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On the problems of transparency of metal–dielectric composite media with dissipative and amplifying components

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***Abstract*. Two models of metal–dielectric composite media are used to study the optical properties of their active (amplifying) components under conditions of compensation for the absorption of external electromagnetic radiation appearing due to the presence of metal inclusions. It is shown that the electrostatic approximation for describing the concentrated composite media (a metal nanosphere in a dielectric shell) and bulk composite media (a system of metal nanocylinders in a dielectric matrix) can be applied only in a small range of geometrical and optical parameters. Precise electrodynamic calculations give much smaller gains in the active component required to compensate for absorption, which can be useful for developing 'transparent' composite materials with unique optical properties or 'invisible' composite particles.**

Keywords: *metal–dielectric composites, heterogeneous media, metamaterials.*

1. Introduction

A number of publications have been devoted in recent years to the unusual properties of metal–dielectric composite media. Along with a great number of works in which the optical properties of photonic crystals and materials with a negative refractive index have been studied, attention should be paid to studies devoted to the problem of obtaining a high [1–6], small [7] or unit [8–10] effective refractive index of the composite medium, the problems of giant absorption or supertransmission [10–14], and the problem of reducing electromagnetic scattering by nanoparticles [15]. It is assumed that composite nanostructure materials with unusual optical properties can be used as classical optical elements such as polarisers, prisms and lenses. The optical properties of such artificial media can be selected by choosing proper materials, geometry of inclusions, and their concentration.

However, practical applications of composite media can be restricted by absorption of incident electromagnetic radiation due to the presence of metal components. It was proposed [2, 3, 8, 10] to use an active (amplifying) matrix to compensate for absorption by metal inclusions. Estimates based on the Maxwell–Garnet theory show that a high gain, close to the limiting value achieved at present [2, 8] or even exceeding it [3], is required to compensate for absorption. In addition, it follows from the Maxwell–Garnet formula that as the concentration of metal nanoinclusions tends to zero, the gain of the dielectric matrix should remain high enough [8]. The latter result is a nonphysical consequence of the inapplicability of the Maxwell–Garnet electrostatic theory in the region of low concentrations of inclusions where electrodynamic delay effects start playing a significant role [16]. In this paper, we consider the optical properties of metal–dielectric composite media with amplifying components and use precise electrodynamic calculations for different component concentrations for studying the behaviour of the gain required for compensating absorption by metal inclusions.

2. A metal nanosphere in an amplifying shell

The simplest model of a composite medium is a nanosphere covered by a shell. At present, the optical resonance properties of a sphere in a shell are interesting because such nanocomposites are widely used as markers in electron microscopy [15] and elementary optical biosensors [17]. Such devices are important for analytic chemistry, biology and medicine. A change in the thickness and permittivity of the shell and in the permittivity of the environment leads to a noticeable shift of a plasmon resonance, allowing a detection of materials under study.

It was shown in [18] that for a certain relation between the core and shell parameters of the sphere, electromagnetic scattering may be considerably suppressed, which leads to 'invisibility' of the object. It was pointed out in [18] that the presence of a small absorption slightly affects the extinction cross section.

In order to determine the gain required for compensating energy dissipation at the core, we consider the model of a sphere with an amplifying shell. It is also necessary to determine the extent to which the results of electrostatic approximation agree with those obtained from the exact electrodynamic theory.

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The scattering properties of a sphere covered by a shell can be described in the electrostatic approximation if its size is much smaller than the electromagnetic radiation wavelength. The dielectric properties of the particle as a whole are characterised in this case by the polarisability α which has the form [19]

$$\alpha = 4\pi a_2^3 \frac{(\varepsilon_2-\varepsilon)(\varepsilon_1+2\varepsilon_2)+\eta_v(\varepsilon_1-\varepsilon_2)(\varepsilon+2\varepsilon_2)}{(\varepsilon_2+2\varepsilon)(\varepsilon_1+2\varepsilon_2)+\eta_v(2\varepsilon_2-2\varepsilon)(\varepsilon_1-\varepsilon_2)}, \quad (1)$$

where a_2 is the external radius of the particle, equal to the sum of the core (sphere) radius and the shell thickness; η_v is the part of the particle volume occupied by the core; and ε_1, ε_2 are the permittivities of the core and the shell, respectively. The particle is located in a medium with permittivity ε. Let us analyse the particular case $\alpha = 0$ when scattering of electromagnetic radiation by a nanosphere covered by a shell is absent and we can speak of the 'invisibility' (the term 'transparency' is also used in the literature) of such a particle [18, 19]. The invisibility condition can be easily obtained from expression (1) by equating the numerator of the fraction to zero:

$$\eta_v \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+2\varepsilon_2} - \frac{\varepsilon-\varepsilon_2}{\varepsilon+2\varepsilon_2} = 0. \quad (2)$$

For definiteness, we assume that the environment is air or vacuum ($\varepsilon = 1$). Silver is chosen as the core material. Because the size of the particle under study is assumed to be much smaller than the electromagnetic radiation wavelength, the core radius a_1 in the optical range should not exceed several tens of nanometers. Let us assume that $a_1 = 20$ nm. Generally speaking, the permittivity of such small particles differs from the bulk permittivity of the medium. We estimate ε_1 by using the classical model taking into account the constraint imposed on the electron mean free path [20]. According to this model, the finite size of a metal sphere leads to a change in the relaxation rate of conduction band electrons. The electron relaxation rate γ in a nanosphere is related to the electron relaxation rate γ_0 in the metal by the expression

$$\gamma = \gamma_0 + \frac{\upsilon_F}{a_1},$$

where υ_F is the mean electron velocity at the Fermi surface, equal to 1.4×10^6 m s^{-1} for silver. The expression for the permittivity of the nanosphere has the form [21]:

$$\varepsilon_1(\omega) = \varepsilon_{\exp}(\omega) + \frac{\omega_p^2}{\omega(\omega+i\gamma_0)} - \frac{\omega_p^2}{\omega(\omega+i\gamma)}, \quad (3)$$

where $\varepsilon_{\exp}(\omega)$ is the experimentally determined permittivity for a bulk sample; ω_p is the plasma frequency, and ω is the frequency of the external radiation. For silver, $\hbar\gamma_0 = 0.02$ eV and $\hbar\omega_p = 9.2$ eV [21]. The values of $\varepsilon_{\exp}$ for a number of radiation frequencies are presented in [22]. Calculations show that the correction taking into account the finite size of the particle leads mainly to a variation of the imaginary part of the permittivity ε_1. For an external radiation wavelength of 400 nm, for example, the value of ε_1 for a bulk silver sample is $-3.72 + 0.29i$, while calculations based on expression (3) give the value $-3.72 + 0.42i$ for a sphere of radius 20 nm.

Our analysis shows that because the permittivity of the metal core is complex, condition (2) cannot be satisfied exactly for a shell with real ε_2. Let us try to compensate the effect of the imaginary part of the metal permittivity on the optical properties of the entire composite medium by choosing an amplifying medium as the shell material. For this purpose, we will simulate the optical parameters of the amplifying medium by adding an imaginary part to the permittivity:

$$\varepsilon_2 = n_2^2 - g^2 - 2in_2 g,$$

where n_2 is the refractive index and g is the gain (extinction coefficient), which is equal to the gain multiplied by $\lambda/2\pi$.

Consider the dependence of the gain, which is required for compensating the absorption of external electromagnetic radiation, on the shell thickness. The imaginary part of the permittivity of the silver core is much smaller than the real part, and therefore, we can assume that the gain g of the shell will also be much smaller than its refractive index. In this case, we obtain

$$g \approx \frac{9\eta_v}{2(1-\eta_v)} \frac{n_2^3 \mathrm{Im}(\varepsilon_1)}{(\mathrm{Re}(\varepsilon_1)+2n_2^2)^2 + 2\eta_v(\mathrm{Re}(\varepsilon_1)-n_2^2)^2}, \quad (4)$$

where n_2 is obtained from Eqn (2) by the replacement $\varepsilon_1 \to \mathrm{Re}(\varepsilon_1)$, $\varepsilon_2 \to n_2^2$. The dashed curves in Fig. 1 show the dependence of the refractive indices n_2 and gain g of the shell on its radius a_2 obtained from (2) and (4) for the shell

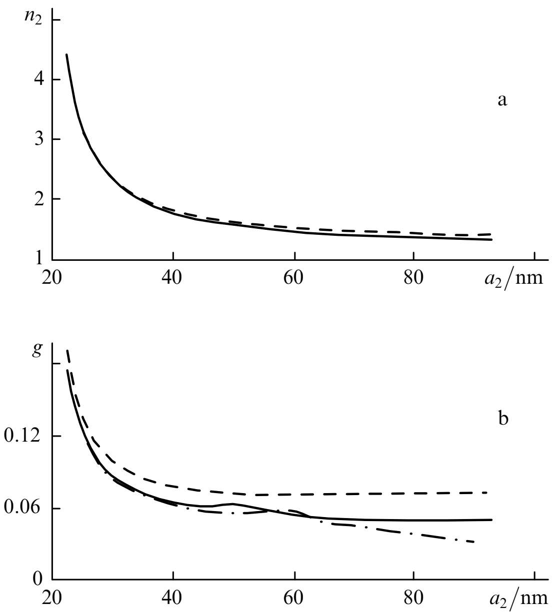

Figure 1. Refractive index (a) and the gain (b) of the shell of a composite spherical particle as functions of its radius, calculated on the basis of electrostatic approximation condition (2) (dashed curves) and satisfying the condition of minimum scattering (solid curves), as well as the gain of the shell of a composite spherical particle as a function of its radius, calculated on the basis of exact electrodynamic theory for values of n_2 satisfying condition (2) (dot-and-dash curve); the computational parameters are $\varepsilon_1 = -3.72 + 0.42i$, $a_1 = 20$ nm, $\lambda = 400$ nm.

radius varying in the interval 22.5–93 nm, which corresponds to a relative core volume $\eta_v = 0.7 - 0.01$. One can see from Fig. 1 that the condition $g \ll n_2$ is satisfied quite well for any value of a_2. Note that for $a_2 > 50$ nm ($\eta_v < 0.065$), the gain takes a fixed value of about 0.07.

Consider now the exact solution of the problem of scattering of optical radiation by a sphere with a shell. The system consisting of a sphere covered by a shell is convenient from the theoretical point of view because an exact calculation of the electromagnetic properties is possible in this case. The corresponding theoretical approaches are described in [18, 19]. We will calculate the required gain for a known value of n_2 by using the condition of equality of the extinction and scattering cross sections Q_{ext} and Q_{sca}, respectively, described by the expressions [19]

$$Q_{\text{sca}} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2 + |b_n|^2),$$
$$Q_{\text{ext}} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}(a_n + b_n). \tag{5}$$

Here $k = 2\pi/\lambda$. For nonmagnetic materials, the coefficients have the form

$$a_n = \{\psi_n(y)[\psi'_n(m_2 y) - A_n\chi'_n(m_2 y)] - m_2\psi'_n(y)[\psi_n(m_2 y)$$
$$-A_n\chi_n(m_2 y)]\}\{\xi_n(y)[\psi'_n(m_2 y) - A_n\chi'_n(m_2 y)]$$
$$-m_2\xi'_n(y)[\psi_n(m_2 y) - A_n\chi_n(m_2 y)]\}^{-1},$$

$$b_n = \{m_2\psi_n(y)[\psi'_n(m_2 y) - B_n\chi'_n(m_2 y)] - \psi'_n(y)[\psi_n(m_2 y)$$
$$-B_n\chi_n(m_2 y)]\}\{m_2\xi_n(y)[\psi'_n(m_2 y) - B_n\chi'_n(m_2 y)]$$
$$-\xi'_n(y)[\psi_n(m_2 y) - B_n\chi_n(m_2 y)]\}^{-1};$$

$$A_n = \frac{m_2\psi_n(m_2 x)\psi'_n(m_1 x) - m_1\psi'_n(m_2 x)\psi_n(m_1 x)}{m_2\chi_n(m_2 x)\psi'_n(m_1 x) - m_1\chi'_n(m_2 x)\psi_n(m_1 x)};$$

$$B_n = \frac{m_2\psi_n(m_1 x)\psi'_n(m_2 x) - m_1\psi_n(m_2 x)\psi'_n(m_1 x)}{m_2\chi'_n(m_2 x)\psi_n(m_1 x) - m_1\psi'_n(m_1 x)\chi_n(m_2 x)};$$

$x = ka_1$; $y = ka_2$; m_1, m_2 are the refractive indices of the core and the shell, respectively, relative to the environment; $\chi_n(z) = -zy_n(z)$ and $\psi_n(z) = zj_n(z)$ are the Riccati–Bessel functions;

$$j_n(z) = \sqrt{\frac{\pi}{2z}}J_{n+1/2}(z) \text{ and } y_n(z) = \sqrt{\frac{\pi}{2z}}Y_{n+1/2}(z)$$

are spherical Bessel functions; and J_υ, Y_υ are the Bessel functions of the first and second kind, respectively.

The dot-and-dash curve in Fig. 1b shows the results of our calculations of the gain. Calculations were made for the values of the refractive index n_2 obtained from condition (2). One can see that the values of the gain required for compensation of absorption are much smaller than those obtained in the electrostatic approximation.

A rigorous approach to the solution of the scattering problem also requires the inclusion of field inhomogeneities inside nanoparticles. Already for $a_2 > \lambda/10$, the field inhomogeneity is such that expression (2) of the electrostatic approximation is not applicable for analysing the conditions of radiation scattering minimum and for calculating the refractive index n_2. For this reason, the conditions of scattering minimum were obtained by direct numerical computations based on formula (5).

The dependences of the refractive index of the shell, providing the conditions of radiation scattering minimum, on its radius are shown by solid curves in Fig. 1a. The values of the gain corresponding to these refractive indices are shown in Fig. 1b. One can see that according to the exact calculations, the gain takes a fixed value of about 0.05 for $a_2 > 70$ nm. Although this value is slightly higher than the values of g calculated for n_2 satisfying condition (2), it is still considerably lower (by about 30 %) than the gain $g \approx 0.07$ obtained in the electrostatic approximation.

Thus, calculations made for a sphere encased in a shell indicate that the values of the gain of the active component necessary for compensation of electromagnetic radiation absorption obtained under electrostatic approximation should be refined. Exact electrodynamic calculations of the optical properties of such a composite gives a qualitative picture of the dependence of refractive indices and gain of the active component on the external radius of the particle, which is analogous to the electrostatic case, but at a much lower value of the gain.

The results of exact electrodynamic calculations presented here suggest that the value of the gain in the active component of bulk composite media providing the compensation of absorption of external radiation can be reduced. The conditions for energy loss compensation are analysed in the next section of this paper.

3. A system of nanocylinders in an amplifying matrix

To our knowledge, no appropriate analytic theory has been developed so far for determining the effective optical parameters of bulk composite media with an active matrix. The methods for calculating the optical properties of photonic crystals, such as the scattering matrix method [23] and the Korringa–Kohn–Rostoker (KKR) method [24], either need to be refined substantially or are entirely unsuitable for our purposes. For this reason, we determined the effective parameters of composite media by using the method of finite elements. The FEMLAB 3.0a software developed by Comsol was used for numerical simulation.

To elucidate general properties only and to reduce the calculation time, we restricted our analysis to the case when a plane–parallel layer of a composite medium contained only one row of parallel infinitely long nanocylinders (nanowires) located in the same plane and separated by the same distance from one another. The analysis was performed for the normal incidence of an electromagnetic wave with polarisation vector directed normally to the nanocylinder axes.

Following the approach developed in [3], we obtain an expression for the effective permittivity ε_{eff} of the composite medium formed by nanocylinders, which is analogous to the

Maxwell–Garnet equation. The corresponding calculation in the electrostatic approximation involves an iterative procedure in which the expression for $\varepsilon_{\rm eff}$ is obtained as the first approximation relative to the contribution from nanoinclusions. Taking into account that the polarisability of the unit length of a cylinder of radius $a_{\rm b}$ with permittivity $\varepsilon_{\rm b}$ placed in a medium with permittivity $\varepsilon_{\rm m}$ in an external field perpendicular to the cylinder axis is described by the expression [25]

$$\alpha = \frac{a_{\rm b}^2}{2}\frac{\varepsilon_{\rm b}-\varepsilon_{\rm m}}{\varepsilon_{\rm b}+\varepsilon_{\rm m}},$$

we obtain the equation

$$\frac{\varepsilon_{\rm eff}-\varepsilon_{\rm m}}{\varepsilon_{\rm eff}+\varepsilon_{\rm m}} = \eta\frac{\varepsilon_{\rm b}-\varepsilon_{\rm m}}{\varepsilon_{\rm b}+\varepsilon_{\rm m}}, \tag{6}$$

where η is the volume concentration of nanocylinders (filling factor).

The effective refractive index $n_{\rm eff}$ and absorption coefficients $\varkappa_{\rm eff}$ of a plane–parallel layer of the composite medium were numerically calculated by analysing the dependence of the layer transmittance and reflectance on the refractive index of the environment (for a fixed layer thickness). It is known that the maximum ratio of the transmittance T to reflectance R is achieved when the refractive index of the environment is equal to $(n_{\rm eff}^2+\varkappa_{\rm eff}^2)^{1/2}$. In the cases under study, $\varkappa_{\rm eff} \ll n_{\rm eff}$, and the approximate equality $(n_{\rm eff}^2+\varkappa_{\rm eff}^2)^{1/2} \approx n_{\rm eff}$ is satisfied to a high degree of accuracy. Thus, the effective refractive index of a composite medium can be determined quite easily in numerical calculations by analysing the dependences of R and T on the refractive index of the environment.

In order to determine the effective absorption coefficient of the layer, we analysed the energy losses during the propagation of a wave through the layer. For this purpose, we substituted the values of transmittance and reflectance, obtained by numerical simulation, into the known analytic expressions for the transmittance and reflectance of a constant-thickness layer [26], and obtained $\varkappa_{\rm eff}$ from the known values of $n_{\rm eff}$ and the layer thickness h.

The effective refractive index and absorption coefficient of a composite monolayer differ generally from those in the bulk of a composite medium away from the interface. This conclusion can be drawn easily by analogy with the propagation of an electromagnetic wave through an ultrathin layer taking the discrete atomic structure into account [27]. This difference was estimated from numerical calculations of a composite layer containing nanocylinders of a very small radius $a_{\rm b} = 5$ nm separated from one another by a distance of 13 nm. Because the length 400 nm of the external electromagnetic wave greatly exceeds these parameters, the electrostatic approximation conditions are satisfied in this case to a high degree of accuracy. The results of this numerical simulation were compared with the results of calculations by expression (6) describing the effective optical parameters of the composite medium away from its surface. Taking the obtained results into account, it can be expected that the calculated effective optical parameters of the composite layer, which are presented below, differ from analogous values in the bulk of the composite by no more than 10 %.

As the radius of nanocylinders increases, the difference in the values of the effective refractive index obtained by exact electrodynamic calculations and in the electrostatic approximation (6) becomes larger. Figure 2 shows the dependences of the effective refractive index $n_{\rm eff}$ on the refractive index $n_{\rm m}$ of the matrix for various filling factors for $a_{\rm b} = 10$ nm under the assumption that absorption by nanocylinders is compensated by amplification in the matrix. Regions with a negative effective permittivity of the composite medium correspond to refractive indices of the matrix for which the values of $n_{\rm eff}$ are not given on the plots. For these values of $n_{\rm m}$, an external electromagnetic wave cannot propagate in a composite medium and is totally reflected from it. In particular, it is shown within the framework of the electrostatic approximation that the composite will not transmit light for any values of $n_{\rm m}$ in the range 1–2.5 for a filling factor $\eta = 0.7$ (Fig. 2d).

One can see from Fig. 2 that despite a small value of the nanocylinder radius compared to the wavelength, the result of exact electrodynamic calculations differs substantially from that obtained in the electrostatic approximation. For moderate volume concentrations of nanocylinders ($\eta = 0.1 - 0.3$), this difference is only quantitative (Figs 2b and c). Qualitative differences appear for a relatively low

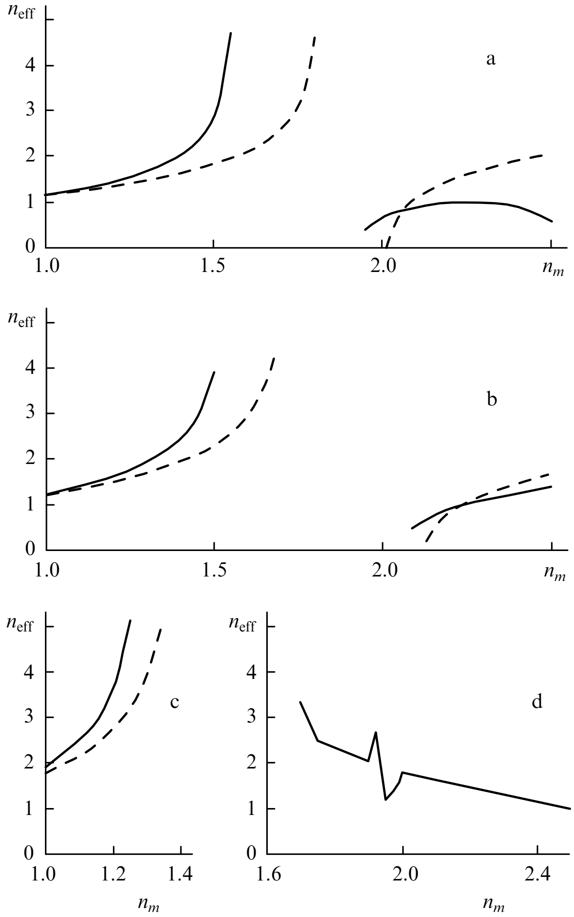

Figure 2. Effective refractive indices of a composite medium as functions of the refractive indices of the matrix for a filling factor $\eta = 0.05$ (a), 0.1 (b), 0.3 (c), 0.7 (d); the solid curves show the results of simulation based on the finite elements method, while the dashed curves are calculated on the basis of the electrostatic theory; the computational parameters are $a_{\rm b} = 10$ nm, $\varepsilon_{\rm b} = -3.72 + 0.42{\rm i}$, $\lambda = 400$ nm.

($\eta < 0.1$) or high ($\eta \approx 0.7$) concentration of nanocylinders. For small values of η, this is manifested in a decrease in the effective refractive index of the composite with increasing the refractive index of the matrix in the range $n_m > 2.3$ (Fig. 2a). For large values of η (Fig. 2d), the dependence of n_{eff} on n_m becomes substantially different from that in the electrostatic case; consequently, a transmitted wave is formed in the composite medium. In the former case, the differences mentioned above can be attributed to delay effects in the interaction of nanocylinders, while in the latter case, they can be related to excitation of multipole moments of closely spaced nanocylinders.

Figure 3 shows the dependences of the absorption coefficient $\varkappa_{eff}$ of the composite layer on the refractive index of the matrix for the same parameters as in Fig. 2 for the case when the matrix is not amplifying. One can see that for small n_m, the results of numerical calculations coincide to a high accuracy with the results obtained in the electrostatic approximation for filling factors $\eta \approx 0.05 - 0.3$. For large values of n_m, the exact electrodynamic calculation gives smaller values of $\varkappa_{eff}$ than in the case of the electrostatic theory. The difference in the values of $\varkappa_{eff}$ obtained by these two different methods becomes significant for a low concentration of nanocylinders: for $\eta = 0.05$ and $n_m = 2.5$ (see Fig. 3a), the absorption coefficients obtained from (6) and exact calculations are 0.11 and 0.029, respectively, i.e., differ by a factor of 3.8.

Our calculations showed that for a composite matrix with a large refractive index ($n_m \approx 2.5$) situations may arise when $n_{eff} = 1$ for a relatively small absorption coefficient $\varkappa_{eff} \sim 0.05$ (for $\eta \approx 0.06$) or for $n_{eff} \ll 1$ and $\varkappa_{eff} \sim 0.01$ (for $\eta \approx 0.04$). The situations when the effective refractive index becomes much larger than unity for a small absorption coefficient are not realised in this system for the values of parameters we used.

One can see from Fig. 3 that a metal–dielectric composite exhibits a noticeable absorption even for small η and large n_m. We shall try to compensate for the absorption by using an amplifying medium as the material for the matrix. Let us calculate the gain required for this purpose. Similarly to the case of a metal sphere in a shell under the condition $\mathrm{Im}(\varepsilon_b) \ll \mathrm{Re}(\varepsilon_b)$, we obtain from (6) an approximate expression for the gain of the matrix under conditions of compensated absorption in the composite medium:

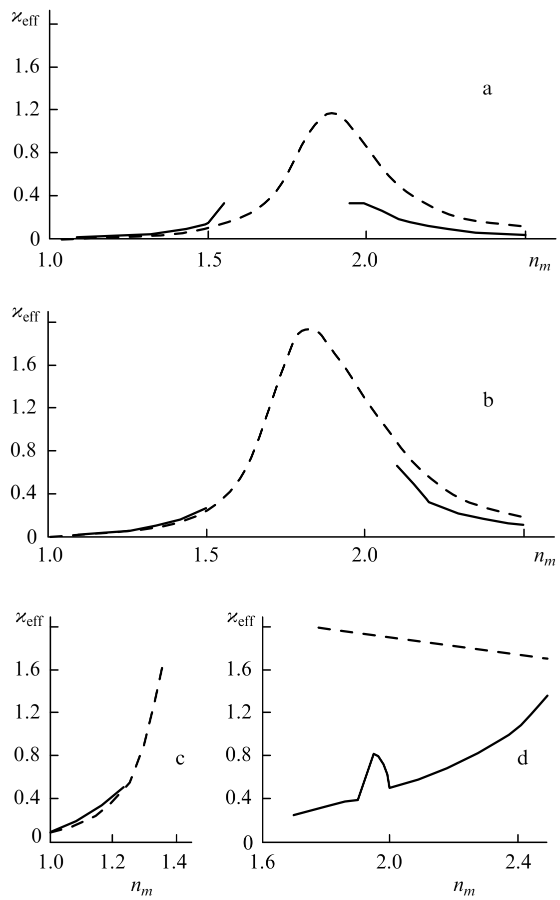

Figure 3. Effective absorption coefficients of a composite medium as functions of the refractive indices of the matrix for a filling factor $\eta = 0.05$ (a), 0.1 (b), 0.3 (c), 0.7 (d); the solid curves show the results of simulation based on the finite elements method (simulation was not carried out for the region in which the electromagnetic wave cannot propagate), while the dashed curves are calculated on the basis of the electrostatic theory; the computational parameters are the same as in Fig. 2.

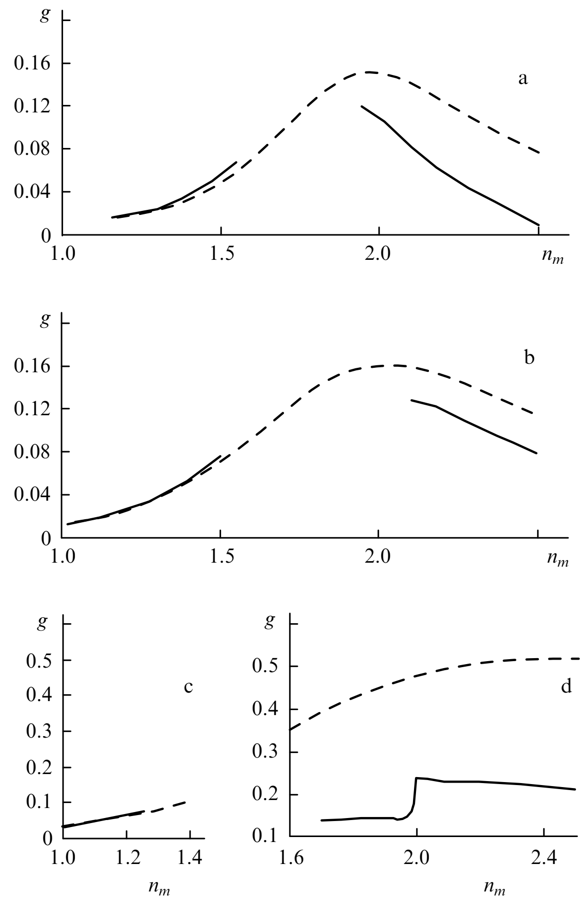

Figure 4. The gain of the matrix as a function of its refractive index in the absence of radiation absorption by a composite medium for a filling factor $\eta = 0.05$ (a), 0.1 (b), 0.3 (c), 0.7 (d); the solid curves show the results of simulation based on the finite elements method (simulation was not carried out for the region in which the electromagnetic wave cannot propagate), while the dashed curves are calculated on the basis of the electrostatic theory; the computational parameters are the same as in Fig. 2.

$$g \approx -\frac{2\eta n_{\mathrm{m}}^3 \mathrm{Im}(\varepsilon_{\mathrm{b}})}{4\mathrm{Re}(\varepsilon_{\mathrm{b}})\varepsilon_{\mathrm{m}}\eta - [\mathrm{Re}(\varepsilon_{\mathrm{b}}) + \varepsilon_{\mathrm{m}}]^2 + [\mathrm{Re}(\varepsilon_{\mathrm{b}}) - \varepsilon_{\mathrm{m}}]^2\eta^2}, \quad (7)$$

where $n_{\mathrm{m}} \approx \sqrt{\varepsilon_{\mathrm{m}}}$.

For comparison, Fig. 4 shows the dependences of the gains obtained from (7) and from exact numerical simulations. One can see that for $n_{\mathrm{m}} = 1 - 1.5$, the gains obtained by these methods coincide. For larger values of n_{m}, however, the required gain is much lower than the value obtained in the electrostatic approximation. The difference in the gains increases with decreasing concentration of nanocylinders. Thus, for $\eta = 0.05$ and $n_{\mathrm{m}} = 2.5$, the values of g obtained by the two methods differ almost by an order of magnitude. According to the electrostatic calculations, the gain under these conditions should be $(2\pi/\lambda)g \approx 12 \times 10^3$ cm^{-1}, which is an unattainable value at present. According to the electrodynamic calculations, however, this value can be lowered to 1.7×10^3 cm^{-1}, which is although a large but attainable value in principle.

Note that the gain of the matrix required for compensating the absorption (Fig. 4) is much smaller than the absorption coefficient of the composite (Fig. 3).

A comparison of Figs 3d and 4d shows that the excitation of multipole moments in closely spaced nanocylinders leads to quantitative as well as qualitative differences in the above dependences. Although the gain required for compensating absorption of radiation in the composite acquires slightly higher values in this case, its magnitude still remains much smaller than the value obtained in the electrostatic approximation.

4. Discussion

Exact electrodynamic calculation of the optical properties of composite media for a nanosphere with a shell and a system of metal nanocylinders in a dielectric matrix shows that the difference from the results obtained for electrostatic approximation may be quite significant in some cases. Such a situation arises, for example, if one of the characteristic sizes of the system (thickness of the shell surrounding the nanosphere or the spacing between nanocylinders) becomes comparable with the wavelength in the corresponding material and the delay effects become appreciable. The role of the delay effects also increases for the optically denser matrix in a bulk composite medium. Note that for a certain relation between the parameters of the composite, the results of exact calculations may differ from the results obtained in the electrostatic approximation by an order of magnitude.

The models of composite media considered in the absorption–compensation regime predict a lower value of the gain obtained in exact numerical calculations compared to that obtained in the electrostatic theory (see Fig. 1b and Fig. 4). The exact calculation shows that for large values of the refractive index of the active matrix, the gain required for compensation of absorption in the composite assumes values that are attainable in actual practice. This result is quite interesting from the point of view of obtaining transparent or weakly absorbing composite materials with extremely large, small or unit refractive index.

Because the values of the gain of the composite are quite small (0.01–0.2), amplification leads only to a small variation of the field in the medium as compared to the case of a nonamplifying matrix. This may form the basis of a new method of describing the optical properties of composite media with amplifying components, where the gain is used as a perturbing parameter. However, it should be noted that in the case of plasmon resonance, the presence of the amplifying medium may lead to a considerable increase in the local field strength [28]. The field distribution in the composite medium should be determined using the exact electrodynamic calculations.

In our numerical calculations, we assumed that $\lambda =$ 400 nm. Such a wavelength was chosen because the requirements imposed on the gain become less stringent for shorter wavelengths of the visible range, and the problem of obtaining nonabsorbing or weakly absorbing composite media is simplified for these wavelengths.

Note that we considered in this paper the normal incidence of an electromagnetic wave on the composite layer. Generally speaking, the optical properties of the layer also depend on the angle of incidence because of retardation effects [16]. A separate study will be devoted to an analysis of this problem, as well as to generalisation of the results presented here to other composite media.

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