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Oscillation dynamics of a phase-locked three-channel holographic Nd : YAG laser system

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***Abstract*. A method of phase locking of a parallel multichannel laser system by holographic gain gratings in the active laser medium at a long-range holographic coupling of laser channels is developed. The oscillation dynamics of a three-channel holographic Nd : YAG laser system is simulated by considering the phase-locking conditions depending on the relative gain mismatch in laser channels. The conditions of lasing with the gain-selected control laser channel are found under which controlled channels are phase-locked even in the case of the threefold difference in their gains.**

Keywords: *holographic coupling, lasing conditions, multichannel laser system, phase locking.*

We proposed in [1] the method for phase locking of optically coupled loop laser resonators by gain gratings written upon four-wave mixing directly in the active elements of the laser. The principal possibility of the self-phase locking of a Nd : YAG laser system with two parallel coupled channels was demonstrated experimentally. Because this method does not require additional nonlinear media and external laser pumping, we propose to use it in high-power multichannel holographic laser systems for the long-range coupling of laser channels (the coupling between all laser channels) [2] in the common active element.

In this paper, we simulated numerically the lasing dynamics in the case of a long-range diffraction coupling of three holographic lasers in order to study the possibility of phase locking for the development of a branched series-parallel multichannel laser system. Experimental results are presented in [3].

Consider the development of oscillations in a laser with the degenerate multiwave mixing (DMWM) in the active medium by the method proposed in our papers [1, 4]. We will consider only transmission gain gratings because their diffraction efficiency exceeds that of reflection gratings by a factor of G [4] (G is the single-pass gain in the active medium). To simplify the model, we neglect refractive index gratings because, according to [5], for the gain in active elements $\alpha \leqslant 0.4$ cm^{-1}, the imaginary part of the nonlinear susceptibility is twice and more as large as the real part, and therefore refractive index gratings do not play a significant role in the intracavity DMWM [6]. We also assume that the DMWM time is small compared to the lifetime T_1 of the upper laser level. This is well consistent with pulse durations upon self-modulation ($\sim 10^{-7}$ s [1, 7]) and upon Q-switching ($\sim 10^{-8}$ s [8, 9]), which are small compared to the time T_1 for neodymium active media ($\sim 10^{-8}$ s). Then, the gain of the active medium for the field amplitude is described by the expression [10]

$$\alpha(t) = \alpha_0 \exp\left[-\frac{U_{\rm tot}(t)}{U_{\rm s}}\right], \tag{1}$$

where α_0 is the small-signal gain for the field amplitude; $U_{\rm s} = 0.55$ J cm^{-2} is the gain saturation energy density in a Nd : YAG crystal;

$$U_{\rm tot}(t) = \int_0^t I_{\rm tot}(t')\,{\rm d}t' \tag{2}$$

is the total energy density; $I_{\rm tot}(t)$ is the total radiation intensity in a medium at a certain instant. Therefore, upon DMWM in an amplifying medium, the gain (1) will be modulated as

$$\alpha = \alpha_0 \exp(-\sigma)\exp\left[-\sum_\tau g_\tau \cos(\boldsymbol{K}_\tau \boldsymbol{r} - \varphi_\tau)\right], \tag{3}$$

$$\sigma = \frac{1}{U_{\rm s}}\int_0^t I_{\rm mean}\,{\rm d}t', \tag{4}$$

$$g_\tau = \frac{1}{U_{\rm s}}\int_0^t I_{\rm coh}^\tau\,{\rm d}t', \tag{5}$$

where τ is the number of a transmission holographic grating (when several gratings are recorded); g_τ is the coherent parameter describing each holographic grating; $\boldsymbol{K}_\tau$ is the grating vector; φ_τ is the grating phase; σ is the average (incoherent) saturation parameter; $I_{\rm mean}$ is the average intensity in the medium; and $I_{\rm coh}^\tau$ is the field intensity caused by the interference of pairs of waves writing the given transmission grating.

To describe a three-channel laser system with one common active element for the interchannel coupling, we consider the simultaneous writing of three transmission gratings ($\tau = 1, 2, 3$) by three pairs of waves interfering

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upon DMWM in the active (amplifying) medium. By using the first two terms

$$\exp(-y\cos\theta) = I_0(y) + 2\sum_{k=1}^{\infty}(-1)^k I_k(y)\cos(k\theta) \approx I_0(y) - 2I_1\cos\theta \quad (6)$$

from the expansion in [11], where I_0 and I_k are the modified Bessel functions of the first kind of the zero and kth orders, we obtain the parameters of the spatial modulation of the gain in the harmonic approximation, which is valid for the energy flux density small compared to U_s:

$$\alpha = a + \sum_{\tau} 2b_{\tau}\cos(\boldsymbol{K}_{\tau}\boldsymbol{r} - \varphi_{\tau}), \quad (7)$$

$$a = \alpha_0\exp(-\sigma)I_0(g_1)I_0(g_2)I_0(g_3), \quad (8)$$

$$b_1 = -\alpha_0\exp(-\sigma)I_1(g_1)I_0(g_2)I_0(g_3), \quad (9)$$

$$b_2 = -\alpha_0\exp(-\sigma)I_0(g_1)I_1(g_2)I_0(g_3), \quad (10)$$

$$b_3 = -\alpha_0\exp(-\sigma)I_0(g_1)I_0(g_2)I_1(g_3), \quad (11)$$

where a is the average value of the gain in the active medium and $2b_{\tau}$ is the modulation amplitude of the gain corresponding to each gain grating.

Figure 1 presents the optical scheme of the three-channel holographic laser system with a long-range diffraction coupling between laser channels. Each laser channel is a laser with a loop self-pumped phase-conjugate resonator. A transmission gain grating providing a feedback to obtain a dynamic resonator is written in each active holographic element (AHE) [7]. Nonreciprocal elements direct lasing to the output feedback mirror, preserve the saturation of the gain in the common active interchannel coupling element (ACE) until the instant of lasing in laser channels, and equalise the intensities of the waves writing gain gratings in AHEs. The laser channels are coupled linearly via beamsplitters B2 and B3 to write gain gratings in active elements of each of the channels and also nonlinearly due to the diffraction exchange of laser radiation between channels inside the ACE, which also represents an intraloop amplifier in each individual laser channel. The reflection coefficients of beamsplitters B2 and B3 directing a part of radiation to the second and third channels, respectively, are selected to provide the uniform distribution of radiation over laser channels. Upon summation of radiation from laser channels to one collinear beam according to the scheme in Fig. 1, we obtain the reflection coefficient $R_i = 1/i$, where i is the number of a laser channel. Then, the reflection coefficients R_2 and R_3 of beamsplitters B2 and B3 are equal to 1/2 and 1/3, respectively.

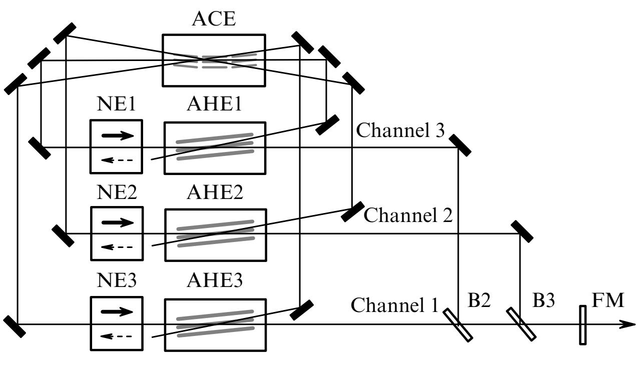

Figure 1. Optical scheme of the three-channel laser system: AHE1, 2, 3 are active holographic Nd : YAG elements; ACE is the active interchannel coupling Nd : YAG element; NE1, 2, 3 are nonreciprocal Faraday elements; B2, 3 are beamsplitters for the second and third laser channels; FM is the feedback mirror.

Let us construct a mathematical model of the oscillation dynamics for this laser. We assume that not only the eigenmode is developed in each channel but a part of radiation is also injected from one channel to another ('foreign' mode is injected) through the ACE. We will seek the regime of generation of phase-locked single-frequency radiation upon mode selection. In this connection we can neglect the phase relations responsible for mode beats and, assuming a strong selection of longitudinal modes, will consider only the generation of the mode of the control (strong) channel [1]. The possibility of the existence of the 'foreign' frequency in channels is explained by the self-tuning of the lengths of dynamic resonators to this frequency [12], which is inherent in self-phase-conjugation (SPC) gain-grating lasers.

It follows from the optical scheme that the linear coupling between channels via beamsplitters is performed only to write SPC mirrors in all the active elements by the degenerate waves of the control channel. Therefore, only the frequency-degenerate multiwave interaction is taken into account. Let us write the systems of coupled equations describing the DMWM dynamics on mixed holographic gratings in the active medium of a laser. The system of equations [13] for the AHE has the form

$$\begin{aligned}
&\left(\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_1 = aE_1 + b_1E_4,\\
&\left(-\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_2 = aE_2,\\
&\left(-\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_3 = aE_3,\\
&\left(\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_4 = aE_4 + b_1E_1.
\end{aligned} \quad (12)$$

For the ACE, the system of equations has the form

$$\begin{aligned}
&\left(\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_1 = aE_1 + b_1E_6 + b_2E_4,\\
&\left(-\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_2 = aE_2,\\
&\left(-\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_3 = aE_3,\\
&\left(\frac{\partial}{\partial z} + \upsilon^{-1}\frac{\partial}{\partial t}\right)E_4 = aE_4 + b_2E_1 + b_3E_6,
\end{aligned} \quad (13)$$

$$\left(-\frac{\partial}{\partial z}+v^{-1}\frac{\partial}{\partial t}\right)E_5 = aE_5,$$

$$\left(\frac{\partial}{\partial z}+v^{-1}\frac{\partial}{\partial t}\right)E_6 = aE_6 + b_1E_1 + b_3E_4.$$

Here, v is the speed of light; E_{1-6} are the moduli of slowly varying amplitudes of plane waves involved in DMWM; b_1 is the coupling coefficient describing the efficiency of a hologram written by a pair of waves E_2 and E_3 propagating in the negative $(-z)$ direction [the direction of lasing is taken as the positive $(+z)$ direction]. The system of equations (13) for the ACE contains two additional equations compared to (12) because holograms are written by all combinations of pairs of the waves propagating in the negative direction: pairs of waves E_2 and E_3 (the coupling coefficient b_1 describes coupling between channels 1 and 2), E_2 and E_5 (b_2 describes coupling between channels 1 and 3), E_3 and E_5 (b_3 describes coupling between channels 2 and 3).

The systems of equations (12) and (13) can be used to study the dynamics of the development of diffraction-coupled lasing in a three-channel laser system by specifying the boundary conditions corresponding to the optical scheme in Fig. 1.

Consider the synchronisation conditions for laser channels when the gain mismatch $\Delta\alpha_{ij}$ between the ith and jth channels is introduced. Let us introduce the synchronisation (pulse overlap) factor f_{ij} describing the overlap of the output laser pulses for each pair of channels 1 and 2, 1 and 3, and 2 and 3 [1]:

$$f_{ij} = \frac{\int 2(I_iI_j)^{1/2}\mathrm{d}t}{\int (I_i + I_j)\mathrm{d}t}, \qquad (14)$$

where I_i and I_j are the output pulse intensities at a certain instant in channels i and j, respectively. The synchronisation factor normalised to unity ($f_{ij} = 1$) corresponds to the complete synchronisation (overlap) of the ith and jth pulses, while $f_{ij} = 0$ corresponds to their complete mismatch.

Figure 2 presents the results of the numerical solution demonstrating the dynamics of the development of the laser pulse I_i in channels 1, 2, and 3 of the laser system and diffraction efficiencies η_i of holograms in the ith laser channel (in the AHE), as well as diffraction efficiencies of holograms (in the ACE) for coupling channels 1 and 2 (η_{12}), 1 and 3 (η_{13}), and 2 and 3 (η_{23}). The system had the following input parameters: the active element length was $L = 10$ cm, the initial field-amplitude gain in the AHE1 and ACE was $\alpha_1 = 0.205$ cm^{-1} (single-pass small-signal gain $G_0 = 60.3$), the gain mismatch in laser channels was $\Delta\alpha_{12} = \Delta\alpha_{23} = 0.003$ cm^{-1}, $\Delta\alpha_{13} = \Delta\alpha_{12} + \Delta\alpha_{23} = 0.006$ cm^{-1}, i.e. the initial gain in the AHE2 was $\alpha_2 = 0.202$ cm^{-1} ($G_0 = 56.8$), and the initial gain in the AHE3 was $\alpha_3 = 0.199$ cm^{-1} ($G_0 = 53.5$); the contrast of the nonreciprocal element was $K = T_{\rm NR}/t_{\rm NR} = 80$ ($T_{\rm NR} = 0.8$ and $t_{\rm NR} = 0.01$ are the transmission coefficients of the nonreciprocal element in open and closed directions, respectively; the reflection coefficient of the output feedback mirror was $R = 0.05$, and the length of each resonator was 3 m.

The radiation intensity is shown in Fig. 2 for pulses in each channel at the laser output, i.e. incident on the feedback mirror. One can see that the output pulses of the laser system are shifted in time with respect to each other. In other words, in the presence of even such a small gain mismatch as $\Delta\alpha_{ij}$, the time delay between pulses in the ith and jth channels reduces the efficiency of their interference summation. In this case, the synchronisation factors between channels (14) are $f_{12} = 0.94$, $f_{13} = 0.81$, and $f_{23} = 0.96$.

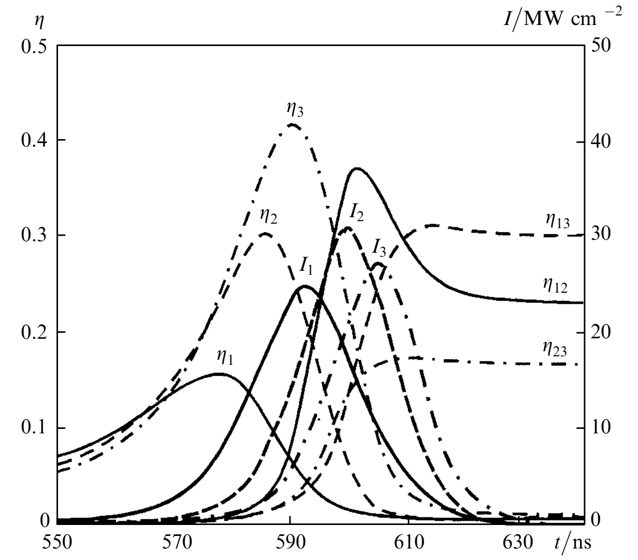

Figure 2. Calculated diagrams of the lasing dynamics for the relative gain mismatch in lasing channels $\Delta\alpha_{12} = \Delta\alpha_{23} = 0.003$ cm^{-1} and $\Delta\alpha_{13} = \Delta\alpha_{12} + \Delta\alpha_{23} = 0.006$ cm^{-1}; $\eta_1 - \eta_3$ are the diffraction efficiencies of holograms in the AHE1, AHE2, and AHE3, respectively; η_{12} is the diffraction efficiency in the ACE of coupling holograms in the first and second channels; η_{13} is the diffraction efficiency in the ACE of coupling holograms in the first and third channels; η_{23} is the diffraction efficiency in the ACE of coupling holograms of the second and third channels; $I_1 - I_3$ are radiation intensities in the first, second, and third channels, respectively.

It also follows from Fig. 2 that for equal relative gain mismatches between channels 1, 2 and 2, 3 ($\Delta\alpha_{12} = \Delta\alpha_{23}$), the shift $\Delta t_{12} = 8$ ns of laser pulses of the control (first) and controlled (second) channel nearest in gain exceeds the shift $\Delta t_{23} = 3$ ns for a pair of controlled second and third channels. This means that the first laser channel is the control channel. It triggers lasing in controlled (second and third) laser channels and provides their better synchronisation due to the long-range coupling of laser channels because controlled channels are coupled not with the control channel but also with each other. In this case, the peak intensity of radiation I_1 in the first (control) channel with a higher gain is lower than the peak intensities of radiation I_2 and I_3 in the second and third (controlled) channels with a lower gain (see Fig. 2). This is explained by the fact that in the case of a long-range coupling between laser channels, due to the high diffraction efficiency η_{12} and η_{13} of coupling holograms, radiation injected from the first channel to the second and third channels intensifies the development of lasing in them, resulting in a more rapid increase in the diffraction efficiencies η_2 and η_3 of saturation-gain gratings in AHEs 2 and 3 and a more intense nonlinear stage of the development of lasing with peak intensities $I_{2,3} > I_1$. The maximum of the diffraction efficiency η_3 proves to be greater than the maximum of the diffraction efficiency η_2, which is explained by the additional injection of radiation from the second to third channel due to the coupling between controlled channels with the diffraction efficiency η_{23}. This provides a decrease in the shift of pulses Δt_{23} compared to Δt_{12}, i.e. a better synchronisation of controlled channels. However, the gain $\alpha_3 < \alpha_2$, and for this reason, due to the development of lasing, the pulse intensities satisfy the inequality $I_3 < I_2$.

We found that the time shift Δt_{23} of a pair of the output pulses of controlled laser channels depends not only on the gain mismatch $\Delta\alpha_{23}$ but also on the gain mismatch $\Delta\alpha_{12}$ of controlled channels with respect to the control channel. Let us introduce the critical overlap factor $f_{\mathrm{cr}\,ij} = 0.5$ for each pair of output laser pulses. In the case of Gaussian radiation pulses, their relative time shift is $\Delta t_{ij} = \tau$, where τ is the full width at the half-maximum of the pulse. Then, we can determine the critical values of the gain mismatch $\Delta\alpha_{\mathrm{cr}23}$ for $f_{\mathrm{cr}23}$ depending on the gain mismatch $\Delta\alpha_{12}$ in another pair of channels.

Figure 3 shows the calculated dependence of the critical mismatch $\Delta\alpha_{\mathrm{cr}23}$ of the gain between the second and third channels, which corresponds to the criterion $f_{\mathrm{cr}23} = 0.5$, on the gain mismatch $\Delta\alpha_{12}$ between the first and second channels. The last value $\Delta\alpha_{12} = 0.017$ cm^{-1} in the plot corresponds to the critical gain mismatch $\Delta\alpha_{\mathrm{cr}12}$ between the first and second channels (when f_{12} decreases down to $f_{\mathrm{cr}12}$). One can see that critical values $\Delta\alpha_{\mathrm{cr}23}$ exceed $\Delta\alpha_{\mathrm{cr}12}$ for any mismatch. As $\Delta\alpha_{12}$ is increased, the value of the admissible critical mismatch $\Delta\alpha_{\mathrm{cr}23}$ increases, which indicates that the requirements to the relative equating of the gain in controlled channels are reduced. This becomes an important advantage of the laser system with increasing the number of channels, when its output energy is mainly determined by a set of controlled laser channels.

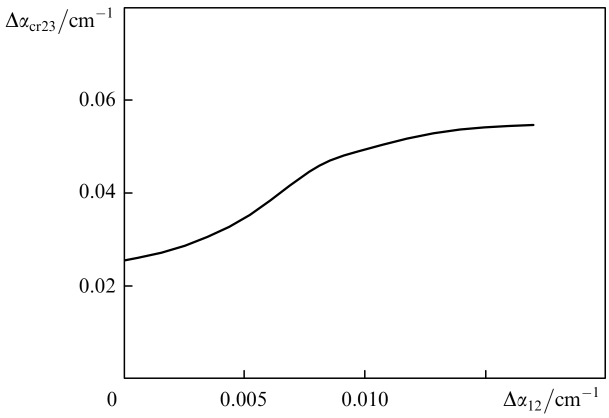

Figure 3. Calculated dependence of the critical gain mismatch $\Delta\alpha_{\mathrm{cr}23}$ between controlled (second and third) channels (critical mismatch corresponds to the criterion $f_{\mathrm{cr}23} = 0.5$) on the gain mismatch $\Delta\alpha_{12}$ between the control (first) and controlled (second) channels.

In the case of the maximum (critical) value $\Delta\alpha_{12} = 0.017$ cm^{-1}, the admissible ratio of single-pass small-signal gains in controlled channels can achieve even three [$\exp(2 \times 0.055L) = 3$]. This can be explained by the exponential growth of the diffraction efficiency η_{ij} of the coupling holograms of channels (see Fig. 2) after the appearance of lasing in the first (control) channel.

Thus, in the case of a long-range coupling in a multichannel laser system, it is most important to match the gains only between the control channel and a set of controlled laser channels. However, as follows from calculations, as the number of channels is further increased, this requirement becomes less strict. The small-signal gain can differ already by a factor of 1.4 [$\exp(2 \times \Delta\alpha_{\mathrm{cr}12}L) = 1.4$] because in the case of a large gain mismatch $\Delta\alpha_{12}$ between the control and controlled lasers, the value $\Delta\alpha_{\mathrm{cr}23}$ increases and controlled channels can be better synchronised with each other. In addition, the increase in the number of channels of a holographic laser system results in a stronger dependence of the radiation parameters of controlled laser channels on the radiation parameters of the control laser channel. This expands the range of the self-phase-locking of a parallel multichannel holographic laser system with a long-range coupling.

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