OPTICAL FIELD PARAMETERS

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Estimate of the minimal coherence length of probe optical radiation in interferometry

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***Abstract*. The minimal coherence length of probe optical radiation sufficient for formation of a homogeneous interference structure is estimated. The estimate is based on the analysis of the interference structure in the intensity distribution of the field scattered by rough surfaces and point objects and also formed in interferometers. Analysis was performed for the field intensity detected for the time $T > 10\tau_c$ (under the condition that the coherence time of the probe radiation is $\tau_c > 3/\omega_0$, where ω_0 is the central frequency of the emission spectrum). It is shown that the minimal coherence length L_c of the probe radiation, at which the homogeneous stratified interference structure of the scattered field can be still formed, is 8λ (λ is the central wavelength). The possibility of using this result for determining the maximal information content of the method of low-coherence optical tomography is analysed.**

Keywords: *optical coherence, speckle contrast in scattered field, visibility and contrast of interference fringes.*

1. Introduction

Phenomena related to optical coherence have been long investigated beginning from paper [1], where they have been explained for the first time by the interference of light. In particular, upon scattering of light by rough surfaces, these effects are observed in the form of random speckles appearing due to the interference of light beams scattered by different sites of the surfaces [2]. Although such phenomena are assumed investigated in detail [2 – 6], nevertheless a number of problems that have not been discussed earlier were considered in [7, 8]. First of all, it is interrelations between chromatic characteristics of probe optical radiation, coherent properties of light fields scattered by rough surfaces being probed, which are manifested in the formation of a speckle structure of these fields, and geometrical characteristics of scattering surfaces. The coherence length of the probe radiation $L_c = c/\Delta\omega$ (where c is the speed of light and $\Delta\omega$ is the width of the probe radiation spectrum) plays a main role in these interrelations. In [7, 8], the coherence of probe radiation was related for the first time to the conditions of formation of homogeneous interference stratified structures by the scattered field, i.e. structures with distinct and approximately identical extrema observed in the intensity distribution. It is these structures (for example, speckles in the case of rough surfaces) that are used, as a rule, in classical, holographic, and speckle interferometry [9].

It was shown that the scattered-field intensity distribution $\bar{I}(\boldsymbol{\rho})$ [$\boldsymbol{\rho}$ is the radius vector in the receiving aperture plane (Fig. 1)] averaged over time $T > 10\tau_c$ (τ_c is the coherence time of the probe radiation) is statistically homogeneous if the width of the probe radiation spectrum is $\Delta\omega = 1/\tau_c \leqslant 0.125\omega_0\pi^{-1}M^{-1/2}$, i.e. when the coherence length is $L_c \geqslant 4\lambda M^{1/2}$. Here, $M = (d_{\rho h}d)^2/(2\lambda r_c)^2$ is the number of speckles in the scattered field on the receiving aperture within the homogeneity region; $d_{\rho h}$ is the size of this region; ω_0 is the central frequency of the probe radiation spectrum; λ is the corresponding wavelength; d is the transverse size of the backscattering region; and r_c is the distance between the receiving aperture and surface. Such a probe radiation was defined as a narrowband radiation. Upon scattering by rough surfaces, this radiation produces a speckle pattern on the receiving aperture, which has almost invariable contrast in the homogeneity region.

It is obvious that, while the number of extrema in the homogeneous region is small, the interference structure of the scattered field outside this region is strongly inhomogeneous, which should affect the accuracy of interferometric

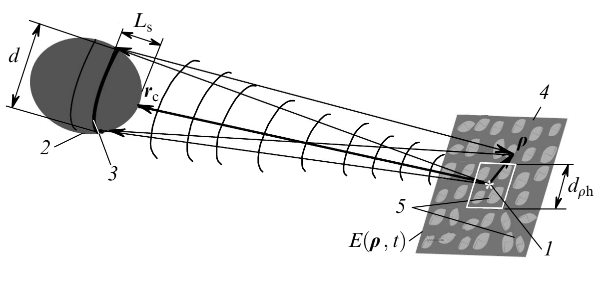

Figure 1. Scattering of light by a rough surface: (*1*) point probe radiation source; (*2*) rough scattering surface; (*3*) backscattering region boundary; (*4*) receiving aperture; (*5*) speckle pattern; $E(\boldsymbol{\rho}, t)$ is the field scattered by an object; L_s is the backscattering region depth. Upon illumination of a rough surface by probe radiation with the minimal coherence length $L_{c\,min} = 8\lambda$, a region (separated by a white contour) is located at the centre of the receiving aperture, which contains four speckles where the scattered field is homogeneous.

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measurements. Because of this, the question about the minimal possible coherence length of probe optical radiation at which the homogeneous structure of the scattered field can still be formed remained open. The aim of our paper is to determine this length and to analyse the possibility of applying this result in optical low-coherence tomography with a Michelson interferometer [10].

2. Minimal possible coherence length of probe optical radiation scattered by rough surfaces

According to [6], we recall first that the intensity distribution $\bar{I}(\boldsymbol{\rho})$ of the field scattered by a rough object is a random process whose contrast $C(\boldsymbol{\rho}) = [\langle \bar{I}^2(\boldsymbol{\rho})\rangle_r - \langle \bar{I}(\boldsymbol{\rho})\rangle_r^2]\times\langle \bar{I}(\boldsymbol{\rho})\rangle_r^{-2}$ can be conveniently analysed by measuring the spatial contrast

$$C_s(\boldsymbol{\rho}) = \frac{\langle \bar{I}^2(\boldsymbol{\rho})\rangle_s - \langle \bar{I}(\boldsymbol{\rho})\rangle_s^2}{\langle \bar{I}(\boldsymbol{\rho})\rangle_s^2} \quad (1)$$

introduced in [7, 8], where $\langle P(\boldsymbol{\rho})\rangle_s = (1/d_\rho)^2 \int P(\boldsymbol{\rho})d\boldsymbol{\rho}$, d_ρ is the receiving aperture size, and angle brackets $\langle\rangle_r$ denote averaging over different realisations of the heights of roughnesses of the scattering surface. It is assumed that the surface is probed by a point source emitting the signal $E_0U(t)\exp(i\omega_0 t)$, where E_0 is the amplitude of the source field and $U(t)$ is relatively slowly varying nonperiodical modulation function characterised by the coherence time $\tau_c = 1/\Delta\omega \geqslant 3/\omega_0$. For a determinate process $U(t)$, the parameter τ_c is the probe-pulse duration, and for a random process – the time interval of its correlation. For $M \geqslant 400$, the condition $C(\boldsymbol{\rho}) \approx C_s(\boldsymbol{\rho})$ is fulfilled [8]. For $M < 400$, the contrast $C(\boldsymbol{\rho})$ can be determined by measuring different functions $C_{sj}(\boldsymbol{\rho})$ by several different statistically independent realisations $\bar{I}_j(\boldsymbol{\rho})$ as the arithmetical mean of these functions:

$$C(\boldsymbol{\rho}) \approx C_s(\boldsymbol{\rho}) = \frac{1}{N_r}\sum_{j=1}^{j=N_r} C_{sj}(\boldsymbol{\rho}),$$

where N_r is the number of realisations.

Let us select some, for example, central region of size $d_{\rho h}$ on the receiving aperture. It follows from [8] that, under the condition $L_c \geqslant 4\lambda M^{1/2}$, the contrast $C(\boldsymbol{\rho})$ of the scattered field within this region is virtually invariable. This means that the scattered-field intensity distribution in this region is homogeneous and, hence, the intensity distribution in speckles has on average approximately identical maxima and minima, which demonstrates the coherence of the probe radiation. The minimal number M_{min} of speckles in the field scattered by the rough surface at which the averaged intensity $\bar{I}(\boldsymbol{\rho})$ has the stratified spatial structure, which allows one to determine reliably whether or not the distribution $\bar{I}(\boldsymbol{\rho})$ is homogeneous, is equal to four ($M_{min} = 4$). Therefore, the minimal coherence length $L_{c\,min}$ of the probe radiation, at which a region with the homogeneous distribution $\bar{I}(\boldsymbol{\rho})$ can be still formed, is equal to $4\lambda M_{min}^{1/2} = 8\lambda$. This means that the size of this region is $d_{\rho h} \approx 4\rho_c = 4(\lambda r_c)d$, where $\rho_c = (\lambda r_c)/d$ is the correlation radius of speckles in the scattered field [6]. This region is separated by a white contour in Fig. 1. The homogeneous distribution $\bar{I}(\boldsymbol{\rho})$ within this region is formed for $L_c \geqslant 4\lambda M_{min}^{1/2} = 8\lambda$. Therefore, $L_{c\,min} = 8\lambda$ is the minimal coherence length at which a region with the stratified interference structure with the homogeneous distribution $\bar{I}(\boldsymbol{\rho})$ can be still formed. The distribution $\bar{I}(\boldsymbol{\rho})$ outside this region is inhomogeneous, so that it is reasonable to consider radiation probing of a rough surface coherent if $L_c \geqslant L_{c\,min} = 8\lambda$ and incoherent if $L_c < L_{c\,min} = 8\lambda$.

We determined above the minimal possible coherence length of probe radiation scattered by rough surfaces. It is also interesting to determine this length in the case of scattering of probe radiation by other objects. The simplest of them is a two-point object B_1B_2* (Fig. 2). We will show below that the structure of the field scattered by this object is similar to that of the field scattered by rough surfaces and the minimal possible coherence length of probe radiation in this case is also equal to 8λ.

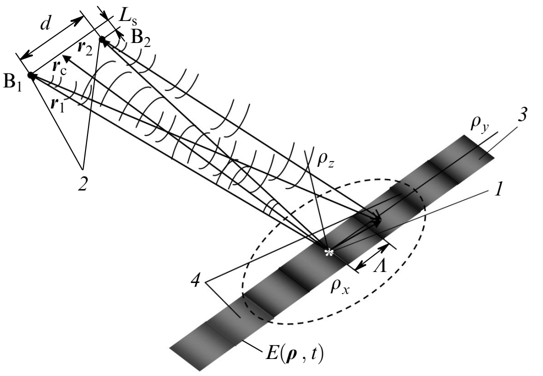

Figure 2. Scattering of light by a two-point object B_1B_2: (*1*) point probe radiation source; (*2*) two-point scattering object; (*3*) receiving aperture; (*4*) interference fringes. Upon illumination of the two-point object by probe radiation with the minimal coherence length $L_{cm} = 8\lambda$, four virtually identical interference maxima are formed at the centre of the receiving aperture (shown by a dotted oval).

3. Minimal possible coherence length of probe radiation scattered by a two-point object

Let us analyse the structure of the field scattered by a two-point object B_1B_2, which is illuminated by a point source emitting a signal in the form $E_0U(t)\exp(i\omega_0 t)$ (Fig. 2). The field scattered by the object has the form

$$E(\boldsymbol{\rho}, t) = E_1(\boldsymbol{\rho}, t) + E_2(\boldsymbol{\rho}, t),$$

where

$$E_j(\boldsymbol{\rho}, t) = \frac{-ik_jS_jE_0}{\lambda r_c}\, U\left(t - \frac{\alpha_j}{c}\right)\exp\left[i\omega_0\left(t - \frac{\alpha_j}{c}\right)\right]$$

in the Fresnel approximation; $\alpha_j = 2|\boldsymbol{r}_j| + \boldsymbol{\rho}\cdot\boldsymbol{r}_j/r_c$; k_1 and k_2 are the amplitude reflection coefficients of point objects B_1 and B_2; S_1 and S_2 are the areas of their effective scattering surfaces; $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the radius vectors of point objects B_1 and B_2. The intensity distribution on the receiving aperture averaged over time $T > 10\tau_c$ has the form of interference fringes

* Here, we call a two-point object B_1B_2 the object consisting of two small (point) scattering objects B_1 and B_2 for which the backscattering intensity is virtually constant within the receiving aperture.

$$\bar{I}(\boldsymbol{\rho}) = \langle I(\boldsymbol{\rho}, t)\rangle_t = \frac{1}{T}\int_{t_0}^{t_0+T} I(\boldsymbol{\rho}, t)\mathrm{d}t$$

$$\sim 1 + \frac{K_1K_2}{K_1^2+K_2^2}|G(\alpha_1,\alpha_2)|\cos\left[\varphi + \arg G + \frac{2\pi\rho_y}{\Lambda}\right], \quad (2)$$

where

$$I(\boldsymbol{\rho}, t) = |E(\boldsymbol{\rho}, t)|^2;$$

$$G(\alpha_1,\alpha_2) = \frac{1}{T}\int_{t_0}^{t_0+T} U\left(t - \frac{\alpha_1}{c}\right)U^*\left(t-\frac{\alpha_2}{c}\right)\mathrm{d}t;$$

$$K_j = \frac{k_jS_jE_0}{\lambda r_c}; \quad \varphi = \frac{4\pi(r_1 - r_2)}{\lambda};$$

and $\Lambda = r_c\lambda/d$ is the period of interference fringes. It is easy to show that

$$G(\alpha_1,\alpha_2) = \gamma_{12}(\alpha_1,\alpha_2) = \frac{E_1E_2^*}{\bar{I}},$$

where γ_{12} is the complex degree of the mutual coherence of the fields E_1 and E_2 [4, 5]. By assuming that for $T > 10\tau_c$, the function

$$G(\alpha_1,\alpha_2) = G(w),$$

where

$$w(\rho_y) = -\frac{(\alpha_1-\alpha_2)^2}{L_c^2} = -\left[\frac{2(r_1-r_2)+d\rho_y/r_c}{L_c}\right]^2,$$

$L_c = c\tau_c$ is the coherence length of the radiation of a point source [8] and taking into account relation (2), we obtain that the visibility of the interference fringes is

$$V[w(\rho_y)] = \frac{\bar{I}_{\max}(\rho_y) - \bar{I}_{\min}(\rho_y)}{\bar{I}_{\max}(\rho_y) + \bar{I}_{\min}(\rho_y)} = \frac{2K_1K_2}{K_1^2+K_2^2}G(w), \quad (3)$$

where $\bar{I}_{\max}(\rho_y)$ and $\bar{I}_{\min}(\rho_y)$ are the maximal and minimal values of $\bar{I}(\rho_y)$. If, for example, $G(w) = \exp(-w^2)$, we have $V(w) = [2K_1K_2/(K_1^2+K_2^2)]\exp(-w^2)$. Below, without loss of generality, we will assume that $K_1 = K_2$. Then, $V(w) = \exp(-w^2)$.

Let us return to relation (1) which allows a unified description of the interference fringes formed upon scattering by two-point objects and of speckles formed upon scattering from rough surfaces. Taking relations (1) and (3) into account, it is easy to show that the spatial contrast of interference fringes for the distribution $\bar{I}(\boldsymbol{\rho})$ is $C_s(\rho_y) = V^2(\rho_y)$. It achieves maximum values at the centre of the receiving aperture ($\rho_y = 0$). For example, for $L_c > 20L_s$, the contrast $C_s(\rho_y) \approx \exp\{-2[d\rho_y/(L_cr_c)]^2\}$ and the maximum value is $C_s \approx 1$. In the interval $|\rho_y| \leqslant d_{\rho h}/2$, where $d_{\rho h} = M_f/\Lambda = r_cL_c(4d)^{-1}$ and $M_f = d_{\rho h}/\Lambda$, the contrast $C_s \approx \exp[-8(L_s/L_c)^2]$ and is virtually constant (here, M_f is the number of interference fringes in the interval $|\rho_y| \leqslant d_{\rho h}/2$). This means that, under the condition $|\rho_y| \leqslant d_{\rho h}/2$, the interference pattern is homogeneous and stratified. In particular, for $L_c \geqslant 20L_s$, we have $C_s = 1$ and $e^{-0.12}$ at the centre of the receiving aperture and at its edge, respectively. In this case, the structure of interference fringes is similar to speckles of the field scattered by a rough surface under the condition that L_c considerably exceeds the depth L_s of the backscattering region of the surface [8]. The condition $|\rho_y| \leqslant d_{\rho h}/2$ also means that the maxima and minima of the distribution $\bar{I}(\rho_y)$ within the region of size $d_{\rho h}$ are approximately equal.

The condition $|\rho_y| \leqslant d_{\rho h}/2$ can be rewritten in the form $\Delta\omega \leqslant 0.5\omega_0M_f^{-1}$ and $L_c \geqslant 2M_f\lambda$. The inequality $\Delta\omega \leqslant 0.5\omega_0M_f^{-1}$ resembles the above-mentioned condition $\Delta\omega \leqslant 0.125\omega_0\pi^{-1}M^{-1/2}$ of the narrowness of the spectral band of radiation probing a rough surface. For $L_c < 2M_f\lambda$, the contrast C_s and the maxima and minima of $\bar{I}(\rho_y)$ begin to depend noticeably on ρ_y and rapidly decrease with increasing ρ_y as the periphery of the receiving aperture is approached. Therefore, for $|\rho_y| > d_{\rho h}$, the interference pattern near the periphery of the receiving aperture becomes strongly inhomogeneous.

As the coherence length L_c decreases, the region within which the distribution $\bar{I}(\rho_y)$ is homogeneous, narrows down to the centre of the receiving aperture. The minimal possible coherence length $L_{c\min}$ and the size of this region $d_{\rho h}$ (shown by a dotted oval in Fig. 2) are determined by the minimal number of interference fringes at which the distribution $\bar{I}(\boldsymbol{\rho})$ has the stratified structure from which one still can judge whether or not this structure is homogeneous. This number M_f is equal to four (see Fig. 2), which means that $L_{c\min} = 8\lambda$ and $d_{\rho h} = 4\Lambda$. For any coherence length of the probe radiation no less than L_{cm}, the region of size $d_{\rho h} = 4\Lambda$, within which the distribution $\bar{I}(\boldsymbol{\rho})$ is homogeneous, is always formed at the receiving aperture centre.

It is interesting to note that, although the contrast C_s of interference fringes at the centre of this region achieves its maximum value, this value for $L_s \gg L_c$ can be very small. In this case, the scattered field pattern does not differ from that formed in natural light even at rather high coherence, when, for example, $L_c = 10$ cm and $L_s = 50$ cm. A similar scattered field pattern is also observed in the case of a rough scattering surface when the depth of the backscattering region is $L_s \gg L_c$ [8], the only difference being that the contrast of speckles of this field decreases with decreasing the ratio L_c/L_s much slower than that of the fringes.

4. Minimal possible coherence length of probe radiation in interferometers

Interferometers are widely used for precision measurements of the parameters of various objects. The measurements are performed by analysing the distribution $\bar{I}(\boldsymbol{\rho})$ in interference fringes formed due to the overlap of the waves propagating in the object and reference arms of the interferometer [9]. Let us assume that the minimal number of fringes in the interference pattern, at which these fringes have a stratified structure allowing us to judge whether or not this pattern is homogeneous, is equal to four. Then, as follows from analysis (see section 3), the minimal possible coherence length $L_{c\min}$ of the probe radiation used in the interferometer is also equal to 8λ.

By knowing this length, we can estimate, for example, the maximum information content of the method of optical low-coherence tomography [10]. The information content is determined by the maximum number of depth-resolved elements of a bulk medium in the distribution of optical microscopic inhomogeneities in the medium obtained by this method. The method is based on the focusing of probe radiation to the medium under study and analysis of interference fringes in the field formed due to summation of backscattered waves propagating in the object arm of a Michelson interferometer and the reference wave propagating in its reference arm.

Because the longitudinal size of the focusing region $d_{\rm len} = f^2\lambda/d_{\rm f}^2$, where $d_{\rm f}$ is the size of the focusing system aperture, f its focal distance, and the resolution over depth is equal to the coherence length $L_{\rm c\,min}$ of probe radiation [10], the maximum number of the depth-resolved elements of the medium falling within the focal region is $N_{\rm max} = d_{\rm len}/L_{\rm c\,min} = f^2/(8d_{\rm f}^2)$. For example, if $d_{\rm f}/f = 0.125$, then $N_{\rm max} = 8$.

5. Conclusions

Analysis of the time-averaged intensity distribution $\bar{I}(\boldsymbol{\rho})$ in the fields formed by the mixing of waves coming from the reference and object interferometer arms and in the fields scattered by rough surfaces and two-point objects leads to the following conclusion. Depending on the coherence length $L_{\rm c}$ of probe radiation, these fields behave similarly and can form homogeneous interference stratified structures when $L_{\rm c}$ is no less than 8λ. Therefore, the minimal possible coherence length $L_{\rm c\,min}$ of probe radiation used in interferometry beginning from which its coherence is manifested, i.e. the ability to form such structures, is 8λ.

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