \tag{5}$$

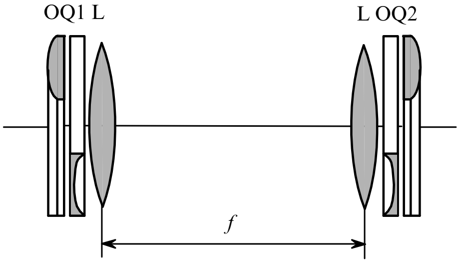

Figure 2. Optical scheme of a tunable π/2 mode converter. Optical quadrupoles OQ1 and OQ2 and spherical lenses L (with the focal distance f) form a Fourier transformer.

* Note that an error is committed in the expression for the beam radius in [8]: the factor 2 is absent in the numerator of the fraction under the root.

for the device in Fig. 2. By assuming that the Rayleigh length of the beam at the OQ1 input is MF_1^* [$w_{in} = (\lambda M F_0/\pi \cos 2\varphi_1)^{1/2}$] and the wave front is plane, we obtain

$$w_{out} = f\left[\frac{\lambda(M^2+1)}{\pi M F_1^*}\right]^{1/2}, \tag{6}$$

$$\rho_{out\,xz,yz} = \mp\frac{1}{F_2^*} \pm \frac{M^2 F_1^*}{f^2(M^2+1)} \tag{7}$$

for the beam at the OQ2 output. Here, M is the magnification factor. It follows from (7) that the wave front of the beam at the OQ2 output will be also plane if

$$(M^2+1)f^2 = M^2 F_1^* F_2^* \tag{8}$$

or

$$\cos 2\varphi_1 \cos 2\varphi_2 = \frac{M^2 F_0^2}{f^2(M^2+1)}. \tag{9}$$

And because for the accumulated Gouy phase** of a beam of radius w_{in} we have

$$\tan\psi_{xz,yz} = \mp\frac{1}{M}, \tag{10}$$

the optical system presented in Fig. 2 will be a π/2 converter (i.e., $\Delta\psi = \psi_{xz} - \psi_{yz} = \pi/2$) only when $M \equiv 1$. In this case, the orders of the Fourier transform in the xz and yz planes are 1.5 and 0.5, respectively. Figure 3a shows the dependences of φ_2 on φ_1 for $M = 1$ and different ratios F_0/f.

According to (8), the dependence of the radius w_{out} of the output beam of the converter normalised to $w_0 = (\lambda F_0/\pi)^{1/2}$ on the normalised input beam radius w_{in} is described by hyperbola for each value of the parameter F_0/f (Fig. 3b). For $F_0/f = \sqrt{2} = 1.414$, the plot is represented by the only point (1, 1) corresponding to the rotation of cylindrical lenses in both OQs through the angle $\varphi_1 = \varphi_2 = 0$. The restrictions to the parameter F_0/f are caused by the necessity of fulfilment of relations (8) and (9), which determines the range of variation of φ_2 and φ_1. Taking this into account, for example for $F_0/f = 1$, the entire range of variation in the input beam size is from 1 to $\sqrt{2}$, and for $F_0/f = 0.5$, the beam size can change from 1 to $\sim 2\sqrt{2}$.

4. Peculiarity of Fourier transforms used in a tunable $\pi/2$ mode converter with a fixed distance between reference planes

Note that matrices (5) differ from the *ABCD* matrices for integer or fractional Fourier transforms used earlier [2, 3, 7, 9, 10] in π/2 converters. In the general form, matrices (5) are matrices of the scaled fractional Fourier transformation (SFFT) [11]

$$T_{SFrFT} = \begin{pmatrix} k\cos\theta & F\sin\theta \\ -\sin\theta/F & \cos\theta/k \end{pmatrix}, \tag{11}$$

** The value of the accumulated Gouy phase at the output of the optical system described by the *ABCD* matrix for an arbitrary Gaussian beam of radius w at the system input and the wave-front curvature ρ is determined by the relation $\tan\psi' = B\lambda/(A + B\rho)\pi w^2$.

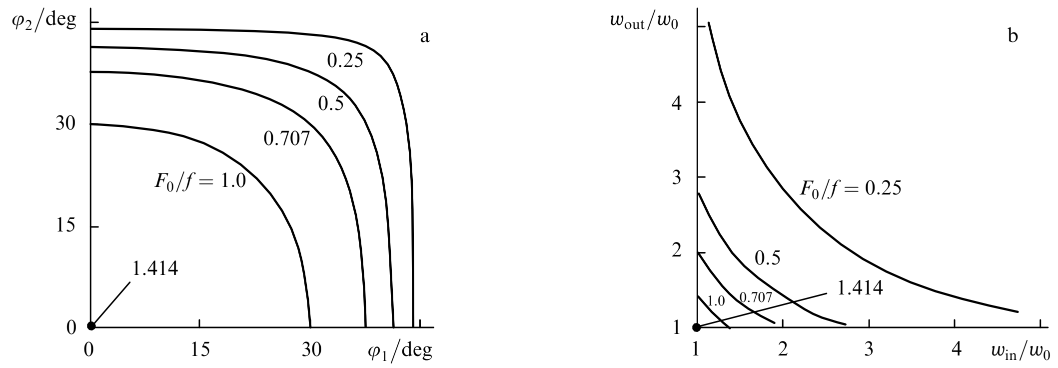

Figure 3. Dependences of the rotation angle φ_2 of cylindrical lenses of the optical quadrupole OQ2 on the rotation angle φ_1 of lenses of the quadrupole OQ1 (a) and the ratio of the normalised radii of the input and output beams of the π/2 converter (b) for different ratios F_0/f.

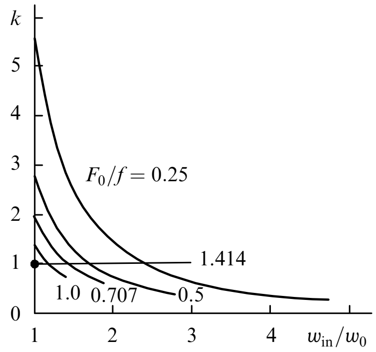

Figure 4. Dependences of the scaling factor of fractional Fourier transforms on the normalised input beam radius in the π/2 converter for different ratios F_0/f.

in which $A/D = k^2 \neq 1$ (k is the scaling factor). In this case, the wave-front curvature of the eigenbeam is nonzero:

$$\rho_{eig} = \frac{C(1-k^2)}{2kFS} \qquad (12)$$

($C = \cos\theta,\ S = \sin\theta$). The radius of the eigenbeam is

$$w_{eig} = \left(\frac{\lambda F}{\pi}\right)^{1/2} \left\{ \frac{2kS}{\left[4k^2 - C^2(k^2+1)^2\right]^{1/2}} \right\}^{1/2}. \qquad (13)$$

Taking (5) into account, expressions (12) and (13) for the π/2 converter for $M \equiv 1$ are transformed to

$$\rho_{eig} = \mp\frac{1}{2}\left(\frac{1}{F_2^*} - \frac{1}{F_1^*}\right),$$

$$w_{eig} = \left(\frac{\lambda F}{\pi}\right)^{1/2} \left\{ \frac{2F_1^* F_2^*}{\left[4(F_1^* F_2^*)^2 - f^2(F_1^* + F_2^*)^2\right]^{1/2}} \right\}^{1/2}, \qquad (14)$$

i.e. for $F_1^* \neq F_2^*$, the eigenbeam of the SFFT converter has the saddle wave front and, as can be easily verified, $\Delta\psi \neq \pi/2$ for this converter. A comparison of (5) and (11) shows that $k^2 = F_2^*/F_1^* = \cos 2\varphi_1/\cos 2\varphi_2$ and $F = f\sqrt{2}$. Dependences of the scaling factor k on the input beam radius for different ratios F_0/f are presented in Fig. 4. Note that for each ratio F_0/f, there exists the ratio w_{in}/w_0 for which $k = 1$. These are the cases when $F_1^* = F_2^*$ ($\varphi_1 = \varphi_2$), matrices (5) describe the fractional Fourier transform, and the output-beam parameters (6) and (7) coincide with the eigenbeam parameters (14). In addition, the relations $w_{out}/w_{in} = k$ and $w_{out}w_{in} = w_F^2$ [where $w_F = (\lambda F/\pi)^{1/2} = (\lambda f\sqrt{2}/\pi)^{1/2}$] are satisfied at the converter input and output.

5. Conclusions

The scheme of a tunable astigmatic π/2 mode converter with a fixed distance between the input and output reference planes in which beams have plane wave fronts has been considered. As a whole this converter is a Fourier transformer located between two optical quadrupoles. The converter can be tuned to the required parameters of the input beam only by rotating the cylindrical components of optical quadrupoles around the optical axis of the converter without moving any elements along the optical axis. The restrictions imposed on the admissible values of parameters of FT lenses, optical quadrupoles, and the rotation angles of their components are presented.

It is shown that, unlike astigmatic π/2 converters considered earlier, the Gouy phase difference in orthogonal planes required for mode conversion is achieved in the converter proposed in the paper due to the use of a special type of the Fourier transform – scaled fractional Fourier transformations of the appropriate orders.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant No 05-02-16818).

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