OPTICAL SYSTEMS

PACS numbers: 42.25.Bs; 42.30. –d; 42.60.Jf
DOI: 10.1070/QE2007v037n03ABEH013223

Calculation of the illuminance distribution in the focal spot of a focusing system taking into account aberrations in this system and divergence of a focused laser beam

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***Abstract*. The dependence of the focal-spot size of a 'deep' parabolic mirror reflector on the laser-beam divergence is analysed by the method of elementary reflections. The dependence of the focal-beam diameter of an ideal focusing optical system on the laser-beam parameters is described. The expression is obtained for calculating the illumination distribution in the focal spot of a 'deep' mirror reflector which takes into account both aberrations and light-gathering power of the reflector and the divergence of a focused laser beam.**

Keywords: *Wigner function, laser-beam divergence, parabolic reflector, focusing.*

1. Introduction

The interaction of light with matter depends both on the laser radiation power and possibility to concentrate radiation on an area of minimum size. Radiation is concentrated by using wide-aperture focusing optical systems, as a rule, a 'deep' parabolic reflector. The methods of calculating the illumination distribution in the focal spot of a parabolic reflector applied at present in laser technologies take into account aberrations of the reflector [1, 2] and polarisation of the focused radiation [3, 4] but neglect the main factor – the divergence of the focused wave beam.

The necessity of considering the divergence of a light beam in calculations of the transfer and concentration of light energy in optical systems is presented in a bitter polemic form by Slyusarev in his book 'On Possible and Impossible in Optics' [5]. The radiometric approach proposed in [5] emphasises that only a parallel pencil of light rays (plane wave) can be focused to a point, but it cannot carry energy. The energy can be carried by a diverging pencil of rays, but it cannot be focused to a point. From this radiometric point of view, nearly diffraction-limited wave beams emitted by lasers can carry energy only due to their divergence.

Traditional radiometry uses the theoretical concepts of a variety of technical disciplines: the energy calculations of optical systems, lighting technology, the theory of thermal radiation, etc. [6]. In addition, many theoretical concepts of traditional radiometry are based on *a priori* assumption of the Lambert radiation pattern of sources. To order logically the construction of the radiometry theory and generalise its results to sources with a non-Lambert radiation pattern (in particular, lasers), the basic relations of this theory were derived in [7] from Hamiltonian optics. This approach is called in the literature [8] 'the description of radiation energy transfer in the phase space'. Another, alternative approach to the construction of the theory of generalised radiometry of non-Lambert sources, in particular, laser beams based on Fourier optics, the theory of a partial coherence, and the mathematical properties of the Wigner function was proposed by Walther [9]. It is important that the mathematical formalism of both theories of the so-called generalised radiometry was borrowed from mechanics and therefore they mutually supplement and enrich each other. The Hamiltonian radiometry allows the use of the mathematical apparatus of calculation optics (the eikonal theory) for energy calculations, while the generalised Walther radiometry refines the region of applicability of relations of the Hamiltonian radiometry in the case of diffraction-limited laser wave beams.

In this paper, the influence of the laser-beam divergence on the focal-spot size in the focus of an optical system is analysed based on the concepts of generalised radiometry and the expression is derived for calculating the illumination distribution in the focal spot, which takes into account both aberrations of a wide-aperture mirror reflector and the divergence of the focused laser beam.

2. Method of elementary reflections

Let us estimate the influence of the divergence of a wave beam on the focal-spot size produced by a 'deep' mirror reflector. The dimensions of a laser reflector expressed in wavelengths are approximately the same as those in microwave antennas, and therefore a laser wave beam can be represented in calculations of reflectors as a pencil of light rays with a low divergence satisfying the laws of geometrical optics [10, 11].

Before proceeding to the solution of the problem under study, we consider the solution of the inverse problem of estimating the influence of the size of a spherical luminous body on the divergence of a beam produced by a 'deep' mirror reflector. In lighting and laser technologies, a parabolic reflector is widely used as a 'deep' mirror reflector. The specific property of this reflector is that the pencil of rays from a point source located at the focus F of the reflector is

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Received 16 May 2006
Kvantovaya Elektronika **37** (3) 307 – 312 (2007)
Translated by M.N. Sapozhnikov

reflected parallel to its optical axis [12]. The meridional cross section of this reflector is a parabola, which is described in the polar coordinate system $R\varphi$ centred at the point F by the equation

$$R(\varphi) = \frac{2f}{1+\cos\varphi},$$

where f is the focal distance of the parabolic reflector. The influence of the size of a spherical luminous body on the divergence of a beam reflected from a mirror reflector is calculated in lighting technology by the method of elementary reflections [13], in which each point P of the mirror reflector is considered as the centre of two elementary homocentric pencils of rays: incident from the spherical source and reflected to space (Fig. 1). According to the law of reflection, the solid angles of these homocentric pencils are equal; therefore, if a point source located at the focus of the parabolic reflector is replaced by a spherical source, the point $P(R, \varphi)$ of this reflector will emit the homocentric pencil of rays with the axis directed along the symmetry axis of the parabola and the divergence

$$\Theta(\varphi) \approx \frac{d}{R(\varphi)} = \Theta_0(1+\cos\varphi),$$

where $\Theta_0 \equiv d/(2f)$ is the divergence of a paraxial pencil of rays and d is the diameter of the spherical source.

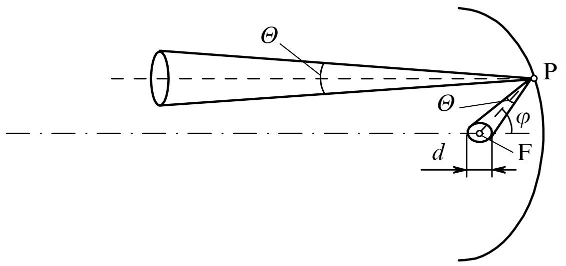

Figure 1. Scheme of elementary reflections in the case of a parabolic reflector.

Let us return to the initial problem when a pencil of uniformly diverging rays ($\Theta_0 = \text{const}$) is incident on the parabolic mirror reflector. In this case, the size $d(\varphi)$ of the focal beam produced by a thin pencil of parallel rays incident on the parabolic reflector in the vicinity of the point $P(R, \varphi)$ increases with distance from the point P to the paraboloid focus F, i.e. with increasing the angle φ:

$$d(\varphi) \approx \frac{2f\Theta_0}{1+\cos\varphi}.$$

The change in the focal-spot size is so large that, unlike a lens, a 'deep' wide-aperture reflector cannot produce the distinct image of an object. (The focal spot at the focus of the parabolic reflector represents a superposition of focal spots formed by the elementary pencils of parallel rays.)

3. The Wigner function and phase brightness

The parameters of laser beams, in particular, their divergence are measured at present according to international standards [14–16] based on the mathematical properties of the Wigner function. Let us describe the relation of the divergence and the possibility of laser-beam focusing with the properties of the Wigner function in more detail.

Consider for simplicity the one-dimensional case in the scalar quasi-monochromatic approximation, i.e. neglect polarisation effects and the spectrum of radiation. Let us assume that the z axis approximately coincides with the propagation direction of radiation and the x axis is perpendicular to it. Under these assumptions, the wave field in the laser-beam cross section is described by the complex amplitude $U(x)$, where x is the spatial coordinate. The spatial coherence of this light wave is characterised by the two-point correlation function – the mutual intensity

$$\Gamma(x_1, x_2) \equiv \langle U(x_1) U^*(x_2) \rangle.$$

Here, the angle brackets denote averaging over an ensemble, and the asterisk * means complex conjugation. By passing to the average variable $x = (x_1 + x_2)/2$ and the difference variable $\zeta = x_2 - x_1$ and using the Fourier transform over the difference variable, we can obtain the characteristic of the spatial coherence of radiation – the Wigner distribution function [9, 17]

$$W(x,u) \equiv \int_{-\infty}^{\infty} \Gamma\left(x+\frac{\zeta}{2}, x-\frac{\zeta}{2}\right) \exp(-\mathrm{i}\zeta u)\mathrm{d}\zeta \qquad (1)$$

(where u is the spatial frequency), which is mathematically equivalent to the mutual intensity but is more convenient.

For example, if the wave field in the cross section of the diffraction-limited Gaussian beam is described by the expression [17]

$$U(x) = \left(\frac{2}{\rho^2}\right)^{1/4} \exp\left[-\frac{\pi}{\rho^2}(x-x_0)^2 + \mathrm{i}u_0 x\right],$$

where ρ is the positive quantity, the Wigner function W^{G} of this wave field takes the form

$$W^{\mathrm{G}}(x,u) = 2\exp\left[-\frac{2\pi}{\rho^2}(x-x_0)^2 - \frac{\rho^2}{2\pi}(u-u_0)^2\right]. \qquad (2)$$

The level line at the 1/e height of the function W^{G} has the shape of an ellipse with the centre at the point x_0, u_0 in the xu plane and symmetry axes directed parallel to the coordinate axes x and u. The projections of the function W^{G} in planes Wx and Wu have the Gaussian shape, i.e. the root-mean-square width of the beam is

$$\sigma_x^{\mathrm{G}} = \left(\frac{1}{2}\frac{\rho^2}{2\pi}\right)^{1/2},$$

and the root-mean-square width of the spectrum is

$$\sigma_u^{\mathrm{G}} = \left(\frac{1}{2}\frac{2\pi}{\rho^2}\right)^{1/2}.$$

It follows from the symmetry of the widths σ_x^{G} and σ_u^{G} that the identity

$$\sigma_x^{\mathrm{G}}\sigma_u^{\mathrm{G}} = \frac{1}{2} \qquad (3)$$

is valid for a Gaussian wave beam.

The Wigner function of the wave field in an arbitrary cross section of a real laser beam in the xu plane also usually has the shape of an ellipse, but the product of its root-mean-square sizes (3) is greater than (or equal to) 1/2, while its symmetry axes doe not coincide in the general case with the coordinate axes x and u.

It is known that if a plane monochromatic wave with the wavelength λ in vacuum is incident normally on a sinusoidal diffraction grating with the period d_0, the diffracted wave is deflected through the angle θ (Fig. 2). In this case, the projection $k_\perp = kn\sin\theta$ of the wave vector $\boldsymbol{k}$ on the diffraction grating in an optically homogeneous medium with the refractive index n is equal to the spatial frequency $u = 2\pi/d_0$ of this grating [18]:

$$u = k_\perp = kp, \tag{4}$$

where $p = n\sin\theta$ is the 'momentum' (optical unit vector) and $k \equiv |\boldsymbol{k}| = 2\pi/\lambda$ is the wave number. Thus, for a fixed wavelength in vacuum $\lambda = 2\pi$ (here λ is normalised for convenience so that $k = 1$) and $k > u$, the 'momentum' p of the diffracted wave is proportional to the spatial frequency u of the diffraction grating. This proportionality, which is known as the 'dual meaning of spatial frequencies' [19], allows one to go over easily from the concepts of wave optics to the concepts of Hamiltonian optics and radiometry.

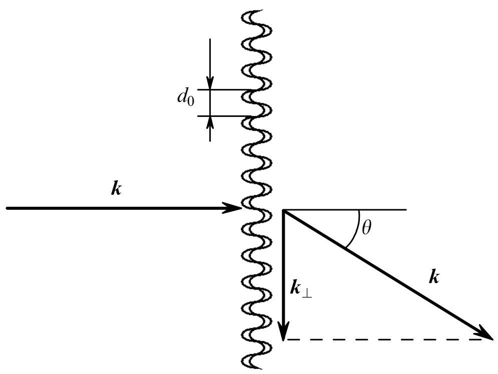

Figure 2. Dual meaning of spatial frequencies.

The radiation flux (power) Φ from a source in the one-dimensional Hamiltonian radiometry is distributed over the coordinate x and 'momentum' $p \in [-n, n]]$. The distribution of the radiation flux Φ in the phase xp plane is described by the phase brightness [7]:

$$\mathrm{d}^2\Phi = \mathscr{B}(x,p)\mathrm{d}x\mathrm{d}p. \tag{5}$$

Expression (4) relates the phase brightness $\mathscr{B}(x,p)$ in (5) with the Wigner function $W(x,u)$ (1) by the similarity transformation

$$\mathscr{B}(x,p) = kW(x,kp)\chi_n(p), \tag{6}$$

where

$$\chi_n(p) \equiv \begin{cases} 1 & \text{for } |p| \leqslant n, \\ 0 & \text{for } |p| > n. \end{cases}$$

The proportionality between p and u allows us to use the phase brightness instead of the Wigner function.

Note that the dual meaning of spatial frequencies described by expression (4) allows us to write identity (3), which is valid for diffraction-limited Gaussian wave beams, not in the coordinate-frequency representation but in the coordinate-momentum representation:

$$\sigma_x^{\mathrm{G}}\sigma_p^{\mathrm{G}} = \frac{\sigma_x^{\mathrm{G}}\sigma_u^{\mathrm{G}}}{k} = \frac{1}{2k} = \frac{\lambda}{4\pi}. \tag{7}$$

Consider the matrices of the first and second moments of the phase brightness [16–18]

$$\begin{pmatrix} \bar{x} \\ \bar{p} \end{pmatrix} = \int_{-\infty}^{\infty}\int_{-n}^{n}\begin{pmatrix} x \\ p \end{pmatrix}\mathscr{B}(x,p)\mathrm{d}x\mathrm{d}p\left[\int_{-\infty}^{\infty}\int_{-n}^{n}\mathscr{B}(x,p)\mathrm{d}x\mathrm{d}p\right]^{-1},$$

$$\Sigma \equiv \begin{pmatrix} \sigma_x^2 & m_{xp} \\ m_{px} & \sigma_p^2 \end{pmatrix} \tag{8}$$

$$= \int_{-\infty}^{\infty}\int_{-n}^{n}\begin{pmatrix} (x-\bar{x})(x-\bar{x})^* & (x-\bar{x})(p-\bar{p})^* \\ (p-\bar{p})(x-\bar{x})^* & (p-\bar{p})(p-\bar{p})^* \end{pmatrix}\mathscr{B}(x,p)\mathrm{d}x\mathrm{d}p$$

$$\times\left[\int_{-\infty}^{\infty}\int_{-n}^{n}\mathscr{B}(x,p)\mathrm{d}x\mathrm{d}p\right]^{-1}.$$

Note that square roots from the elements of the leading diagonal of the beam matrix Σ characterise the beam width σ_x and its divergence σ_p.

In matrix optics [20, 21], the position and orientation of a light beam in the meridional plane of an optical system is described by the column matrix and its transformation in the optical system is described by the $ABCD$ matrix:

$$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} x \\ p \end{pmatrix}. \tag{9}$$

Hereafter, primed variables are related to the image space of the optical system, while non-primed variables are related to the object space. After the propagation of the wave beam through the optical system, the phase brightness of the beam and the corresponding matrix Σ change. The transformation of the matrix of second moments Σ (8) after the propagation of the beam trough the $ABCD$ system (9) is described by the expression [16–18]

$$\begin{pmatrix} \sigma_{x'}^2 & m_{x'p'} \\ m_{p'x'} & \sigma_{p'}^2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \sigma_x^2 & m_{xp} \\ m_{px} & \sigma_p^2 \end{pmatrix}\begin{pmatrix} A & C \\ B & D \end{pmatrix}. \tag{10}$$

Note that, because the determinant of the $ABCD$ matrix is unity [20, 21],

$$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det\begin{pmatrix} A & C \\ B & D \end{pmatrix} = 1,$$

the determinant of the matrix Σ of second moments of the phase brightness is the same in an arbitrary cross section of the wave beam [18]:

$$\det\begin{pmatrix} \sigma_{x'}^2 & m_{x'p'} \\ m_{p'x'} & \sigma_{p'}^2 \end{pmatrix} = \det\begin{pmatrix} A & B \\ C & D \end{pmatrix}\det\begin{pmatrix} \sigma_x^2 & m_{xp} \\ m_{px} & \sigma_p^2 \end{pmatrix}\times$$

$$\times \det\begin{pmatrix} A & C \\ B & D \end{pmatrix} = \det\begin{pmatrix} \sigma_x^2 & m_{xp} \\ m_{px} & \sigma_p^2 \end{pmatrix} = I^2 = \text{const.}$$

Thus, the determinant I of the matrix of second moments is an invariant of the laser beam (analogue of the Lagrange–Helmholtz invariant [21] in classical optics).

It is known that the wave-beam waist is formed in the vicinity of the image focal point of a focusing optical system. The waist region has the mirror-symmetry plane $\tilde{z}$ in which the so-called generalised beam waist is located [16]. Consider the level line at the 1/e height of the phase brightness of a real laser beam in the phase plane $\tilde{x}\tilde{p}$ corresponding to the generalised beam waist. This level line for Wigner function (2) of a Gaussian wave beam has the shape of an ellipse with symmetry axes coinciding with the coordinate axes of the phase plane $\tilde{x}\tilde{p}$, and therefore the corresponding matrix Σ of second orders is diagonal:

$$I = \det\begin{pmatrix} \sigma_x^2 & m_{xu} \\ m_{ux} & \sigma_p^2 \end{pmatrix} = \det\begin{pmatrix} \sigma_{\tilde{x}}^2 & 0 \\ 0 & \sigma_{\tilde{p}}^2 \end{pmatrix} = \sigma_{\tilde{x}}^2\sigma_{\tilde{p}}^2. \quad (11)$$

Thus, it is in this symmetry plane $\tilde{z}$ that the invariant I of the laser beam can be calculated most simply. In particular, according to equality (7), the invariant of the Gaussian wave beam in the symmetry plane $\tilde{z}$ is

$$I^{\mathrm{G}} = \sigma_{\tilde{x}}\sigma_{\tilde{p}} = \frac{\lambda}{4\pi}. \quad (12)$$

4. The M^2 factor

Because a diffraction-limited Gaussian laser beam is the ideal beam, its invariant I^{G} is always smaller than that for a real laser beam:

$$I \geqslant I^{\mathrm{G}} = \frac{\lambda}{4\pi}. \quad (13)$$

Because of this, it is convenient to characterise a real wave beam by the M^2 factor [15, 16] representing the ratio of the invariant I of a real laser beam to the invariant I^{G} of the reference diffraction-limited Gaussian beam:

$$M^2 \equiv \frac{I}{I^{\mathrm{G}}} = \left(\frac{\det\Sigma}{\det\Sigma^{\mathrm{G}}}\right)^{1/2} = \frac{4\pi}{\lambda}(\det\Sigma)^{1/2} = \frac{4\pi}{\lambda}\,\sigma_{\tilde{x}}, \sigma_{\tilde{p}}. \quad (14)$$

In the small-angle approximation ($p \approx n\theta$), expression (14) can be written in a more customary form [14]

$$M^2 \equiv \frac{I}{I^{\mathrm{G}}} \approx \frac{\pi n}{\lambda}\frac{d\Theta}{4}. \quad (15)$$

In the one-dimensional case under study, the 'generalised diameter' [16] d and the 'generalised angular divergence' Θ are determined by the expressions

$$d = 4\sigma_x, \quad \Theta \approx \frac{4}{n}\sigma_p.$$

Let us show that the M^2 factor characterises the possibility of focusing a real laser beam. For this purpose, we consider the ideal optical system with the focal distance f, in which a real wave beam and, for comparison, a diffraction-limited beam of the same diameter D are focused. The divergence of the wave beam determines the ratio of the beam diameter D to the object focal distance f of the optical system, and therefore the divergences of the real ($d = 4\sigma_{x'}$) and Gaussian ($d^{\mathrm{G}} = 4\sigma_{x'}^{\mathrm{G}}$) beams are the same (Fig. 3):

$$4\sigma_{p'} = 4\sigma_{p'}^{\mathrm{G}} \approx n\Theta \approx \frac{D}{f}. \quad (16)$$

In this case, the M^2 factor is equal to the ratio of the 'generalised diameters' of focal spots of the real ($d = 4\sigma_{x'}$) and Gaussian ($d^{\mathrm{G}} = 4\sigma_{x'}^{\mathrm{G}}$) laser beams:

$$M^2 = \frac{d}{d^{\mathrm{G}}}. \quad (17)$$

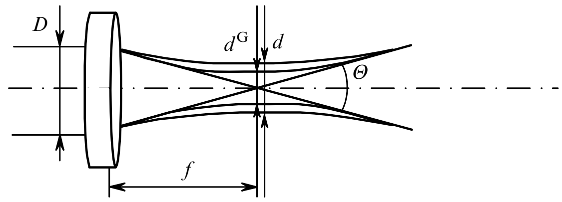

Figure 3. Effect of the M^2 factor on the focal-spot size.

The product $d\Theta$ of the beam parameters used to calculate the M^2 factor can be measured by two methods.

Single-lens measurement method. The required waist region of a wave beam is formed in the vicinity of the image focal plane of the ideal optical system. The diameter d of the wave beam and its angular divergence Θ can be measured in the symmetry plane $\tilde{z}$ of the waist [14–16] (Fig. 4).

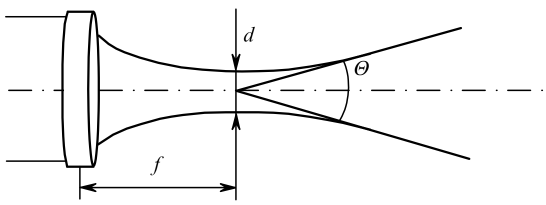

Figure 4. Scheme illustrating the single-lens method for measuring $d\Theta$.

Two-lens measurement method. To simplify the measurement of the angular divergence Θ of the laser beam in the symmetry plane of the waist, we place the second optical system behind the first focusing system so that the object focal plane of the second optical system would coincide with the symmetry plane of the waist. It is known that the illumination distribution in the image focal plane of the second optical system produced by a diverging wave beam is the same as that produced by a point source with the same divergence located at the back node plane N$'$ of this optical system [22, 23]. Therefore, by measuring the wave-beam diameter d_{f} in the image focal plane of the second optical system (Fig. 5), we obtain the required angular divergence Θ from the expression

$$\Theta = \frac{d_{\mathrm{f}}}{f_2}. \quad (18)$$

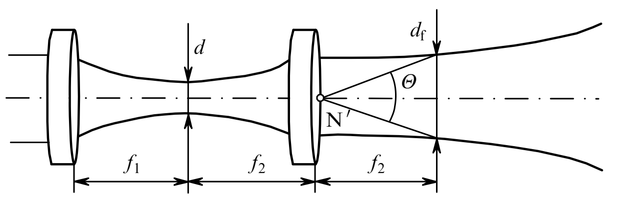

Figure 5. Scheme illustrating the two-lens method for measuring $d\Theta$.

The wave-beam diameter d is measured, as before, in the symmetry plane of the waist formed by the first focusing system [24] (Fig. 4).

5. Calculation of the illumination distribution in the focal spot of a wide-aperture focusing optical system

The size of the focal spot of a real focusing system, for example, of a 'deep' parabolic reflector depends not only on the divergence of the laser beam being focused but also on its aberration. Therefore, the M^2 factor is useless for the calculation of the focal-spot size of a wide-aperture mirror reflector. Let us derive the expression for calculating the illumination distribution in the focal spot of a wide-aperture mirror reflector.

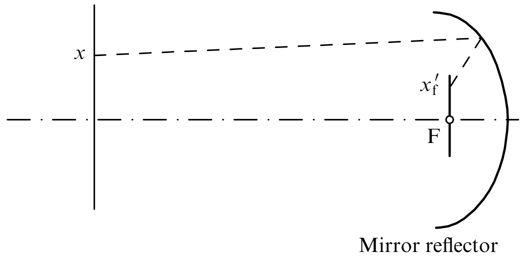

Figure 6. Coordinates x and x'_f of a light beam in the object plane and the focal plane of a mirror reflector, respectively.

$$\Sigma \equiv \begin{pmatrix} \sigma_x^2 & m_{xu} & m_{xp} & m_{xq} \\ m_{yx} & \sigma_y^2 & m_{yp} & m_{yq} \\ m_{px} & m_{py} & \sigma_p^2 & m_{pq} \\ m_{qx} & m_{qy} & m_{qp} & \sigma_q^2 \end{pmatrix} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-n}^{n}\int_{-n}^{n} \begin{pmatrix} (x-\bar{x})(x-\bar{x})^* & (x-\bar{x})(y-\bar{y})^* & (x-\bar{x})(p-\bar{p})^* & (x-\bar{x})(q-\bar{q})^* \\ (y-\bar{y})(x-\bar{x})^* & (y-\bar{y})(y-\bar{y})^* & (y-\bar{y})(p-\bar{p})^* & (y-\bar{y})(q-\bar{q})^* \\ (p-\bar{p})(x-\bar{x})^* & (p-\bar{p})(y-\bar{y})^* & (p-\bar{p})(p-\bar{p})^* & (p-\bar{p})(q-\bar{q})^* \\ (q-\bar{q})(x-\bar{x})^* & (q-\bar{q})(y-\bar{y})^* & (q-\bar{q})(p-\bar{p}) & (q-\bar{q})(q-\bar{q})^* \end{pmatrix}$$

$$\times \mathscr{B}(x,y,p,q)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}p\,\mathrm{d}q\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-n}^{n}\int_{-n}^{n}\mathscr{B}(x,y,p,q)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}p\,\mathrm{d}q\right]^{-1},$$

Let us assume that we know the phase brightness $\mathscr{B}(x,p)$ of a laser beam in the object plane of a mirror reflector. It is known that aberrations of an optical system, in particular, of a mirror reflector are completely characterised by the point eikonal $S(x,x'_f)$ – the optical length of the shortest path connecting a point x in the object plane with a point x'_f in the image focal plane (Fig. 6). The point eikonal has the differential properties [20]:

$$p \equiv -\frac{\partial S(x,x'_f)}{\partial x}, \quad p'_f \equiv -\frac{\partial S(x,x'_f)}{\partial x'_f}. \tag{19}$$

By knowing the point eikonal $S(x,x'_f)$ of the focusing optical system and its aperture angle in the image plane $2\alpha'$ and using expressions (5) and (19), we obtain the required expression for calculating the influence of the Wigner function of the laser beam in the object plane on the distribution of the field E in the focal spot of the focusing optical system [7, 21, 25–27]:

$$E(x'_f) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}x'_f} = \int_{-\infty}^{\infty}\mathscr{B}\left(x, -\frac{\partial S}{\partial x}\right)\chi_{\alpha'}\left(\frac{\partial S}{\partial x'_f}\right)\left|\frac{\partial^2 S}{\partial x\,\partial x'_f}\right|\mathrm{d}x, \tag{20}$$

where

$$\chi_{\alpha'}(p'_f) = \begin{cases} 1 & \text{for } |p'_f| \leqslant \alpha', \\ 0 & \text{for } |p'_f| > \alpha'. \end{cases}$$

6. Generalisation of the results to the two-dimensional case

Consider the two-dimensional generalisation of the obtained results to the case of an axially symmetric optical system and a light beam with coordinates x and y in the cross section z. The propagation direction of the beam is described by two optical unit vectors ('momenta') $p = n\sin\theta_x$ and $q = n\sin\theta_y$. In this case, the first- and second-order matrices of the phase brightness $\mathscr{B}(x,y,p,q)$ take the form

$$\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{p}\\ \bar{q}\end{pmatrix} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-n}^{n}\int_{-n}^{n}\begin{pmatrix}x\\ y\\ p\\ q\end{pmatrix}\mathscr{B}(x,y,p,q)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}p\,\mathrm{d}q$$

$$\times\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-n}^{n}\int_{-n}^{n}\mathscr{B}(x,y,p,q)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}p\,\mathrm{d}q\right]^{-1},$$

respectively. The determinant of the second-order matrix of the Hamiltonian brightness is the invariant of the laser beam, and therefore its M^2 factor can be calculated by the expression

$$M^2 = \frac{4\pi}{\lambda}(\det\Sigma)^{1/4}$$

or by expression (15), if the 'generalised diameter' and the 'generalised angular divergence' of the beam are calculated by the expressions [14–16]

$$d = 2\sqrt{2}\left(\sigma_x^2 + \sigma_y^2\right)^{1/2}, \quad \Theta \approx 2\sqrt{2}\,n^{-1}\left(\sigma_p^2 + \sigma_q^2\right)^{1/2}.$$

The two-dimensional generalisation of expression (20) can be easily obtained in the form

$$E(x'_{\mathrm{f}}, y'_{\mathrm{f}}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{B}\left(x, y, -\frac{\partial S}{\partial x}, -\frac{\partial S}{\partial y}\right) \times$$

$$\chi_{\alpha'}\left(\left[\left(\frac{\partial S}{\partial x'_{\mathrm{f}}}\right)^2 + \left(\frac{\partial S}{\partial y'_{\mathrm{f}}}\right)^2\right]^{1/2}\right) \det\begin{pmatrix} \dfrac{\partial^2 S}{\partial x\,\partial x'_{\mathrm{f}}} & \dfrac{\partial^2 S}{\partial x\,\partial y'_{\mathrm{f}}} \\ \dfrac{\partial^2 S}{\partial y\,\partial x'_{\mathrm{f}}} & \dfrac{\partial^2 S}{\partial y\,\partial y'_{\mathrm{f}}} \end{pmatrix} \mathrm{d}x\,\mathrm{d}y,$$

where $S(x, y, x'_{\mathrm{f}}, y'_{\mathrm{f}})$ is the point eikonal of a two-dimensional optical system.

Thus, the expression has been proposed in this paper for calculating the radiation intensity distribution in the focal spot of a mirror reflector taking into account its aberrations [the point eikonal $S(x, y, x', y')$], the light-gathering power (the aperture angle $2\alpha'$ in the image space), and the laser-beam divergence [the phase brightness $\mathscr{B}(x, y, p, q)$].

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