Quantum Electronics **37** (4) 358 – 362 (2007)

PACS numbers: 42.70.Qs; 42.50.Gy
DOI: 10.1070/QE2007v037n04ABEH013216

Generation of difference terahertz-frequency signals in a system of compound one-dimensional photonic crystals

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***Abstract*. The possibility of amplification of difference terahertz-frequency signals generated in one-dimensional nonlinear crystals is studied theoretically. It is shown that the intensity of nonlinear terahertz signals can be considerably increased both due to the coherent summation of waves generated in several photonic crystals and the choice of a special configuration of photonic crystals representing a two-periodic structure.**

Keywords: *photonic crystals, difference-frequency generation, terahertz radiation.*

During the past decade, interest in the study of linear and nonlinear optical processes in photonic crystals (PCs) has considerably increased [1], which is explained first of all by the existence of photonic band gaps (PBGs) in PCs – the regions of frequencies or angles of incidence inside which the propagation of radiation inside the crystal is forbidden. A strong spatial dispersion near PBGs allows the compensation of the material dispersion of the PC material, providing the fulfilment of phase-matching [2] or quasi-phase-matching [3, 4] conditions during the generation of nonlinear signals. In addition, a strong localisation of radiation inside a PC when the frequency or the angle of incidence of radiation corresponds to the PBG edge permits the nonphase-matching enhancement of nonlinear signals by increasing the intensity of nonlinear radiation sources. The energy of the filed localised in a periodic structure can increase in this case proportionally to the cube of the number of the structure periods [5]. The possibility of nonphase-matching enhancement upon second harmonic generation was theoretically substantiated in [6] and demonstrated experimentally in [7]. In [8], it was proposed to use this amplification for sum-frequency generation in one-dimensional PCs.

The possibility of nonphase-matching enhancement of nonlinear signals makes PCs promising materials for generating intense coherent signals at the difference terahertz frequency. Terahertz radiation is widely used in spectroscopy and medical diagnostics [9]. The phase-matching condition cannot be fulfilled upon generating difference terahertz-frequency signals due to a considerable difference between the refractive indices of materials in the optical and terahertz frequency regions, and for this reason nonphase-matching enhancement is the most efficient mechanism of amplification of these signals. Thus, the possibility of increasing the intensity of nonsynchronously amplified terahertz signals generated in an isolated one-dimensional PC almost by two orders of magnitude compared to a homogeneous medium was demonstrated in [10]. It was shown recently [11] that the intensity of terahertz signals propagating in a one-dimensional superlattice formed by several PCs can be increased by more than an order of magnitude compared to an isolated PC.

In this paper, we consider the possibility of a considerable (more than by three orders of magnitude compared to a homogeneous medium) increase in the intensity of nonlinear signals upon a successive coherent summation of terahertz waves generated and reflected at the optimal angle from a quasi-two-dimensional superlattice consisting of a number of PCs (Fig. 1).

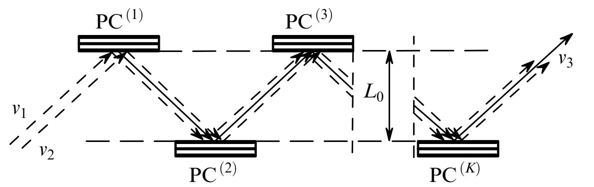

Figure 1. Scheme of the PC superlattice consisting of K crystals and the interaction geometry of radiations.

We will solve the problem of generating terahertz signals in a PC superlattice by the method of radiation-transfer matrices [12] based on the approximations of plane infinite waves and an inexhaustible pump wave. However, taking into account multiple reflections of linear and nonlinear signals from different PCs, this method should be modified. To find forward and backward waves at the fundamental and difference frequencies, it is necessary to solve the nonlinear wave equation

$$\mathrm{rot\,rot}\,\boldsymbol{E}(\boldsymbol{r},t) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{D}(\boldsymbol{r},t) = -\left(\frac{4\pi}{c^2}\right)\frac{\partial^2}{\partial t^2}\boldsymbol{P}_{\mathrm{NL}}(\boldsymbol{r},t), \quad (1)$$

where $\boldsymbol{E}(\boldsymbol{r},t)$ is the electric field vector; $\boldsymbol{D}(\boldsymbol{r},t)$ is the electric induction vector; and $\boldsymbol{P}_{\mathrm{NL}}$ is the nonlinear polarisation vector.

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Received 2 May 2006; revision received 14 October 2006
Kvantovaya Elektronika **37** (4) 358 – 362 (2007)
Translated by M.N. Sapozhnikov

Consider first the linear problem of multiple reflections of radiation from several PCs, each of them consisting of N layers. Each layer has the thickness d_m, the linear refractive index n_m, and the second-order nonlinear susceptibility $\chi_m^{(2)}$, where m is the number of the layer. We assume that $m = 0$ corresponds to a plane–parallel layer of thickness L_0 located between PCs (Fig. 1), and $m = N + 1$ corresponds to a substrate on which a PC is grown. The z axis is directed inside the PC normally to the structure surface. Let a plane monochromatic wave with the frequency ν and amplitude $E_0^{(+)}$ be incident on the PC at the angle θ to the normal to the structure surface. According to the transfer-matrix method, the amplitudes of electric fields of the fundamental waves in the zero and mth layers are related by the expression

$$E_0 = \begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix} = \hat{A}_{01}\hat{A}_{12}\ldots\hat{A}_{m-1\,m}E_m \equiv \hat{M}(0,m)\begin{bmatrix} E_m^{(+)} \\ E_m^{(-)} \end{bmatrix}. \quad (2)$$

The superscripts '+' and '–' refer to the waves propagating forward (transmission) and backward (reflection), respectively. The transfer matrix $\hat{A}_{l\,l+1}$ has the form

$$\hat{A}_{l\,l+1} = \frac{1}{t_{l\,l+1}}\begin{bmatrix} \bar{g}_l & \bar{g}_l r_{l\,l+1} \\ g_l r_{l\,l+1} & g_l \end{bmatrix}, \quad (3)$$

where $g_l = \exp(\mathrm{i}s_l d_l)$; $\bar{g}_l \equiv g_l^{-1} = \exp(-\mathrm{i}s_l d_l)$; $s_l = (2\pi\nu/c)\times(n_l^2 - \sin^2\theta)^{1/2}$ is the z component of the radiation wave vector in the lth layer; $r_{l\,l+1}$ and $t_{l\,l+1}$ are the Fresnel reflection and transmission coefficients, respectively, for the interface of two semi-infinite media with the refractive indices n_l and n_{l+1}.The matrix $\hat{M}(0,m)$ in (2) is the product of matrices $\hat{A}_{l\,l+1}$ for $l = 0,\ldots,m-1$.

Below, all the quantities in the interaction of radiation with different PCs [PC$^{(1)}$, PC$^{(2)}$,... PC$^{(K)}$] are denoted by the corresponding superscripts. The relation between the field amplitudes in the mth layer with the amplitude $E_0^{(+)}$ of the field incident on PC$^{(J)}$ obtained from (2) has the form [12]

$$\big(E_m^{(\pm)}\big)^{(J)} = \big(\Phi^{(\pm)}\big)_m^{(J)}\big(E_0^{(+)}\big)^{(J)}, \quad (4)$$

where

$$\big(\Phi^{(+)}\big)_m^{(J)} = \frac{M_{22}^{(J)} - R^{(J)}M_{12}^{(J)}}{\zeta_m};$$

$$\big(\Phi^{(-)}\big)_m^{(J)} = \frac{R^{(J)}M_{11}^{(J)} - M_{21}^{(J)}}{\zeta_m}; \quad \zeta_m = \det\big[\hat{M}^{(J)}(0,m)\big];$$

$M_{ij}^{(J)}$ are the elements of the matrix $\hat{M}^{(J)}(0,m)$ ($i = 1, 2$, $j =$ 1,2); and $J = 1, 2, \ldots, K$ is PC number. Here, $R^{(J)}$ is the complex reflection coefficient of the PC$^{(J)}$, which can be obtained from (2) for $m = N+1$, taking into account that $(E_{N+1}^{(-)})^{(J)} = 0$ (there is no radiation on the substrate side):

$$R^{(J)} = \frac{M_{21}^{(J)}(0, N+1)}{M_{11}^{(J)}(0, N+1)}. \quad (5)$$

By assuming that the wave is incident on the PC$^{(1)}$ from infinity, we set $g_0^{(1)} = 1$, whereas $g_0^{(J)} = \exp(\mathrm{i}s_0 L_0) \equiv g_0$ ($J = 2, \ldots, K$) for the waves incident on all the successive PCs. The rest of the factors $g_m^{(J)}$ have a similar form for any numbers J and m ($m \neq 0$). Thus, according to (2) and (3), the elements of matrices $\hat{M}^{(1)}(0,m)$ and $\hat{M}^{(J)}(0,m)$ ($J \neq 1$) are related by the expressions $M_{11}^{(J)} = \bar{g}_0 M_{11}^{(1)}$, $M_{12}^{(J)} = \bar{g}_0 M_{12}^{(1)}$, $M_{21}^{(J)} = g_0 M_{21}^{(1)}$, and $M_{22}^{(J)} = g_0 M_{22}^{(1)}$. Taking this into account, expression (5) can be written in the form $R^{(J)} = g_0^2 R^{(1)}$. Because the wave reflected from the PC$^{(J-1)}$ is the incident wave for the PC$^{(J)}$, the relation

$$\big(E_0^{(+)}\big)^{(J)} = \big(E_0^{(-)}\big)^{(J-1)} = \big(E_0^{(+)}\big)^{(J-1)}R^{(J-1)}$$

is fulfilled. Then, taking into account the relation between $R^{(1)}$ and $R^{(J)}$, we obtain from (4) the expression

$$\big(E_m^{(\pm)}\big)^{(J)} = (g_0)^{2J-3}\big(R^{(1)}\big)^{J-1}\big(E_m^{(\pm)}\big)^{(1)}, \quad (6)$$

which takes into account that $g_0 = 1$ for $J = 1$ and $(E_m^{(\pm)})^{(1)} = (\Phi^{(\pm)})^{(1)}E_0^{(+)}$.

As mentioned above, the most efficient amplification mechanism of difference terahertz-frequency signals generated in PCs in the nonphase-matching enhancement, which is determined by the energy of fundamental waves localised inside the PC. The field energy in the mth layer of the PC$^{(J)}$ is determined by the expression $W_m^{(J)} = n_m^2 d_m \times |E_m^{(J)}|^2/2$, where $E_m^{(J)} = (E_m^{(+)})^{(J)}g_m + (E_m^{(-)})^{(J)}\bar{g}_m$ is the total radiation field in the mth layer. Taking (6) into account, we can write the expression

$$E_m^{(J)} = (g_0)^{2J-3}\big(R^{(1)}\big)^{J-1}\big[\big(E_m^{(+)}\big)^{(1)}g_m + \big(E_m^{(-)}\big)^{(1)}\bar{g}_m\big]. \quad (7)$$

Then, by summing energies $W_m^{(J)}$ over all the layers, we obtain the expression relating the total field energy in the PC$^{(J)}$ and PC$^{(1)}$:

$$W^{(J)} = \big|R^{(1)}\big|^{2(J-1)}\sum_{m=1}^{N}\frac{n_m^2 d_m}{2}\big|E_m^{(1)}\big|^2 = \big|R^{(1)}\big|^{2(J-1)}W^{(1)}. \quad (8)$$

Because the frequencies of fundamental waves should lie at the PBG edge to fulfil the nonphase-matching enhancement condition, they will be located near the first (from the PBG centre) transmission resonance (the reflection coefficient minimum). Therefore, $|R^{(1)}|^2 \ll 1$ in (8) when the nonphase-matching enhancement condition is fulfilled, and the total field energy $W^{(J)}$ in each of the structures following after the PC$^{(1)}$ will decrease as a power function. Physically, this occurs because a great part of the fundamental field energy will pass through PCs and only a small part of this energy will be reflected. As a result, the nonphase-matching enhancement efficiency in PCs$^{(J)}$, except the PC$^{(1)}$, will be very low.

To solve this problem, we propose here to use a two-period PC of configuration $(\mathrm{AB})_{N_1}(\mathrm{CD})_{N_2}$, i.e. consisting in fact of two PCs. In this case, the $(\mathrm{AB})_{N_1}$ is the initial PC (containing N_1 AB bilayers, each bilayer consisting of layers A and B) in which the nonphase-matching enhancement takes place (the radiation frequency corresponds to the PBG edge). The thicknesses of layers C and D of the additional PC are chosen so that the PBG centre of this additional structure would correspond to the PBG slope of the main PC, where amplification occurs. In this case, the materials of layers C and D can be the same as those of layers A and B, respectively. Due to such a configuration, the pump wave upon nonphase-matching enhancement will no longer pass completely through the structure because a great part of the wave will be reflected. The field localisation inside the PC will also increase due to increasing coupling between the forward and backward waves, resulting in the additional amplification compared to the initial PC. This is shown in Fig. 2 for the structure with the following parameters: $N_1 = 10$, $d_{1,2} = 3\lambda_0(4n_{1,2})^{-1}$, $n_1 = 2.85$, $n_2 = 1.3$ (the main PC) and variable N_2, $d_{1,2} = 0.2738\lambda_0/n_{1,2}$, $n_1 = 2.85$, $n_2 = 1.3$ (the additional PC), where λ_0 is the reference wavelength and the refractive indices of the layers correspond to ZnTe and DAST. The refractive index of the substrate $n_{\rm sub} = 1$.

Figure 2 shows the dependences of the intensity reflection coefficient $R_0 = |R|^2$ (where R is the amplitude reflection coefficient of the entire structure) and the field energy W on the normalised radiation wavelength for different N_2. The field energy W localised inside the PC is normalised to the maximum energy localised inside the main PC. One can see that, as the number N_2 of bilayers in the additional PC increases, the reflection coefficient R_0 of the entire structure changes considerably, and at the wavelength $\lambda/\lambda_0 = 1.095$ near which the nonphase-matching enhancement condition is fulfilled, reflection with $R_0 >$ 95 % takes place already for $N_2 = 4$ instead of complete transmission. For $N_2 = 3$ or 4, the energy W exceeds the energy for $N_2 = 0$ by more than a factor of four, which considerably enhances the efficiency of the amplification mechanism under study. Thus, the PC configuration proposed above can be efficiently used to generate nonlinear signals upon reflection of not only difference terahertz-frequency signals but also other nonlinear signals, for example, the second harmonic and sum frequency signals.

Consider now briefly the solution of the nonlinear problem of a difference-frequency signal generation by

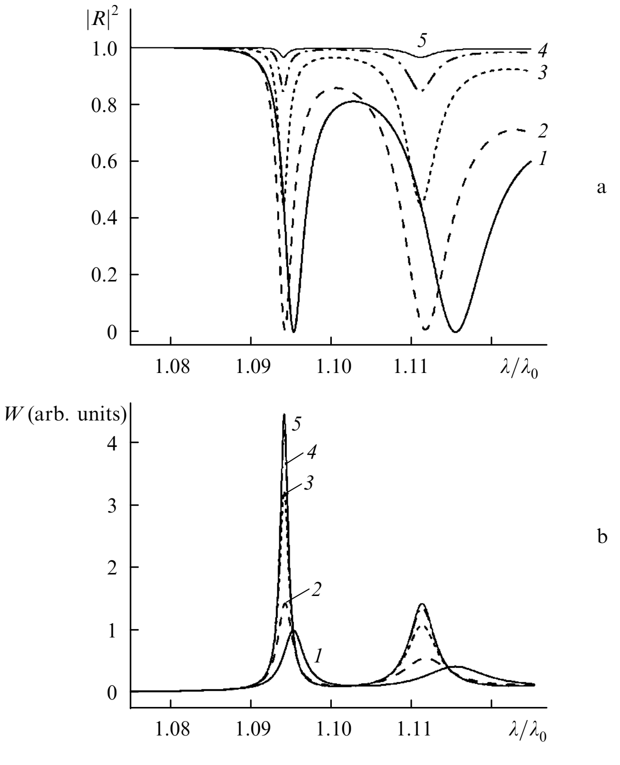

Figure 2. Dependences of the reflection coefficient R_0 (a) and the normalised field energy W localised inside the PC (b) on the normalised wavelength λ/λ_0 for the number of bilayers of the additional PC $N_2 = 0$ (*1*), 1 (*2*), 2 (*3*), 3 (*4*), and 4 (*5*).

the transfer matrix method upon multiple scattering of radiation from PCs forming a superlattice. We will seek a nonlinear wave at the difference frequency in each mth layer of the PC$^{(J)}$ (below, we will omit the superscript J for brevity) in the form of a superposition of the waves corresponding to the homogeneous (superscript u) and inhomogeneous (superscript s) solutions of Eqn (1):

$$E_{3m} = (E_{3m}^{\rm u} + E_{3m}^{\rm s}) \exp[{\rm i}k_{3x}x - 2\pi\nu_3 t)]. \quad (9)$$

Here, the x axis is directed along the PC surface perpendicular to the z axis; the xz plane is the plane of incident radiation; the wave number $k_{3x} = k_3 \sin\theta$ is the projection of the difference-frequency radiation wave vector on the x axis; $k_3 = 2\pi\nu_3/c$; and $\nu_3 = \nu_1 - \nu_2$. Expressions for the fields corresponding to the homogeneous ($E_{3m}^{\rm u}$) and inhomogeneous ($E_{3m}^{\rm s}$) solutions have the form

$$E_{3m}^{\rm u} = E_{3m}^{{\rm u}(+)} \exp({\rm i}s_{3m}z) + E_{3m}^{{\rm u}(-)} \exp(-{\rm i}s_{3m}z), \quad (10a)$$

$$E_{3m}^{\rm s} = E_{3m}^{{\rm s}(+)} \exp[{\rm i}(s_{1m} - s_{2m})z] + E_{3m}^{{\rm s}(-)} \exp[-{\rm i}(s_{1m} - s_{2m})z], \quad (10b)$$

where the coordinate z is measured from the upper boundary of the mth layer; the subscripts 1 and 2 denote quantities corresponding to fundamental waves, and the subscript 3 denotes quantities corresponding to nonlinear waves; and s_{im} are the z components of the radiation waves vectors in the mth layer ($i = 1, 2, 3$). It follows from (1) in the case of generation at the fundamental frequency, i.e. for $P_{\rm NL} = \chi^{(2)} E_1 E_2^*$, that the expression for the radiation amplitudes $E_{3m}^{{\rm s}(\pm)}$ of sources has the form

$$E_{3m}^{{\rm s}(\pm)} = \frac{4\pi\chi_m^{(2)} k_3^2}{(s_{1m} - s_{2m})^2 - s_{3m}^2} E_{1m}^{(\pm)} E_{2m}^{(\pm)*}. \quad (11)$$

The amplitudes $E_{1m}^{(\pm)}$ and $E_{2m}^{(\pm)}$ of the main fields can be found by solving the linear problem [see expression (6)].

To find the field amplitudes of the difference-frequency waves at the PC boundaries, it is necessary to write the continuity conditions for the tangential components of electric fields and their derivatives at each of the $N + 1$ interfaces. However, unlike the transfer matrix method [12], we will not separate some layer as a nonlinear one but will write the boundary conditions for the interface of the mth and $m + 1$th layers taking into account all the homogeneous and inhomogeneous fields in both layers because upon generation of nonlinear signals in the PC superlattice, along with fundamental waves the wave at the difference frequency ν_3 is also incident on the surface of each of the PCs.

Thus, we will write the continuity conditions for the electric fields and their derivatives:

$$\alpha_{1m} g_{3m} E_{3m}^{{\rm u}(+)} + \alpha_{2m} \bar{g}_{3m} E_{3m}^{{\rm u}(-)} + \alpha_{3m} \bar{g}_{1m} g_{2m} E_{3m}^{{\rm s}(+)}$$

$$+ \alpha_{4m} g_{1m} \bar{g}_{2m} E_{3m}^{{\rm s}(-)} = \alpha_{1m+1} E_{3m+1}^{{\rm u}(+)} + \alpha_{2m+1} E_{3m+1}^{{\rm u}(-)}$$

$$+ \alpha_{3m+1} E_{3m+1}^{{\rm s}(+)} + \alpha_{4m+1} E_{3m+1}^{{\rm s}(-)}, \quad (12)$$

where for the σ polarised radiation, $\alpha_{ji} = 1$ ($j = 1, \dots, 4$) for the fields and $\alpha_{1l} = -\alpha_{2l} = s_{3l}$, $\alpha_{3l} = -\alpha_{4l} = s_{1l} - s_{2l}$ for the field derivatives; for the π polarised radiation, $\alpha_{1l} = \alpha_{2l} = s_{3l}/n_{3l}$, $\alpha_{3l} = \alpha_{4l} = (s_{1l} - s_{2l})/n_l^{\rm s}$ for fields and

$\alpha_{1l} = -\alpha_2 = n_{3l}$, $\alpha_3 = -\alpha_4 = n_l^{\rm s}$ for the field derivatives; $l = m$ and $m+1$; $n_l^{\rm s} \equiv [(s_{1l} - s_{2l})^2 + k_{3x}^2]^{1/2}/k_3$; and as before, $g_{im} = \exp(\mathrm{i}s_{im}d_m)$ and $\bar{g}_{im} = \exp(-\mathrm{i}s_{im}d_m)$ ($i = 1, 2, 3$). After algebraic transformations, we obtain for (12) the relation between fields in in the mth and $m+1$th layers:

$$E_{3m}^{\rm u} = \begin{bmatrix} E_{3m}^{\rm u(+)} \\ E_{3m}^{\rm u(-)} \end{bmatrix} = \hat{A}_{m\,m+1}^{(\nu_3)} E_{3m+1}^{\rm u} + B_{m+1} - D_m, \qquad (13)$$

where $\hat{A}_{m\,m+1}^{(\nu_3)}$ is the matrix coinciding in form with (3) where $r_{m\,m+1}$, $t_{m\,m+1}$, and g_m are replaced by the corresponding quantities for radiation at frequency ν_3; columns

$$B_{m+1} = \frac{1}{t_{m+1\,m}^{\rm s}} \begin{bmatrix} \bar{g}_{3m}\left(E_{3m+1}^{\rm s(+)} + r_{m+1\,m}^{\rm s} E_{3m+1}^{\rm s(-)}\right) \\ g_{3m}\left(r_{m+1\,m}^{\rm s} E_{3m+1}^{\rm s(+)} + E_{3m+1}^{\rm s(-)}\right) \end{bmatrix}, \qquad (14a)$$

$$D_m = \frac{1}{t_{mm}^{\rm s}} \begin{bmatrix} \bar{g}_{3m}\left(g_{1m}\bar{g}_{2m}E_{3m}^{\rm s(+)} + \bar{g}_{1m}g_{2m}r_{mm}^{\rm s}E_{3m}^{\rm s(-)}\right) \\ g_{3m}\left(g_{1m}\bar{g}_{2m}r_{mm}^{\rm s}E_{3m+1}^{\rm s(+)} + \bar{g}_{1m}g_{2m}E_{3m}^{\rm s(-)}\right) \end{bmatrix} \qquad (14b)$$

determine the efficiency of nonlinear sources in the $m+1$th and mth layers; $t_{lp}^{\rm s}$ and $r_{lp}^{\rm s}$ [$l = m+1$, $p = m$ in (14a) and $l = p = m$ in (14b)] have the form

$$t_{lp}^{\rm s} = \frac{2s_{3l}\sigma_{lp}^{\rm s}}{s_{3l} + (s_{1p} - s_{2p})(\sigma_{lp}^{\rm s})^2},$$
$$t_{lp}^{\rm s} = \xi\,\frac{s_{3l} - (s_{1p} - s_{2p})(\sigma_{lp}^{\rm s})^2}{s_{3l} + (s_{1p} - s_{2p})(\sigma_{lp}^{\rm s})^2}; \qquad (15)$$

$\xi = 1$, $\sigma_{lp}^{\rm s} = 1$, and $\xi = -1$, $\sigma_{lp}^{\rm s} = n_{3l}/n_p^{\rm s}$ for the σ and π polarised radiations, respectively. Let us introduce the notation $P_{m\,m+1} = B_{m+1} - D_m$. Then, taking (13) into account, we can write the expressions

$$E_{30} = \hat{A}_{01}^{(\nu_3)}E_{31} + P_{01}, \quad E_{31} = \hat{A}_{12}^{(\nu_3)}E_{32} + P_{12}, \ldots,$$
$$E_{3N} = \hat{A}_{N\,N+1}^{(\nu_3)}E_{3N+1} + P_{N\,N+1}.$$

By performing substitutions by turn, we obtain the relation between the fields in the zero and $N+1$th layers of the PC:

$$E_{30} = \hat{M}^{(\nu_3)}(0, N+1)E_{3N+1} + \sum_{l=0}^{N}\hat{M}^{(\nu_3)}(0, l)P_{l\,l+1}, \quad (16)$$

where, as before, $\hat{M}^{(\nu_3)}(0, l) = \hat{A}_{01}^{(\nu_3)}\hat{A}_{12}^{(\nu_3)}\ldots\hat{A}_{l-1\,l}^{(\nu_3)}$ and $\hat{M}^{(\nu_3)}(0, 0)$ is the unit matrix. Taking into account that radiation is not incident on the PC from the substrate side ($E_{3N+1}^{(-)} = 0$), we can represent (16) in the form

$$\begin{bmatrix} E_{30}^{(+)} \\ E_{30}^{(-)} \end{bmatrix} = \begin{bmatrix} M_{11}^{(\nu_3)} & M_{12}^{(\nu_3)} \\ M_{21}^{(\nu_3)} & M_{22}^{(\nu_3)} \end{bmatrix} \begin{bmatrix} E_{3N+1}^{(+)} \\ 0 \end{bmatrix} + \begin{bmatrix} U^{(+)} \\ U^{(-)} \end{bmatrix}, \qquad (17)$$

where $U^{(\pm)}$ are the elements of the column obtained upon summation of the product of matrices $\hat{M}^{(\nu_3)}(0, l)$ and by columns $P_{l\,l+1}$. Finally, the fields of the transmitted and reflected nonlinear waves appearing in the PC on which the fields $E_{i0}^{(+)}$ ($i = 1, 2, 3$) are incident are described by the expressions

$$E_{3N+1}^{(+)} = T_3\left(E_0^{(+)} - U^{(+)}\right),$$
$$E_{30}^{(-)} = R_3\left(E_{30}^{(+)} - U^{(+)}\right) + U^{(-)}. \qquad (18)$$

The amplitude transmission and reflection coefficients for radiation at the difference frequency are determined by the expressions $T_3 = 1/M_{11}^{(\nu_3)}(0, N+1)$ and $R_3 = M_{21}^{(\nu_3)}(0, N+1)/M_{11}^{(\nu_3)}(0, N+1)$, respectively. By calculating nonlinear-radiation fields generated in any $\mathrm{PC}^{(J)}$ by using (18), it is necessary to take into account that the wave with the amplitude $E_{30}^{(+)}$ incident on the $\mathrm{PC}^{(J)}$ surface is the wave reflected by the $\mathrm{PC}^{(J-1)}$. Therefore, for the $\mathrm{PC}^{(1)}$, the wave amplitude is $E_{30}^{(+)} \equiv 0$.

Consider now the results obtained by the transfer matrix method for the structure with parameters indicated in the study of linear effects. We will also take into account the dispersion of ZnTe and DAST: $\Delta n_{1,2} = n_{1,2}(\nu_1) - n_{1,2}(\nu_2) = 0.001$, $n_1(\nu_{\rm TH}) = 3.4$, $n_2(\nu_{\rm TH}) = 3.0$, $\nu_{\rm TH} \equiv \nu_3$ (where $\nu_{\rm TH}$ is the terahertz radiation frequency). It is assumed that even layers are nonlinear, absorption is neglected, and $\nu_1 = \nu_0$, $\nu_2 = 0.99875\nu_0$, and $\nu_0 = c/\lambda_0$.

Figure 3 shows the dependences of the energy W of the filed localised inside the $\mathrm{PC}^{(3)}$ for radiation at the frequency ν_1 and the intensity I of terahertz radiation at the frequency $\nu_{\rm TH}$ generated in a superlattice consisting of three PCs on the angle of incidence θ of radiation for different N_2. The nonphase-matching enhancement occurs for radiation at the frequency ν_1 for $\theta = 38.8°$ and for radiation at the frequency ν_2 for $\theta = 38.4°$. The thickness L_0 of a layer located between PCs was selected optimal for obtaining the moist efficient interference between the incident and terahertz wave produced in these crystals. The refractive index of this layer was set equal to unity. We also assumed that the PC substrate is highly reflecting for terahertz radiation. The intensity I was normalised to the maximum intensity of the same radiation generated in a homogeneous medium of thickness $L = 10d_2$ with the refractive index $n_2(\nu)$.

One can see from Fig. 3a that for $N_2 = 0$ and 1, the nonphase-matching enhancement is virtually absent and a local minimum is observed instead of a maximum of the distribution of W. This results in the increase in the intensity I of the nonlinear terahertz signal only by an order of magnitude, which mainly occurs due to nonphase-matching enhancement in the $\mathrm{PC}^{(1)}$. For $N_2 = 2$, the reflection coefficient of the structure for signals at the fundamental frequencies increases [see curve (*3*) in Fig. 2a], and nonphase-matching enhancement in the $\mathrm{PC}^{(2)}$ and $\mathrm{PC}^{(3)}$ becomes noticeable, further increasing the generated signal intensity by an order of magnitude. In this case, two maxima appear in the angular dependence of the intensity I, which correspond to the most exact fulfilment of the condition of nonphase-matching enhancement for fundamental signals at frequencies ν_2 (the left maximum at $\theta = 38.4°$) and ν_1 (the right maximum at $\theta = 38.8°$). In the case of $N_2 = 3$, the dependence of the energy W on the angle of incidence takes the usual shape – there is no dip at the centre [see curve (*4*) in Fig. 3a], i.e. a considerable nonphase-matching enhancement occurs in each $\mathrm{PC}^{(J)}$. The nonlinear signal intensity in this case is more than 400 times greater than in a homogeneous medium.

The inset in Fig. 3b shows the dependences of the maximum intensity $I^{\max}$ of the generated terahertz signal on the number of PCs forming the superlattice for different

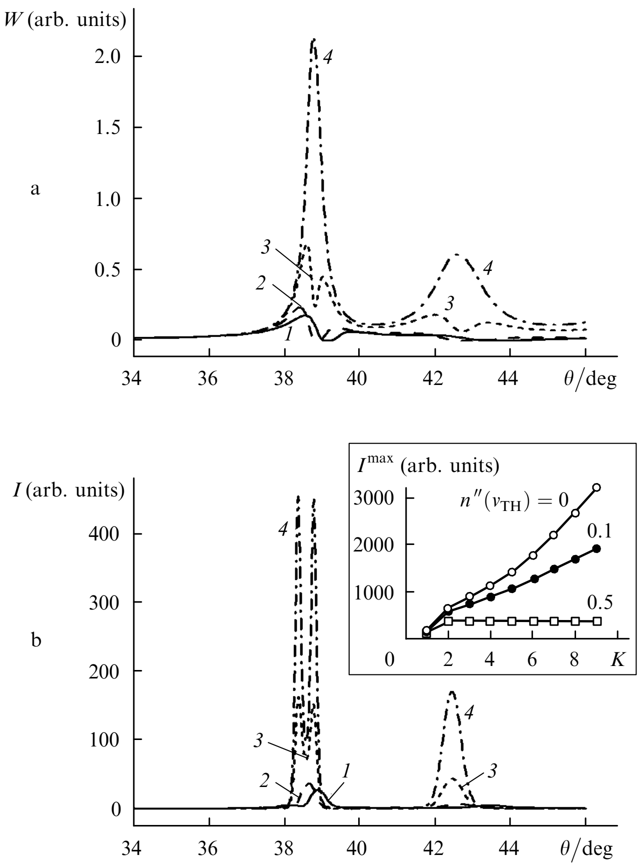

Figure 3. Dependences of the normalised field energy W localised inside the $PC^{(3)}$ (a) and the normalised intensity I of terahertz radiation generated in the $PC^{(3)}$ superlattice (b) on the angle of incidence for the number of bilayers of additional $PCs^{(J)}$ ($J = 1, 2, 3$) $N_2 = 0$ (*1*), 1 (*2*), 2 (*3*), and 3 (*4*). The inset shows the dependences of the maximum normalised intensity I^{max} of terahertz radiation on the number of PCs forming the superlattice for different values of the imaginary part n'' of the refractive indices $n_1(\nu_{TH})$ and $n_2(\nu_{TH})$ in the terahertz region.

absorption coefficients at the terahertz radiation frequency. For each $PC^{(J)}$, $N_2 = 4$. The imaginary parts n'' of the refractive indices $n_1(\nu_{TH})$ and $n_2(\nu_{TH})$ equal to 0, 0.1, and 0.5 for $\nu_{TH} \sim 1$ THz correspond to the absorption coefficients $\delta = 0$, 40, and ~ 200 cm^{-1}. One can see that in the case of the superstructure consisting of four or more PCs, the intensity of generated terahertz radiation can be increase by more than three orders of magnitude compared to a homogeneous medium even if absorption is considerable ($\delta = 40$ cm^{-1}).

Thus, we have shown that the intensity of terahertz signals generated in a quasi-two-dimensional superlattice consisting of four PCs can be increased by more than three orders of magnitude compared to a continuous medium and more than an order of magnitude compared to an isolate PC. This occurs due to the efficient nonphase-matching enhancement and coherent summation of nonlinear signals appearing in different PCs. Note also that the compound PCs described in the paper can be used to increase the generation efficiency of signals at mixed frequencies in the optical range.

Acknowledgements. This work was supported b the Russian Foundation for Basic Research (Grant Nos 04-02-16866 and 06-03-08057).

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