Quantum Electronics **37** (2) 173 – 180 (2007)

PACS numbers: 42.60.Da; 42.60.Fc; 42.55.Rz
DOI: 10.1070/QE2007v037n02ABEH013274

Peculiarities of the fundamental mode structure in stable-resonator lasers upon spatially inhomogeneous amplification

M.V. Gorbunkov, P.V. Kostryukov, L.S. Telegin, V.G. Tunkin, D.V. Yakovlev

***Abstract*. The structure of the fundamental mode of a laser is calculated by the iteration Fox – Li method in the case of inhomogeneous unsaturated amplification produced by axially symmetric longitudinal pumping. The calculation is performed for different parameters g_1 and g_2 of the resonator within the entire stability region. It is shown that in the case of inhomogeneous amplification, the fundamental mode considerably deviates from the Gaussian mode of an empty resonator only in the so-called critical configurations of the resonator, when the quantity $[\arccos(g_1 g_2)^{1/2}]/\pi$ is zero or takes a number of values expressed by irreducible fractions m/n. For the Fresnel number $N_F = 9$, configurations with $m/n = 1/2$, 2/5, 3/8, 1/3, 3/10, 1/4, 1/5, 1/6, 1/8, and 1/10 are pronounced. As N_F increases, the number of critical configurations increases. The expansion in a system of Laguerre – Gaussian beams shows that the fundamental mode in critical configurations is formed by a set of beams with certain radial indices p phased in the active medium.**

Keywords: *longitudinal pumping of lasers, stable resonators, fundamental mode, transverse mode locking.*

1. Introduction

The use of longitudinal pumping, for example, diode pumping of solid-state lasers provides a high excitation efficiency due to the possibility of forming the required spatial distribution of the inversed population. The problem of increasing the efficiency of solid-state lasers due to the optimal matching of the spatial distribution of the longitudinal pump intensity and the fundamental mode was studied in a number of papers [1 – 5], where it was assumed that the resonator mode was Gaussian upon inhomogeneous pumping as well. It was shown theoretically that the lasing efficiency increased and lasing threshold decreased with increasing the parameter $\xi = w/\rho$, where w is the radius of a mode of an empty resonator and ρ is the radius of the amplification region. The efficiency of inhomogeneous pumping ($\xi > 1$) was experimentally confirmed in [5, 6]. In earlier paper [5], an active Nd : YAG crystal was longitudinally pumped by a krypton laser. As ξ increased from ~ 2 to ~ 2.5, the lasing efficiency increased from 26 % to 40 %. The question arises of whether or not this increase in the lasing efficiency is accompanied by the distortion of the spatial structure of laser radiation.

Of course, the lasing efficiency should be estimated taking into account the gain saturation in the active medium. Therefore, strictly speaking, the problem of optimisation of the lasing efficiency should be solved by considering the laser field taking saturation into account. Nevertheless, it is reasonable to calculate the laser field taking into account the profiled amplification but without saturation. Saturation is virtually absent, for example, in repetitively pulsed picosecond lasers operating in the regime of radiation stabilisation with an external negative feedback (NFB) [7]. In this case, the NFB causes the generation of low-energy picosecond pulses (a long train of picosecond pulses). Saturation can be neglected when the NFB circuit is sufficiently sensitive.

The noticeable differences in the spatial structure of radiation of a longitudinally diode-pumped laser ($\xi \approx 5$) from the Gaussian mode of an empty resonator were experimentally observed in [8]. A Nd : YVO_4 crystal was used; the resonator was formed by a plane mirror on the active medium surface and the output spherical mirror with the radius of curvature 80 mm. The resonator length d was varied from 59 to 64 mm. The far- and near-field spatial radiation distributions were measured behind the spherical mirror. When the value of d approached the so-called critical value ($\sim$60 mm), at which the output power decreased and the radiation intensity profile became considerably different from the Gaussian mode of an empty resonator, the near-field spatial intensity distribution narrowed down, while in the far field a complex ring structure was observed. It was found in [8] that the parameters $g_1 = 1 - d/R_1$ and $g_2 = 1 - d/R_2$ of the resonator in the critical configuration satisfy the relation $g_1 g_2 = 0.25$, which corresponds to the frequency degeneracy of the transverse modes of the resonator.

The structure of the fundamental radiation mode was studied in more detail in [9] for an inhomogeneously longitudinally diode-pumped chip Nd : YVO_4 laser. As the resonator length was changed, nearly Gaussian far-field intensity distributions were predominantly observed. However, for some discrete set of resonator lengths, the radiation intensity distribution acquired a ring shape. The authors of [9] supplemented their experimental data by calculations of the mode structure performed by expanding the field in a set of sixty Laguerre – Gaussian beam modes.

P.V. Kostryukov, L.S. Telegin, V.G. Tunkin, D.V. Yakovlev Department of Physics and International Laser Center, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119992 Moscow, Russia;
e-mail: vgtunkin@mail.ru;
M.V. Gorbunkov P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia;
e-mail: vgorbunk@sci.lebedev.ru

Received 31 May 2006; revision received 29 August 2006
Kvantovaya Elektronika **37** (2) 173 – 180 (2006)
Translated by M.N. Sapozhnikov

A direct method for calculating the modes of lasers, in particular, containing elements with an arbitrary and (or) amplitude transfer function is the iteration Fox – Li method [10]. Distortions of the fundamental mode, appearing upon inhomogeneous amplification due to the saturation of the active medium [11] and upon inhomogeneous pumping of a solid-state laser [12], were calculated by the iteration method as early as 1965. It was shown [11] that the mode remained Gaussian when saturation was weak, whereas in the case of sufficiently strong saturation, the mode amplitude preserved the Gaussian shape but strong phase distortions appeared. A similar result was obtained in [12], where it was found that the inhomogeneity of pumping weakly affected the amplitude distribution of the fundamental mode but the phase was distorted noticeably.

The Fox – Li calculations of the modes of resonators with elements described by inhomogeneous transfer functions again attracted attention due to the development of diode-pumped solid-state lasers. The iteration method was used to analyse distortions of the fundamental mode caused by the nonspherical phase transfer function of an aberration thermal lens produced in the active medium [13]. The influence of the inhomogeneous amplification ($\xi \approx 5$) on the fundamental mode of lasers with a plane – spherical resonator (active medium on a plane mirror) was studied in [14]. It was found that in the case of a semi-confocal resonator ($g_1 = 0.5$, $g_2 = 1$), the inhomogeneity of pumping leads to the ring structure of the fundamental mode at the output spherical mirror. Below, we will denote the configuration of a resonator in the form (g_1, g_2). The authors of [15] analysed numerically mode distortions produced in an inhomogeneously pumped $(0.25, 1)$ plane – spherical resonator taking into account saturation. The radiation field distributions on the output end of the active medium with a plane mirror deposited on it were obtained. The calculated intensity distribution consists of a narrow axial peak and a broad pedestal. The pedestal intensity is two – three orders of magnitude lower than that of the peak. The central maximum is $\sim$3.5 times narrower than the empty resonator mode. It is shown that for the selected Fresnel number, the field distribution (amplitude and phase) can be described with good accuracy by a linear combination of phased Laguerre – Gaussian modes with transverse indices $p = 0, 3, 6, \ldots, 18$ and $l = 0$, whose set is frequency-degenerate for $g_1 g_2 = 0.25$.

In [16], an attempt was made to consider in general the spatial structure of the fundamental mode of a laser pumped by a Gaussian beam for a whole family of resonator configurations. The quality of the fundamental mode was characterised by two integral parameters, M^2 and the Petermann K factor [17]. Plane – spherical resonators were studied for which the frequency degeneracy condition

$$\arccos (g_1 g_2)^{1/2} = \pi \frac{m}{n} \tag{1}$$

was fulfilled, where m/n is an irreducible fraction. The consideration was restricted to the cases $m/n = 1/x$, where $2.7 \leqslant x \leqslant 9$. The dependence of M^2 on x was calculated. For $x = 3$, 5, 6, and 8, the increase in M^2 by a few times was observed, and for $x = 4$ – by a factor of $\sim$20. Calculations performed in broad ranges of the gains (no less than 1.1) and parameters ξ (no more than 10) showed that for $x = 4$, the phase of the fundamental mode on a plane mirror has a constant value.

The spatial structure of the fundamental mode field in the case of inhomogeneous amplification was calculated in the papers cited above for individual plane – spherical resonators and fixed Fresnel numbers N_{F}. Strong distortions of the spatial structure of the fundamental mode of the (0.25, 1) resonator upon inhomogeneous amplification was explained in [15] by the synchronisation of the frequency-degenerate modes of an empty resonator. However, the number of degenerate configurations is equal to the number of irreducible fractions m/n. The reasonable question arises: For which resonator configurations the spatially inhomogeneous amplification leads to dramatic distortions of the fundamental mode? In addition, whether or not the values of m and n are related to the type of distortions of the amplitude and phase of the fundamental mode? And finally, how efficiently affects a change in the Fresnel number the degree and type of distortions of the fundamental mode?

These questions require a detailed analysis of the fundamental mode structure depending on the parameters g_1 and g_2 within the entire stability region of resonators taking into account the influence of the Fresnel number. This paper is devoted to the study of the effect of the transversely inhomogeneous unsaturated amplification in a thin active medium on the spatial structure of the fundamental mode of lasers within the entire stability region of resonators. The fundamental mode was calculated by the iteration Fox – Li method.

2. The model and results of calculations

The propagation of an axially symmetric light beam in a free space, whose field is independent of the azimuthal angle, is described by the expression

$$u'(r') = \frac{\mathrm{i}\pi}{\lambda d} \exp(-\mathrm{i}kd) \times \int_0^a J_0\left(\frac{kr'r}{d}\right) \exp\left[-\frac{\mathrm{i}k(r'^2 + r^2)}{2d}\right] u(r) r \mathrm{d}r, \tag{2}$$

where $J_0(x)$ is the zero-order Bessel function; $u(r)$ and $u'(r)$ are the complex amplitudes of the field in front and behind a mirror, respectively; r and r' are the coordinates in the input and output planes; d is the distance between the planes; a is the aperture radius; $k = 2\pi/\lambda$; and λ is the wavelength. Expression (2) is obtained in the approximation $a^2/(d\lambda) \ll (d/a)^2$ [10]. Upon reflection from the mirror, the transformation

$$u'(r) = u(r) \exp\left(-\frac{\mathrm{i}kr^2}{R}\right) \tag{3}$$

takes place, where R is the radius of curvature of the mirror.

As in other papers devoted to calculations of the radiation parameters of longitudinally diode-pumped lasers [9, 13, 15], the active medium was assumed thin. The transformation of the complex amplitude of such an active medium has the form

$$u'(r) = u(r) K(r), \tag{4}$$

where $K(r)$ is the amplitude gain per pass.

An important feature of the fundamental mode is its diffraction losses. They can be determined by calculating the power gains G_+ and G_- for a beam propagated forward and backward in the active medium, respectively:

$$G_{+,-} = \frac{\int_0^a |u'_{+,-}(r)|^2 r\mathrm{d}r}{\int_0^a |u_{+,-}(r)|^2 r\mathrm{d}r}, \quad (5)$$

where $u_{+,-}(r)$ and $u'_{+,-}(r)$ are the complex amplitudes of the field in front and behind the active medium, respectively, and signs '+' and '−' correspond to the forward and backward passes in the resonator, respectively. The obtained values of G_+ and G_- and the total power gain G per round-trip transit in the resonator give the diffraction losses

$$\gamma = 1 - \frac{G}{G_+ G_-}. \quad (6)$$

Expressions (2)–(4) allow one to construct a chain of transformation of the complex amplitude corresponding to the round-trip transit in the resonator – to one iteration. The homogeneous distribution of the complex amplitude is specified as the initial distribution on one of the resonator mirrors. Iterations are repeated until the spatial field distribution is reproduced with the required accuracy. It is this field distribution that represents the fundamental mode [10, 18]. The production of a self-reproducible distribution can be judged, for example, by a decrease in the relative change in diffraction losses from iteration to iteration [19]. In our case, the calculation was stopped when the relative change in diffraction losses was reduced down to $\sim 10^{-10}$.

The fundamental resonator mode was calculated for stable resonators with different values of parameters g_1 and g_2 (Fig. 1). The radii a of mirrors were assumed equal. The active medium was located close to mirror M2. The fundamental mode field was calculated in three planes: behind mirror M1 [$u_1^0(r)$], in front of the active medium [$u_2^0(r)$], and behind it [$u_3^0(r)$]. The gain of the active medium was specified in the form

$$K(r) = 1 + (K_0 - 1)\exp\left(-\frac{r^2}{\rho^2}\right). \quad (7)$$

Note that the gain distribution close to (7) is obtained not only by using radiation from individual diodes [9] or other laser radiation [5] but also upon the transport of radiation from diode arrays in optical fibres [7]. Calculations were also performed for the transfer function corresponding to the Gaussian aperture, for which $K(r) = \exp(-r^2/\rho^2)$. The parameter $\xi = w_2/\rho$ (where w_2 is the radius of the Gaussian mode on mirror M3 in an empty resonator) was set equal to 3, which is close to the value at which highly efficient lasing was observed in [6]. The fundamental mode of a laser with a spatially inhomogeneous amplitude transfer function was calculated by assuming that the axial power gain K_0^2 per pass in the active medium was 1.5^2. Such a gain was achieved, for example, in a picosecond Nd:YAG laser pumped by a beam of diameter 0.8 mm from a 35-W diode array [7].

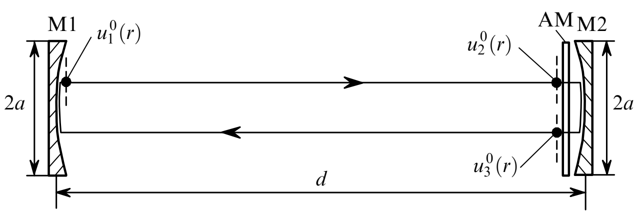

Figure 1. Scheme of the laser resonator: (M1, M2) mirrors; (AM) active medium.

In our case, it is reasonable to compare the fundamental resonator mode with the Gaussian mode of an empty resonator with the help of the parameter

$$\beta_0 = 2\pi \int u_G^*(r) u_m(r) r\mathrm{d}r, \quad (8)$$

where $u_G(r)$ and $u_m(r)$ are the field amplitudes of the Gaussian mode of the empty resonator and the fundamental mode of the resonator with the inhomogeneous amplification satisfying the normalisation conditions

$$2\pi \int |u_G(r)|^2 r\mathrm{d}r = 2\pi \int |u_m(r)|^2 r\mathrm{d}r = 1. \quad (9)$$

The quantity $|\beta_0|^2$ is the fraction of the power of the non-Gaussian fundamental mode contained in the Gaussian mode of the empty resonator. Integral (8) is calculated in the input plane of the active medium. Note that the parameter $|\beta_0|^2$ used here differs from the parameter $|\alpha_0|^2$ proposed in [20], which represents the fraction of energy contained in a Gaussian beam approximating the initial beam with the minimum root-mean-square deviation.

The non-Gaussian distribution of the fundamental mode in the case of inhomogeneous amplification is illustrated in Fig. 2 for different resonator configurations. The values of $|\beta_0|^2$ for $N_F = 9$, $K_0 = 1.5$, and $\xi = 3$ are represented in the coordinates g_1 and g_2 for their positive values in the stability region ($0 < g_1 g_2 < 1$). We will call critical the resonator configurations for which the values of $|\beta_0|^2$ are noticeably lower than unity, which clearly demonstrates the difference of the fundamental mode from the Gaussian mode of the empty resonator. One can see from Fig. 2 that regions near the coordinate axes g_1 and g_2 and on a number of curves representing the parts of hyperbolas correspond to the pronounced critical configurations. The values of $|\beta_0|^2$ considerably decrease for hyperbolas $g_1 g_2 = 0.096$, 0.146, 0.25, 0.346, 0.5, 0.655, 0.75, 0.854, 0.905, and 1.

The results of calculations have shown that $|\beta_0|^2$ does not change when the signs of g_1 and g_2 are changed

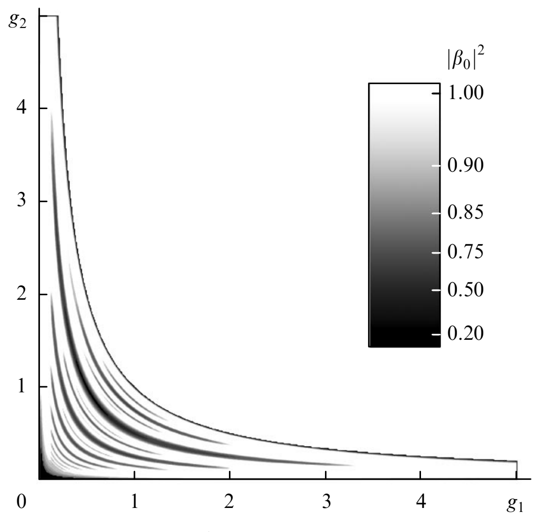

Figure 2. Dependences of $|\beta_0|^2$ on g_1 and g_2 for $N_F = 9$, $K_0 = 1.5$, and $\xi = 3$.

simultaneously. This is consistent with the results of theoretical papers [21, 22], in which the question of the equivalence of resonators with an intracavity element described by the real non-Gaussian transfer function was considered for a rigid aperture [21] and in the general case in [22]. It was shown that when the signs of parameters g_1 and g_2 were simultaneously changed, the distributions of the mode amplitude on mirrors change to complex conjugate distributions. The value of $|\beta_0|^2$ is preserved upon complex conjugation, and therefore the dependence of $|\beta_0|^2$ on (g_1, g_2) is symmetric with respect to the point (0, 0).

The dependence of $|\beta_0|^2$ on the configuration of plane–spherical ($g_2 = 1$) resonators for $K_0 = 1.5$, $\xi = 3$ and $N_F = 3$, 5, 30 are shown in Fig. 3a. For $N_F = 3$, the three critical configurations with $g_1 g_2 = 0$, 0.5, and 1 are most pronounced. For $N_F = 5$, the number of critical configurations increases up to 5 due to the addition of configurations with $g_1 g_2 = 0.25$ and 0.75. As N_F is increased up to 30, the number of critical configurations increases approximately by an order of magnitude. The similar dependences of $|\beta_0|^2$ on g_1 for symmetric ($g_1 = g_2$) resonators are shown in Fig. 3b. Here, a peculiarity near the confocal configuration is most pronounced. As N_F is increased, the number of critical configurations also increases. Calculations performed for a Gaussian aperture showed the absence of peaks in the dependence of $|\beta_0|^2$ on g_1 (dot-and-dash lines in Fig. 3).

Consider the distribution of the amplitudes and phases of the fields $u_1^0(r)$, $u_2^0(r)$, and $u_3^0(r)$ of the fundamental mode of resonators with characteristic critical configurations. Figure 4 shows the structure of the fundamental mode of the confocal and plane–spherical resonators of different critical configurations for $N_F = 30$, $K_0 = 1.5$, and $\xi = 3$. The distributions of the phase of the fundamental mode on the surface of mirrors are also presented, i.e. the differences of the phase distributions from the parabolic distribution corresponding to the mode of an empty resonator are shown in fact. The dashed curves show the distributions of the Gaussian mode amplitude of an empty resonator for resonators of the (0, 0) and (0.5, 1) configurations. This arrangement of the figures showing the mode structure will be used below.

The fundamental-mode amplitude distributions considerably differ from the amplitude distribution of the Gaussian mode of the empty resonator. For critical configurations, the distributions $|u_2^0(r)|$ consist of an axial peak and a rather broad pedestal. This peak is especially pronounced in the configurations (0, 0), (0.5, 1), (0.75, 1), and (0.25, 1). According to the type of the amplitude distribution $|u_1^0(r)|$ of the fundamental mode on mirror M2, resonators can be divided into two groups. Configurations for which $g_1 = g_2 = 0$, $g_1 g_2 = 0.5$ and 0.75 (Figs. 4a–c) belong to the first group. These configurations are characterised by substantial differences between the distributions $|u_1^0(r)|$ and $|u_3^0(r)|$. The width of the amplitude distribution of the fundamental mode in the confocal resonator on mirror M1 noticeably exceeds the width of the Gaussian mode of the empty resonator. The distribution $|u_1^0(r)|$ does not contain the axial peak. A weak ring structure of the pedestal in distributions $|u_2^0(r)|$ and $|u_3^0(r)|$ is accompanied by drastic phase jumps by $\pm\pi$ (Fig. 4a, column IV). Unlike the confocal configuration, the amplitude distributions on mirror M1 for other resonators of the first group have a ring structure, which is most pronounced for $g_1 g_2 = 0.5$ (Fig. 4b, column I).

The amplitude distributions of the fundamental mode for resonators of the second group (Figs 4d, e) are similar on both mirrors in coordinates normalised to the corresponding radii of the Gaussian mode of the empty resonator. As for the phase distribution, we can point out a dynamically stable resonator ($g_1 g_2 = 0.5$) as a special case, for which a constant phase distribution is realised on a plane mirror (Fig. 4b, column IV), in accordance with calculations performed in [16]. A constant phase distribution is also realised on mirror M1 in the case of the confocal resonator (Fig. 4a, column II). For resonators of other configurations, the phase on the mirror is not constant and depends in a complicated way on the distance from the resonator axis.

3. Discussion

Substantial differences between the structures of the fundamental and Gaussian modes are caused by the peculiarities of the transfer function of the active medium in the case of inhomogeneous amplification (7). Of great importance is a finite value of the transfer function in the regions outward from the axis. Note that if the Gaussian aperture is used, the fundamental mode has the Gaussian

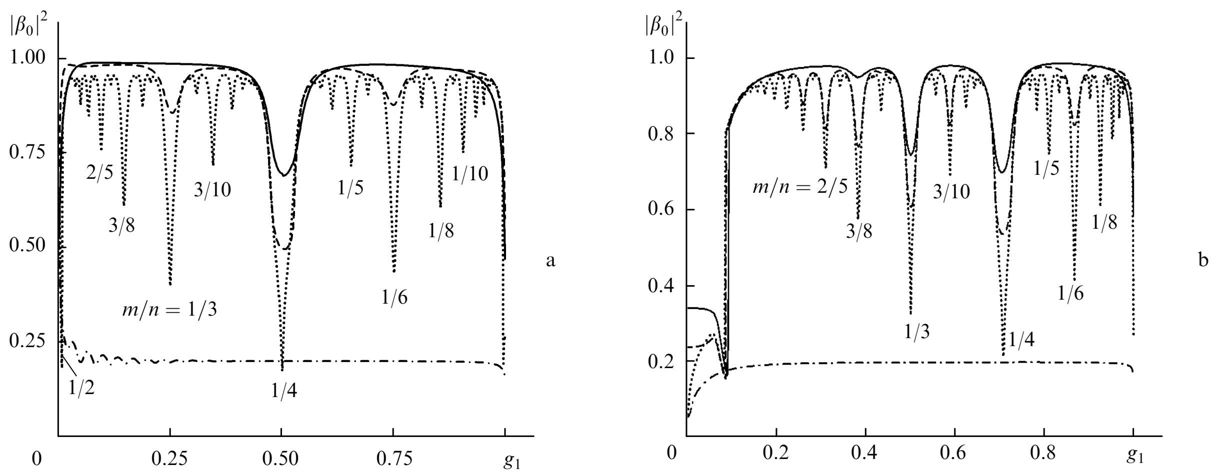

Figure 3. Dependences of $|\beta_0|^2$ on g_1 for plane–spherical ($g_2 = 1$; a) and symmetric ($g_1 = g_2$; b) resonators for $K_0 = 1.5$, $\xi = 3$, $N_F = 3$ (solid curves), 5 (dashed curves), 30 (dotted curves), and for a Gaussian aperture (dot-and-dash curves). Fractions $m/n = [\arccos(g_1 g_2)^{1/2}]/\pi$ are presented (see section 4).

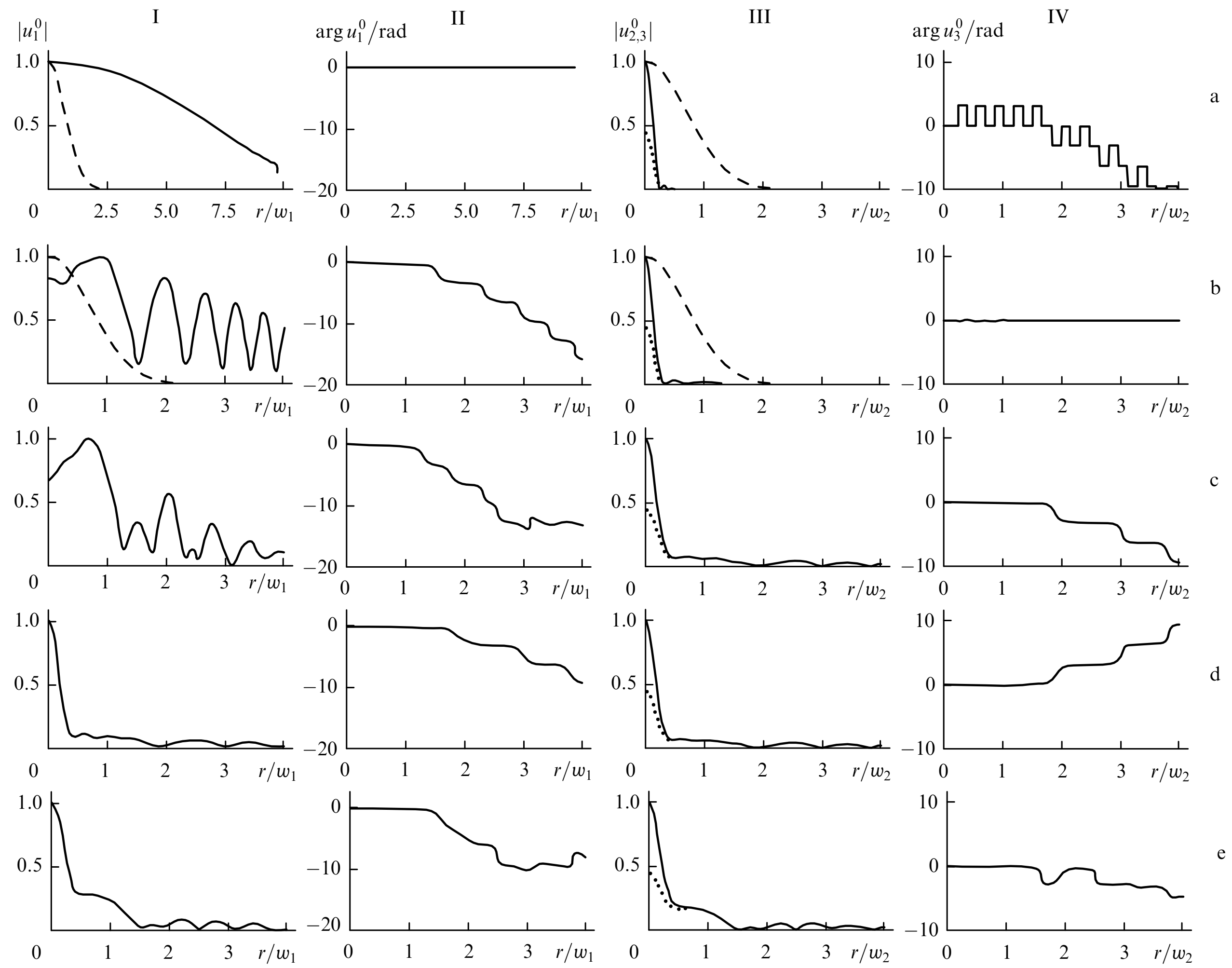

Figure 4. Distributions of the fundamental-mode amplitude and phase on surfaces of the mirrors of the confocal resonator of the (0, 0) configuration (a) and plane–spherical resonators of critical (0.5, 1) (b), (0.75, 1) (c), (0.25, 1) (d), and (0.6545, 1) (e) configurations; column I: $|u_1^0(r)|$; II: $\arg u_1^0(r)$; III: $|u_2^0(r)|$ (dashed curve), $|u_3^0(r)|$ (solid curve); IV: $\arg u_3^0(r)$. Dashed curves are the amplitudes of the Gaussian mode of an empty resonator; $w_{1,2}$ are the radii of the Gaussian mode on mirrors M1 and M2, respectively; $N_{\mathrm{F}} = 30$, $K_0 = 1.5$ and $\xi = 3$.

structure in each section of the resonator, but the complex parameter differs from the value corresponding to the mode of an empty resonator (without aperture). In-homogeneous amplification leads to a different type of the fundamental mode. Our calculations show that in the case of inhomogeneous amplification, the structure of the fundamental mode considerably differs from that of the Gaussian mode of an empty resonator only in the case of certain (critical) configurations of the resonator. In critical configu-rations, as follows from the calculations, the fundamental-mode structure does not change qualitatively in a rather broad domain of variation of K_0 (from 1.01 to 2.5).

The examples of the most pronounced critical configurations of stable resonators are configurations with $g_1 g_2 = 0.5$, 0.25, and 0.75. The amplitude distributions $u_{2,3}^0(r)$ for these configurations exhibit the narrow axial peak located on a relatively broad pedestal. The width of the peak is considerably smaller than the width of the Gaussian mode of the empty resonator. It is clear that a set of the Laguerre–Gaussian beam modes forming the fundamental mode contains, in particular, higher-order beams. Another specific feature of critical configurations is the reproduction of the mode amplitude distribution: the preservation of the peak and the features of the pedestal during the propagation of radiation over the part of the resonator not containing the active medium, i.e. from the active medium to mirror M1 and backward (Fig. 4). This means that the differences of the phase shifts of the Laguerre–Gaussian beams forming the fundamental mode are multiple of 2π.

If we choose the value of the complex parameter of a set of the Laguerre–Gaussian beams that is realised on mirror M2 in the absence of amplification, the expression for the phase shift ψ_{pl} of a beam with the radial and angular indices p and l, propagated in the part of a stable resonator ($0 < g_1 g_2 < 1$) not containing the active medium, has the form (see, for example, [23])

$$\psi_{pl} = 2kd - 2(2p + l + 1)\arccos\left[\pm (g_1 g_2)^{1/2}\right]. \qquad (10)$$

The sign at the radical is chosen to coincide with the sign of $g_1(g_2)$. To form a set of the Laguerre–Gaussian beams in such a way that the self-reproducibility condition (differences of the phase shifts $\psi_{pl} - \psi_{p'l'}$ are multiple of 2π) will be satisfied for their arbitrary superposition is possible only if $\arccos (g_1 g_2)^{1/2} = \pi m/n$. In this case, the expression for the phase shifts of the Laguerre–Gaussian beams has the form

$$\psi_{pl} = 2\pi\left[\frac{kd}{\pi} - (2p + l + 1)\frac{m}{n}\right]. \quad (11)$$

We consider here the fields independent of the azimuthal angle, and therefore the angular index l will be further assumed zero. The differences of phase shifts will be multiple of 2π for the beams with indices p different by $\Delta p = n$ for odd n or by $\Delta p = n/2$ for even n. An arbitrary combination of the beams of such a set in the absence of losses completely reproduces itself after the propagation over the part of the resonator not containing the active medium. To find out a set of the Laguerre–Gaussian beams forming the fundamental mode of the laser in the case of inhomogeneous amplification, we expanded the fundamental mode in the Laguerre–Gaussian functions

$$u_3^0(r) = \sum_p \beta_p u_p^{\mathrm{LG}}(r), \quad (12)$$

where

$$\beta_p = 2\pi \int u^*(r) u_p^{\mathrm{LG}}(r)\mathrm{d}r.$$

The values of $|\beta_p|^2$ are the fraction of the energy contained in the Laguerre–Gaussian beam with the index p.

Fractions m/n equal to 1/2, 2/5, 3/8, 1/3, 3/10, 1/4, 1/5, 1/6, 1/8, and 1/10 correspond to the most pronounced critical configurations with $g_1 = g_2 = 0$, $g_1 g_2 = 0.096$, 0.146, 0.25, 0.346, 0.5, 0.655, 0.75, 0.854, and 0.905. The values of m/n for these configurations are presented in Fig. 3a. Figures 5a–e present the values of $|\beta_p|^2$ and $\arg\beta_p$ for different critical configurations for $N_{\mathrm{F}} = 30$, $K_0 = 1.5$, and $\xi = 3$. Expansion (12) was performed for the field $u_2^0(r)$. In the case of a confocal resonator ($m/n = 1/2$), the expansion contains all the beams (Fig. 5a). For a resonator with $m/n = 1/4$, the terms corresponding to beams with even indices p are significant in expansion (12) (Fig. 5b). The fundamental mode of the resonator with $m/n = 1/3$ is predominantly formed by the beams with $p = 3j$, where $j = 0, 1, 2, 3, \ldots$ (Fig. 5c). For resonators with $m/n = 3/8$ and 2/5, the fundamental mode mainly consist of the beams with indices $p = 4j$ and $5j$, respectively (Figs 5e,f). For all the resonators studied, a set of the beams making a significant contribution to the fundamental mode is phased on mirror M3, i.e. the phase locking of the beam modes occurs.

Expansion (12) shows unambiguously that the fundamental mode in the critical configuration is predominantly formed by a set of the phased Laguerre–Gaussian beams with radial indices differing by $\Delta p = n$ for odd n and by $\Delta p = n/2$ for even n, which will be called below the basic set. In the case of even n for two adjacent beams of the basic set, the difference of phase shifts on passing from one mirror to another is $m\pi$, as follows from expression (11). If n is even, m should be odd, and if the fundamental mode on mirror M2 is the superposition $\sum_i \beta_{in/2} u_{in/2}^{\mathrm{LG}}(r)$ of beams, by neglecting edge effects on mirror M1, the mode will have the form $\sum_i (-1)^i \beta_{in/2} u_{in/2}^{\mathrm{LG}}(r)$. As a result, the amplitude distributions on the opposite mirrors should be noticeably different.

A prominent example of resonators with even n is a confocal resonator ($m/n = 1/2$). Indeed, as follows from expansion (12), the phases of the beams with even and odd p on mirror M1 for fields $u_1^0(r)$ and $u_{2,3}^0(r)$ differ by π, whereas all the beams are phased on mirror M2. This leads to a considerable difference in the amplitude distributions on the opposite mirrors. The Laguerre–Gaussian beams on mirror M2 are summed in phase, resulting in the formation of a narrow axial peak (Fig. 4a, column III). On mirror M1, the odd beams are summed with even beams out-of-phase, the width of the total amplitude distribution exceeding that of the zero Laguerre–Gaussian beam (Fig. 4a, column I). Noticeable differences in the amplitude distribution on

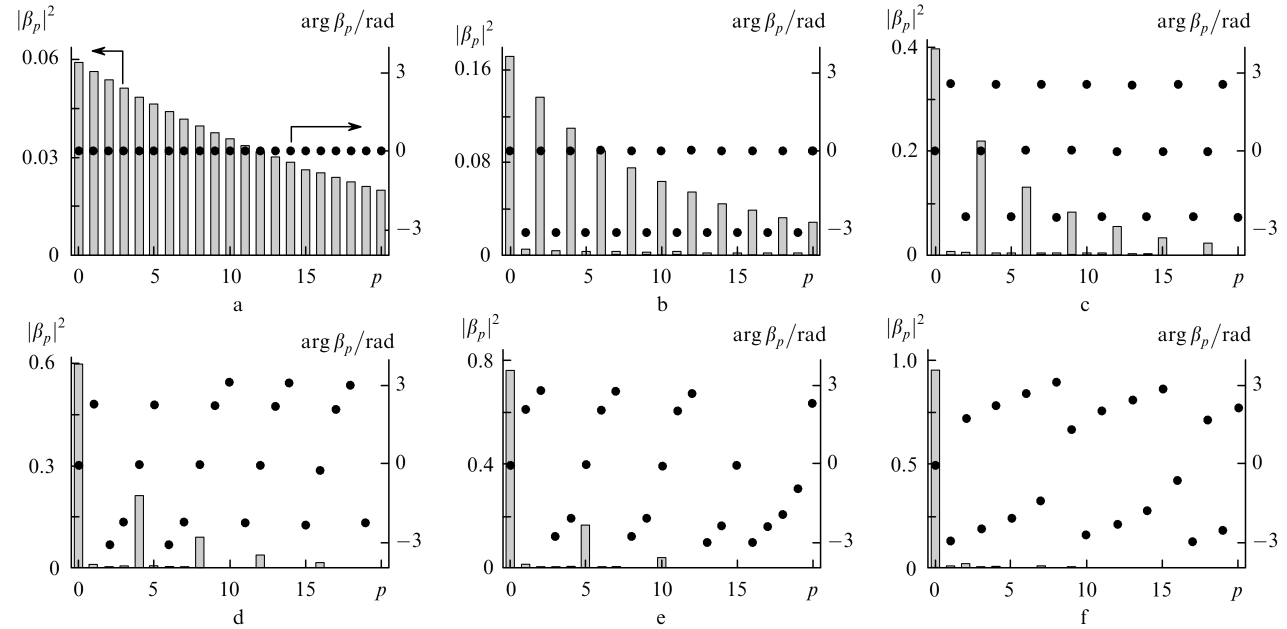

Figure 5. Coefficients of expansion of the field $u_2^0(r)$ in the Laguerre–Gaussian functions for resonators of critical configurations (0, 0) (a), (0.5, 1) (b), (0.25, 1) (c), (0.1464, 1) (d), (0.096, 1) (e), and the resonator of the noncritical (0.596, 1) configuration (f) for $m/n = 1/2$ (a), 1/4 (b), 1/3 (c), 3/8 (d), and 2/5 (e), $N_{\mathrm{F}} = 30$, $K_0 = 1.5$, $\xi = 3$. The vertical columns are $|\beta_p|^2$, circles are $\arg\beta_p$.

the opposite mirrors are also observed for resonators with $g_1 g_2 = 0.5$ ($m/n = 1/4$) and 0.75 ($m/n = 1/6$), whose modes are presented in Figs 4b, c, respectively. The amplitude distributions $u_{2,3}^0(r)$ exhibit the characteristic narrow axial peak on a broad pedestal while the amplitude distribution $u_1^0(r)$ on mirror M1 has a ring structure without the axial peak.

The difference of phase shifts of the Laguerre–Gaussian beams of the basic set for odd n in passing from one mirror to another is $2m\pi$, and therefore the mode amplitude distributions on the opposite mirrors should be similar. Indeed, fundamental modes in resonators with $g_1 g_2 = 0.25$ ($m/n = 1/3$) and 0.6545 ($m/n = 1/5$), shown in Figs 4d, e, respectively, have close amplitude distributions on the opposite mirrors.

Along with the beams of the basic set, expansion (12) (see Fig. 5) also contains other beams as a weak background. The calculation shows that for $g_1 g_2 = 0.5$ ($m/n = 1/4$), as K_0 is decreased from 1.5 to 1.05, the relative intensity of the background beams with odd p decreases from $\sim 10^{-2}$ to $\sim 10^{-4}$. For the value of $k = 2\pi/\lambda$ that is resonant for the basic set, the phase shifts of background beams after the round-trip transit in the resonator are not multiple of 2π. The background beams exist in the resonator due to the energy redistribution in the set of Laguerre–Gaussian beams during their propagation in the active medium. First, the energy exchange in the active medium together with edge effects on mirrors affects the formation of the stationary energy distribution over the beams, i.e. determines the values of β_p, and, second, it provides the multiplicity of the phase shifts of background beams per round-trip transit in the resonator to 2π. The required phase shift occurs due to the addition of fields from the phased basic set to the field of each of the background beams. For example, for $m/n = 1/4$, all the beams have the same phases at the output from the active medium, whereas phases of the beams of the basic set (with even p) and background beams (odd p) at the input to the active medium differ by π.

Aside from the properties of resonators with critical configurations considered above, note that the phase of the fundamental mode for resonators with $g_1 g_2 = 0.5$ ($m/n = 1/4$) on mirror M2 (Fig. 4b, column IV) and the confocal resonator ($m/n = 1/2$) on mirror M1 (Fig. 4a, column II) is constant. In the first case, the fundamental mode on mirror M2 consists predominantly of the phased Laguerre–Gaussian beams with even indices p. The mode of the confocal resonator on mirror M1 is formed by the Laguerre–Gaussian beams with phases differing by π for even and odd p. The constant phase distributions for confocal and semi-confocal resonators are related to the properties of the Laguerre–Gaussian functions: in a sum of the Laguerre–Gaussian beams with even p, phase jumps are absent even when $|\beta_{\rm p}|^2$ decreases comparatively slowly with increasing p. Similarly, phase jumps are also not observed for the difference of the sums of beams with even and odd p.

The properties of critical configurations discussed here are determined only by the product $g_1 g_2$. Therefore, during the movement along the hyperbola $g_1 g_2 = \text{const}$ by increasing one of the parameters g (and decreasing correspondingly another), the structure of the mode amplitude distributions on mirrors in coordinates normalised to the corresponding radii of the Gaussian mode of an empty resonator are preserved qualitatively. In the case of the sufficient deviation from the symmetric configuration ($g_1 = g_2$), diffraction effects on a mirror with the lower value of one of the parameters g prevent the creation of the synchronised set of the Laguerre–Gaussian beams. This is manifested in an increase in $|\beta_0|^2$ on hyperbolas $g_1 g_2 = \text{const}$ corresponding to critical configurations away from the symmetrical configuration (see Fig. 2).

As $N_{\rm F}$ is increased, the number of Laguerre–Gaussian beams, for which aperture effects are insignificant, increases. Thus, for large $N_{\rm F}$ in the case of inhomogeneous amplification, basic sets with large values of Δp can be manifested. This is reflected in the appearance of critical configurations to which fractions m/n with increasing values of n correspond.

The structure of the fundamental mode can be naturally made close to Gaussian by passing from the critical to noncritical configuration. The calculation shows that in the case of a moderate gain per round-trip transit in the resonator ($K_0 \leqslant 1.5$), for non-critical configurations the deviation of $|\beta_0|^2$ from unity does not exceed 10 %. The value of $|\beta_0|^2$ also increases with decreasing the Fresnel number; if the resonator length is fixed, this can be achieved, in particular, by placing an aperture on one of the mirrors.

Figure 6a presents the structure of the fundamental mode of a semi-confocal resonator of the (0.5, 1) configuration, for which $|\beta_0|^2$ substantially differs from unity ($|\beta_0|^2 = 0.17$). Figure 6b shows the mode of the same resonator with an aperture placed on mirror M1. The radius $R_{\rm d}$ of the aperture exceeds by a factor of 1.5 the radius of the Gaussian mode of an empty resonator ($R_{\rm d}/w_1 = 1.5$). The mode amplitude distributions on mirror M1 [$|u_1^0(r)|$] and at the input to the active medium [$|u_2^0(r)|$] are close to Gaussian amplitude distributions of the mode in an empty resonator. The use of the aperture results in the increase in $|\beta_0|^2$ up to 0.97. When an aperture for which $R_{\rm d}/w_2 = 1.5$ is mounted on mirror M2 (Fig. 6c), the value of $|\beta_0|^2$ increases up to 0.95. When the aperture was mounted on mirror M1 or M2, the diffraction losses for the fundamental mode were 5.3 % or 4.1 %. The fundamental mode of the resonator of the noncritical (0.596, 1) configuration, closest to the (0.5, 1) configuration, is shown in Fig. 6d. The parameter $|\beta_0|^2$ in this case is 0.95 and diffraction losses are 3.1×10^{-4} %. The values of $|\beta_p|^2$ and $\arg\beta_p$, obtained by expanding (12) for the resonator of the (0.596, 1) configuration, are presented in Fig. 5f.

4. Conclusions

We have performed numerical simulations of the structure of the fundamental mode of stable-resonator lasers in the case of spatially inhomogeneous amplification. The fundamental-mode field has been studied by the iteration Fox–Li method in the case of inhomogeneous unsaturated amplification produced by the axially symmetric longitudinal pumping. The study has been performed for different parameters g_1 and g_2 of the resonator within the entire stability region. The calculations have been carried out for the active medium located near one of the mirrors. It has been shown that in the case of inhomogeneous amplification, the fundamental mode deviates considerably from the Gaussian mode of an empty resonator only in the so-called critical configurations of the resonator, when the quantity $[\arccos(g_1 g_2)^{1/2}]/\pi$ is either zero or takes the values expressed by irreducible fractions m/n. For the values of ξ of practical interest ($\xi \approx 3$) for the Fresnel number

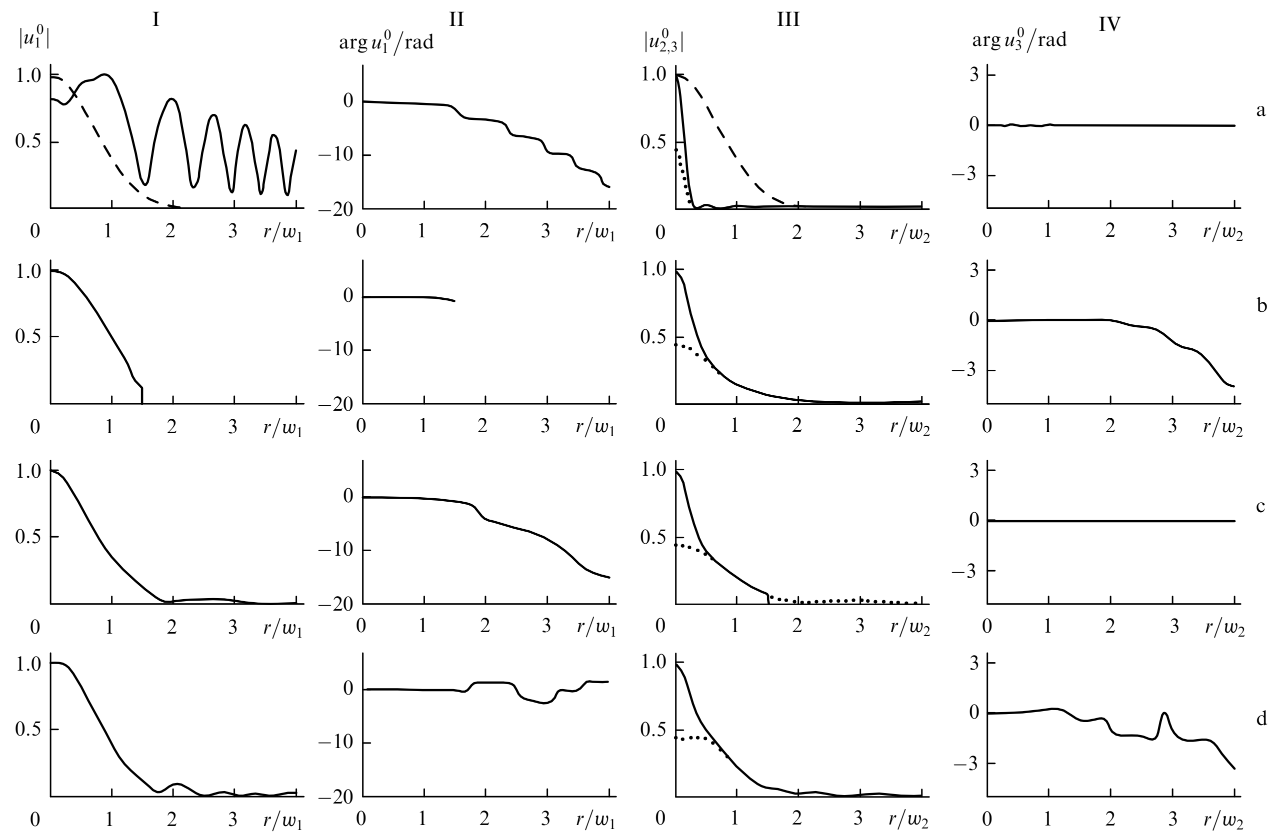

Figure 6. Distributions of the fundamental-mode amplitude and phase on surfaces of the mirrors of the semi-confocal resonator of the (0.5, 1) configuration (a), semi-confocal resonator with an aperture on mirrors M1 (the aperture radius is $R_d = 1.5w_1$) (b) and M2 ($R_d = 1.5w_2$) (c), and of the resonator of the noncritical (0.596, 1) configuration (e) for $N_F = 30$, $K_0 = 1.5$, $\xi = 3$. The arrangement of the figures and notation are as in Fig. 4.

$N_F = 9$, the configurations with $m/n = 1/2$, 2/5, 3/8, 1/3, 3/10, 1/4, 1/5, 1/6, 1/8, and 1/10 are pronounced. As N_F is increased, the number of critical configurations increases. The expansion in the Laguerre–Gaussian function showed that the fundamental mode in critical configurations is formed by a set of phased beams in the active medium with the radial indices $p = nj/2$ ($j = 0, 1, 2, 3, ...$) for even n and $p = nj$ for odd n. Aside from these beams, which predominantly form the fundamental mode, beams with other values of p are also present as a weak background.

Acknowledgements. The authors thank A.V. Petukhov for useful discussions. This work was partially supported by the Russian Foundation for Basic Research (Grant No. 05-02-17448a), a Grant No. NSh 1583.2003.2 of the President of the Russian Federation for Support of the Leading Scientific Schools, and the program of fundamental studies of the Department of Physical Sciences of RAS 'Laser Systems Based on New Active Materials and Optics of Structured Materials'.

References

1. Laporta P., Brussard M. *IEEE J. Quantum Electron.*, **27**, 2319 (1991).
2. Chen Y., Liao T., Kao C., Huang T., Lin K., Wang S. *IEEE J. Quantum Electron.*, **32**, 2010 (1996).
3. Zhang X., Zhao S., Wang Q., Ozygus B., Weber H. *IEEE J. Quantum Electron.*, **35**, 1919 (1999).
4. Hajiesmaeilbaigi F., Koohian A., Mahdizadeh M. *J. Opt. A: Pure and Appl. Opt.*, **4**, 52 (2002).
5. Hall D.G., Smith R.J., Rice R.R. *Appl. Opt.*, **19**, 3041 (1980).
6. Chen Y.F., Kao C.F., Wang S.C. *Opt. Commun.*, **133**, 517 (1997).
7. Gorbunkov M.V., Konyashkin A.V., Kostryukov P.V., Morozov V.B., Olenin A.N., Rusov V.A., Telegin L.S., Tunkin V.G., Shabalin Yu.V., Yakovlev D.V. *Kvantovaya Elektron.*, **35**, 2 (2005) [*Quantum Electron.*, **35**, 2 (2005)].
8. Wu H.-H., Sheu C.-C., Chen T.-W., Wei M.-D., Hsieh W.-F. *Opt. Commun.*, **165**, 225 (1999).
9. Martel G., Labbe C., Sanches F., Fromager M., Ait-Ameur K. *Opt. Commun.*, **201**, 117 (2002).
10. Fox A.G., Li T. *Bell Syst. Techn. J.*, **40**, 453 (1960).
11. Statz H., Tang C.I. *J. Appl. Phys.*, **36**, 1816 (1965).
12. Li T., Skinner J.G. *J. Appl. Phys.*, **36**, 2595 (1965).
13. Blows J.L., Forbes G.W., Dawes J.M. *Opt. Commun.*, **186**, 111 (2000).
14. Laptev G.D., Novikov A.A., Chirkin A.S. *Kvantovaya Elektron.*, **35**, 13 (2005) [*Quantum Electron.*, **35**, 13 (2005)].
15. Chen C.-H., Tai P.-T., Chiu W.-H., Hsieh W.-F. *Opt. Commun.*, **245**, 301 (2005).
16. Maes C.F., Wright E.M. *Opt. Lett.*, **29**, 229 (2004).
17. Petermann K. *IEEE J. Quantum Electron.*, **15**, 566 (1979).
18. Kogelnik H., Li T. *Appl. Opt.*, **5**, 1550 (1966).
19. Chen J., Fujita M., Asakawa M., Imasaki K., Nakai S., Yamanaka C. *Opt. Commun.*, **119**, 563 (1995).
20. Silichev O.O. *Kvantovaya Elektron.*, **17**, 792 (1990) [*Sov. J. Quantum Electron.*, **20**, 715 (1990)].
21. Piche V., Lavigne P., Martin F., Belanger P.A. *Appl. Opt.*, **22**, 1999 (1983).
22. Bykov V.P., Silichev O.O. *Lazernye rezonatory* (Laser Resonators) (Moscow: Fizmatlit, 2004) pp 70, 137.
23. Siegman A.E. *Lasers* (Mill Valley, CA: Univ. Sci. Books, 1986) p. 761.