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Use of the squeezed (sub-Poisson) state of light in small-signal detection with preamplification upon four-wave mixing

A.V. Kozlovskii

***Abstract*. The scheme of an active interferometer for amplification of small optical signals for their subsequent photodetection is proposed. The scheme provides a considerable amplification of signals by preserving their quantum-statistical properties (ideal amplification) and also can improve these properties under certain conditions. The two-mode squeezed state of light produced upon four-wave mixing, which is used for signal amplification, can be transformed to the non-classical state of the output field squeezed in the number of photons. The scheme is phase-sensitive upon amplification of the input coherent signal. It is shown that in the case of the incoherent input signal with the average number of photons $\langle n_s \rangle \sim 1$, the amplification process introduces no additional quantum noise at signal amplification as large as is wished. A scheme is also proposed for the cascade small-signal amplification ($\langle n_s \rangle \sim 1$) in the coherent state producing the amplified signal in the squeezed sub-Poisson state, which can be used for the high-resolution detection of weak and ultraweak optical signals.**

Keywords: *squeezed light, four-wave mixing, photodetection.*

1. Introduction

The problem of measuring the number of photons in weak and ultraweak optical signals with the single-photon resolution remains an urgent problem of quantum optics concerning the fundamentals of the quantum theory of interaction of light with matter. The use of modern photocount schemes with photodetectors of different types is complicated by the fundamental problem of passing from the conditions of microscopic quantum electrodynamics, which describes the interaction of a photon with matter, to the classical description of the results of measurements of the macroscopic photocurrent of a detector containing information on the measured quantum-statistical properties of light.

Because photodetection inevitably involves the stage of amplification of a microscopic photocurrent produced by a weak field with parameters determined by quantum-mechanical laws, the number of electrons in a classical photocurrent measured at the detector output proves to be in the general case not proportional to the number of photons in the measured field due to the random nature of current amplification. In this case, the photocount resolution is considerably larger than one photon. The fundamental difficulty appearing in passing from the quantum-mechanical description of a physical system to classical conditions can be eliminated, in our opinion, by using preamplification of a weak quantum optical signal in optical amplifiers by preserving and possible improvement of its quantum noise properties [1, 2].

The noise properties of an optical signal can be improved by using the known processes of signal transformation to macroscopic quantum states of light having squeezed numbers of photons (non-classical sub-Poisson states). Such a transformation of a weak signal opens up new possibilities upon its direct photodetection. The transformation of the signal to the quadrature-squeezed state also improves the quality of measurements of the squeezed field quadrature during signal homodyning. The theoretical and experimental investigations of the squeezed states of light are reviewed in papers [3, 4].

The preparation of a macroscopic sub-Poisson state of light (close to the Fock state) opens up new possibilities for more accurate photon counting. In this case, the reliable quantum-statistical description of the field requires only a small number of repeated measurements involved in the photocount procedure.

The possibility of preparing a quadrature-squeezed state by the four-wave mixing method was predicted in [5] and experimentally confirmed in [6, 7]. It is known that four-wave mixing also can be used to obtain an electromagnetic field squeezed in the number of photons; however, this phenomenon has not been analysed in detail so far. Such analysis is performed in this paper for the case of degenerate four-wave mixing.

The use of four-wave mixing for preamplification of an optical signal during its detection was first proposed in [1, 2]. Different parametric interaction schemes applied for amplification of optical signals were considered and analysed in [8 – 10].

We proposed the scheme of an active interferometer combining the properties of an optical amplifier and interferometer. The aim of this paper is to demonstrate the principal possibility of obtaining the state of an electromagnetic field with the squeezed number of photons by using our four-wave mixing amplifying interferometer.

A.V. Kozlovskii P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia;
e-mail: kozlovsk@sci.lebedev.ru

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The use of beamsplitters allows one to transform the two-mode squeezed state [3] of the signal and idler modes of light emerging from a four-wave mixer to the single-mode state of the field at the optical system output, which is squeezed in the number of photons. The amplified output signal contains information on the average number of photons in the input signal. The high amplification of a signal permits the high-resolution measurement of the quantum-statistical parameters of a weak input signal by using standard photodetectors. The obtaining of a narrow photon number distribution in the amplified signal allows one to perform the reliable counting of weak input signals containing approximately one photon.

2. Quantum theory of an optical four-wave mixing amplifying interferometer

Figure 1 shows the scheme of the amplifying interferometer considered in the paper. The optical scheme contains a medium with the cubic nonlinearity $\chi^{(3)}$ and two beamsplitters – at the input of optical signals involved in four-wave mixing to the nonlinear medium (BS1) and at the output from the medium (BS2). The optical signal a_{s} being measured is supplied to one of the inputs of BS1, while the electromagnetic field v of vacuum is supplied to its another input. The general scheme of a quantum-mechanical beamsplitter is presented in Fig. 2. Relations between the creation and annihilation operators of the electromagnetic field of the input and output fields of the beamsplitter preserve the canonical permutable relations for the input field operators; in this case, the condition $[\hat{a}, \hat{b}] = [\hat{a}, \hat{b}^{+}] = [\hat{c}, \hat{d}] = [\hat{c}, \hat{d}^{+}] = 0$ is fulfilled (as for the field operators $\hat{a}'$, $\hat{b}'$, $\hat{c}'$, $\hat{d}'$ in the case of the generalised beamsplitter also shown in Fig. 2). The operators of the fields a and d emerging from BS1 are related to the input fields by the expressions

$$\hat{a} = t\hat{a}_{\mathrm{s}} + r\hat{v}, \tag{1}$$

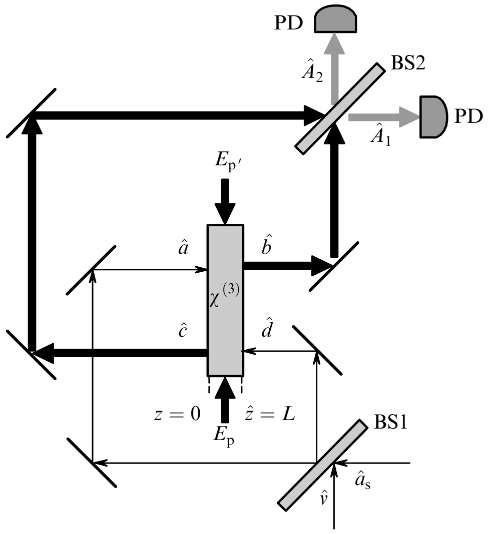

Figure 1. Scheme of photodetection of a quasi-monochromatic signal preliminary amplified in an amplifying interferometer: (BS1 and BS2) input and output beamsplitters; ($\chi^{(3)}$) nonlinear medium with cubic nonlinearity; (PD) photodetector; (L) nonlinear medium length.

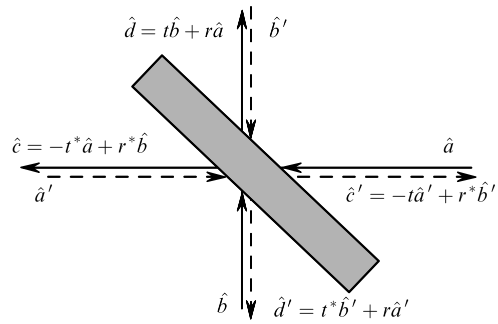

Figure 2. General scheme of a quantum beamsplitter: direct (input and output fields are shown by the solid straight lines) and inverted (input and output fields are shown by the dashed straight lines).

$$\hat{d} = -r^{*}\hat{a}_{\mathrm{s}} + t^{*}\hat{v} \tag{2}$$

(where r and t are the reflection and transmission coefficients) along with the corresponding relations for the Hermitean-conjugate creation operators $\hat{a}^{+}$ and $\hat{d}^{+}$. The operators $\hat{a}$ and $\hat{d}$ are the annihilation operators for the signal and idler modes of the electromagnetic field, respectively.

Then, the counterpropagating collinear fields $\hat{a}$ and $\hat{d}$ enter a medium with the cubic nonlinearity. Two intense counterpropagating classical pump waves E_{p} and $E_{\mathrm{p}'}$ are mixed in the nonlinear medium with the signal ($\hat{a}$) and idler ($\hat{d}$) waves. The frequencies of the four quasi-monochromatic plane waves are assumed equal. It is also assumed that the pump wave intensity greatly exceeds those of the signal and idler waves, and for this reason we will consider pump modes classically and will neglect their depletion in calculations. Note that the fixed-pump approximation remains valid even in the case of high amplification under the condition that the pump wave intensity considerably exceeds the intensity of amplified waves.

The operators $\hat{c}$ and $\hat{b}$ are the annihilation operators of the electromagnetic fields of the reflected and transmitted waves, respectively. The Heisenberg equations of motion for the field annihilation operators $\hat{a}(z)$ and $\hat{d}(z)$ corresponding to the signal field with the initial condition $\hat{a}\ (z = 0)$ and the idler field with the initial value $\hat{d}\ (z = L)$, where L is the length of a nonlinear medium, give the following relations for the output fields [11, 12]:

$$\hat{b} \equiv \hat{a}(z = L) = R\hat{a} + T\hat{d}^{+}, \tag{3}$$

$$\hat{c} \equiv \hat{d}(z = 0) = R\hat{d} + T\hat{a}^{+}, \tag{4}$$

where

$$\hat{a} \equiv \hat{a}(z = 0); \quad \hat{d} \equiv \hat{d}(z = L); \tag{5}$$

$$R \equiv \sec(KL); \quad T \equiv -\zeta \tan(KL); \tag{6}$$

$$\zeta \equiv \mathrm{i}\exp[\mathrm{i}(\phi_{\mathrm{p}} + \phi_{\mathrm{p}'})]; \quad K \equiv \frac{\chi |E_{\mathrm{p}}E_{\mathrm{p}'}|}{c_{\mathrm{m}}}, \tag{7}$$

E_{p} and $E_{\mathrm{p}'}$ are the amplitudes of the pump waves; ϕ_p and $\phi_{\mathrm{p}'}$ are the phases of the pump waves; c_{m} is the speed of light in the medium; and χ is the nonlinearity constant.

Then, the amplified fields b and c arrive at the output beamsplitter BS2, which is identical to the input beamsplitter BS1. By using (1) and (2), we find for the fields emerging from BS2

$$\hat{A}_1 = T(t^2 - |r|^2)\hat{a}_s + R(t^* - t)r\hat{a}_s^+ + Tr(t + t^*)\hat{v} + R(t^2 + |r|^2)\hat{v}^+, \quad (8)$$

$$\hat{A}_2 = -Tr^*(t + t^*)\hat{a}_s + R(t^{*2} + |r|^2)\hat{a}_s^+ + T(t^{*2} - |r|^2)\hat{v} + R(t^* - t)r^*\hat{v}^+. \quad (9)$$

For the output field operator A_1 to contain only the vacuum operators, and the operator A_2 – only signal operators, it is sufficient to fulfil the relation

$$t = t^* = |t| = |r| = \frac{1}{\sqrt{2}}, \quad (10)$$

which is, in particular, valid for a symmetric beamsplitter for

$$t = \frac{1}{\sqrt{2}}, \quad r = \frac{\mathrm{i}}{\sqrt{2}}, \quad |t|^2 + |r|^2 = 1. \quad (11)$$

By using (10), we find from (8) and (9) the relations

$$\hat{A}_1 = \mathrm{e}^{\mathrm{i}\phi_r} T\hat{a}_s + R\hat{a}_s^+, \quad (12)$$

$$\hat{A}_2 = -\mathrm{e}^{-\mathrm{i}\phi_r} T\hat{v} + R\hat{v}^+, \quad (13)$$

where $\phi_r = \mathrm{Arg}\, r$ and, when (11) is satisfied, we have

$$\hat{A}_1 = \mathrm{i}T\hat{a}_s + R\hat{a}_s^+, \quad (14)$$

$$\hat{A}_2 = \mathrm{i}T\hat{v} + R\hat{v}^+. \quad (15)$$

As the output beamsplitter BS2, an inverted beamsplitter can be also used. In this case, the input field b is supplied to the output of the field d, while the input field c is supplied to the output of the field a of the beamsplitter BS1 (see Fig. 2). The output fields A_1 and A_2 in such a configuration (see Fig. 1) can be written as

$$\hat{A}_1 = t^*\hat{b} - r\hat{c}, \quad (16)$$

$$\hat{A}_2 = r^*\hat{b} + t\hat{c}. \quad (17)$$

By substituting expressions (3) and (4) into (16) and (17), we find

$$\hat{A}_1 = T\hat{a}_s - 2t^*rR\hat{a}_s^+ = T\hat{a}_s - \mathrm{e}^{\mathrm{i}\theta}R\hat{a}_s^+, \quad (18)$$

$$\hat{A}_2 = T\hat{v} + \mathrm{e}^{\mathrm{i}\theta}R\hat{v}^+, \quad (19)$$

where $\theta = \mathrm{Arg}(t^*r)$; $|t| = |r| = 1/\sqrt{2}$.

It is obvious that the measurement of the number of photons $\langle N\rangle = \langle\hat{A}_1^+\hat{A}_1\rangle$ of the field A_1 allows one to determine the required number of photons $\langle n_s\rangle = \langle\hat{a}_s^+\hat{a}_s\rangle$ of the signal field a_s.

3. Quantum-statistical properties of a four-wave mixing amplifying interferometer

By using (3), (4) and (12), (13), (14), we find the average values and dispersions of the number of photons in the field A_1 containing information on different quantum states of the input signal a_s, which we assume coherent, chaotic or Fock states with a certain number of photons.

The average number $\langle\hat{A}_1^+\hat{A}_1\rangle$ of photons for the coherent signal $|\alpha\rangle$ at the system input for any configurations of beamsplitters considered above is

$$\langle N\rangle = |\alpha|^2\left[|T|^2 + |R|^2 + 2\left(|T|^2|R|^2\right)^{1/2}\cos\Phi\right] + |R|^2. \quad (20)$$

The root-mean-square deviation (fluctuations) of the number of photons at the system output has the form

$$\left\langle(\Delta N)^2\right\rangle = |\alpha|^2\left[\left(|T|^2 + |R|^2\right)^2 + 4|R|^2|T|^2 + 4\left(|R|^2|T|^2\right)^{1/2} \times\left(|R|^2 + |T|^2\right)\cos\Phi\right] + 2|T|^2|R|^2. \quad (21)$$

When two symmetric beamsplitters are used, the relative phase entering (20) and (21) is

$$\Phi \equiv 2\phi_s - \phi_p - \phi_{p'}, \quad (22)$$

where ϕ_s is the phase of the coherent signal mode. The output beamsplitter BS2 for $|t| = |r| = 1/\sqrt{2}$ can be also an inverted input beamsplitter. In this case,

$$\Phi \equiv 2\phi_s - \phi_p - \phi_{p'} + \phi_t - \phi_r - \frac{\pi}{2}, \quad (23)$$

where $\phi_t = \mathrm{Arg} t$.

It follows from (20) that for $\Phi = \pi$ and

$$|\alpha|^2(|T| - |R|)^2 \approx 0 \quad (24)$$

the average number of photons is

$$\langle N\rangle = |R|^2, \quad (25)$$

and fluctuations of the output field are

$$\left\langle(\Delta N)^2\right\rangle = 2|T|^2|R|^2. \quad (26)$$

Condition (24) is fulfilled at large gains ($|G|^2 \equiv |T|^2 \gg 1$), $|R|^2 = |T|^2 - 1 \approx |T|^2$, and small $|\alpha|^2$. Under such conditions, it is impossible to measure the parameters of the input signal a_s because the signal A_1 transformed by the system does not contain information on the input signal a_s. In this case, the system under study represents a light generator with characteristics determined only by its internal parameters.

In the case of the incoherent (chaotic or Fock) input signal, the dependence of the average number of output photons on the average number of input photons $\langle n_s\rangle = \langle\hat{a}_s^+\hat{a}_s\rangle$ is

$$\langle N\rangle = \left(|T|^2 + |R|^2\right)\langle n_s\rangle + |R|^2. \quad (27)$$

Fluctuations of the number of output photons depend on the parameters $\langle n_s\rangle, \langle(\Delta n_s)^2\rangle$ of the input signal as

$$\left\langle(\Delta N)^2\right\rangle = |R|^2\left\{\left[\left(|R|^2 + |T|^2\right)^2 + 2|R|^2|T|^2\right]\left\langle(\Delta n_s)^2\right\rangle + 2|R|^2|T|^2(\langle n_s\rangle^2 + \langle n_s\rangle + 1)\right\}. \quad (28)$$

Expressions (21) and (24) can be conveniently analysed by using the notation

$$|T| = \cosh s, \quad |R| = \sinh s, \quad |T|^2 - |R|^2 = 1. \quad (29)$$

It follows from (21) that the output signal squeezing in the number of photons is possible only for certain relative phases (22) [or (23) for symmetric beamsplitters]. The sub-Poisson statistics (the Fano factor $F < 1$) is present for $\Phi \approx \pi$, whereas output photons for other values of Φ are described by the super-Poisson statistics ($F > 1$).

When the input signal $\hat{a}_s$ is in the incoherent quantum state, the Fano factor of the output field

$$F = \frac{\langle(\Delta N)^2\rangle}{\langle N \rangle} \quad (30)$$

is expressed in terms of quantum-statistical parameters $\langle(\Delta n_s)^2\rangle$, $\langle n_s \rangle$ of the input field as

$$F = |T|^2 \frac{6\langle(\Delta n_s)^2\rangle + 2(\langle n_s\rangle^2 + \langle n_s\rangle + 1)}{2\langle n_s\rangle + 1} \quad (31)$$

by assuming that $|R|^2 \approx |T|^2 \gg 1$. It follows from (31) that the output field has a high level of the photon noise and is super-Poisson ($F \gg 1$). Thus, in the case of the inverted output beamsplitter and a strong input field ($\langle n_s\rangle \gg 1$), we have $F = |T|^2\langle n_s\rangle$, while for the input field in the single-photon Fock state, we have $F = 2|T|^2$.

For the coherent input signal, it follows from (21) that for the given average number $|\alpha|^2$ of photons in the input signal, the minimal value of the dispersion of output photons is

$$\langle(\Delta N)^2\rangle_{\min} = \frac{1}{8}\left(B^2 + B - \frac{1}{B} - \frac{1}{B^2}\right), \quad (32)$$

where $B = (8|\alpha|^2 + 1)^{1/4}$. Fluctuations are minimal for a certain value of the gain

$$|T|_{sq} = \frac{1}{2}\left(\sqrt{B} + \frac{1}{\sqrt{B}}\right). \quad (33)$$

In this case, the Fano factor of the squeezed output signal can be considerably lower than unity, i.e. the field produced upon amplification is sub-Poisson. At the same time, for large $|\alpha|^2$, the minimal value of fluctuations of the number of photons (32) is rather large ($F \gg 1$), which means that the Fock state of the output field cannot be prepared by using the amplification scheme considered above. A strong squeezing of the number of photons ($F \ll 1$) can be achieved only in the case of a large average number of photons of the input signal. In the case of a high gain and $|\alpha|^2 \gg 32|T|^8$, $|T|^2 \gg 1$, the squeezing with

$$F \approx e^{2s} = \frac{1}{4|T|^2} \ll 1 \quad (34)$$

can be possible, where $\sinh s \approx e^s/2$.

The signal-to-noise ratio $\mathscr{R}$ for the field A_1 transmitted through the beamsplitter BS2 in our scheme is presented in Fig. 3 for different values of the gain $|G|^2 \equiv |T|^2$ and different average numbers a_s of photons incident on the beamsplitter BS1. Figure 3a shows the results for the field a_s in the chaotic quantum state (described by the Bose–Einstein distribution) obtained from (27) and (28). One can see from Fg. 3a that for $\langle n_s\rangle \gg 1$, the value of $\mathscr{R}$ drastically decreases with increasing $|G|^2$ from 1 for $|G|^2 = 1$ to 0.5 for $|G|^2 \approx 4$. At the same time, for $\langle n_s\rangle = 1$, the dependence of $\mathscr{R}$ on the gain is qualitatively different: $\mathscr{R}$ decreases for $1 < |G|^2 < 1.5$ and then increases and achieves unity for $|G|^2 \gg 1$. Thus, the amplifying interferometer considered here is an ideal amplifier of ultraweak chaotic (thermal) light ($\langle n_s\rangle \approx 1$), which does not change the signal-to-noise ratio of the incident field at amplifications as high as is wished.

Figure 3b presents the results of calculations performed for the initial signal in the coherent quantum state by expressions (20) and (21) for the relative phase $\Phi = \pi$. As follows from the curves, the value of $\mathscr{R}$ decreases with increasing the gain for all the values of the average number $\langle n_s\rangle = |\alpha|^2$ of input photons. As $\langle n_s\rangle$ increases, the decrease of $\mathscr{R}$ with increasing gain slows down, and for large $\langle n_s\rangle$; (approximately 10^6) the value of $\mathscr{R}$ will not decrease even for $|G|^2 \gg 1$. Note that in the case of the coherent input field, when the amplification scheme has the phase sensitivity, the amplifier introduces the additional noise to the output field, and $\mathscr{R}$ decreases from 1 to 0.5 with increasing gain.

The calculation performed for the relative phase $\Phi = 0$ showed in this case $\mathscr{R} \approx \langle n_s\rangle$ for any $\langle n_s\rangle$ and $|G|^2$, i.e. for this phase matching the amplification of the coherent signal

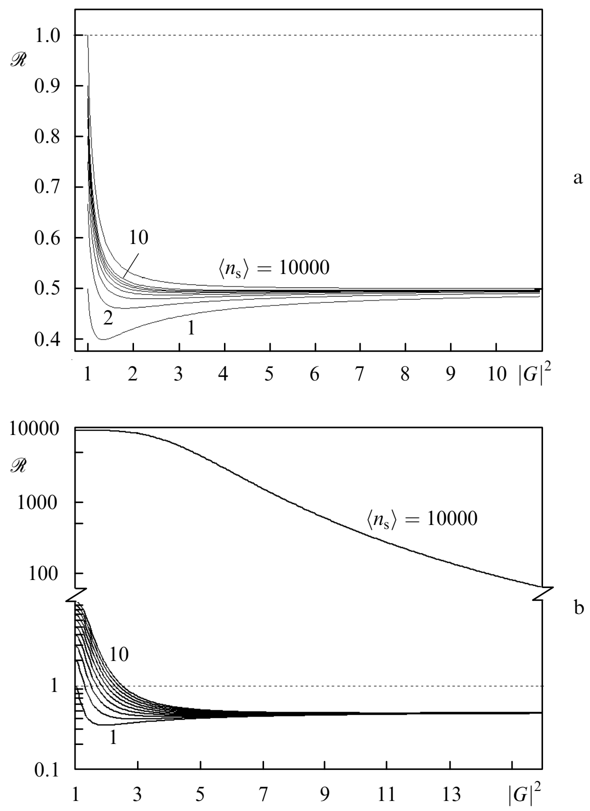

Figure 3. Dependences of the signal-to-noise ratio $\mathscr{R} = \langle N\rangle^2/\langle(\Delta N)^2\rangle$ for the field incident on a photodetector ($\langle N\rangle = \langle A_1^+ A_1\rangle$) on the gain $|G|^2$ for different average numbers $\langle n_s\rangle$ of photons in the input (measured) signal a_s when the input signal is in the chaotic (thermal) quantum state (a) and coherent quantum state (b) (the relative phase is $\Phi = \pi$, $\langle n_s\rangle = 1$, 2, 3, ..., 10, 10000).

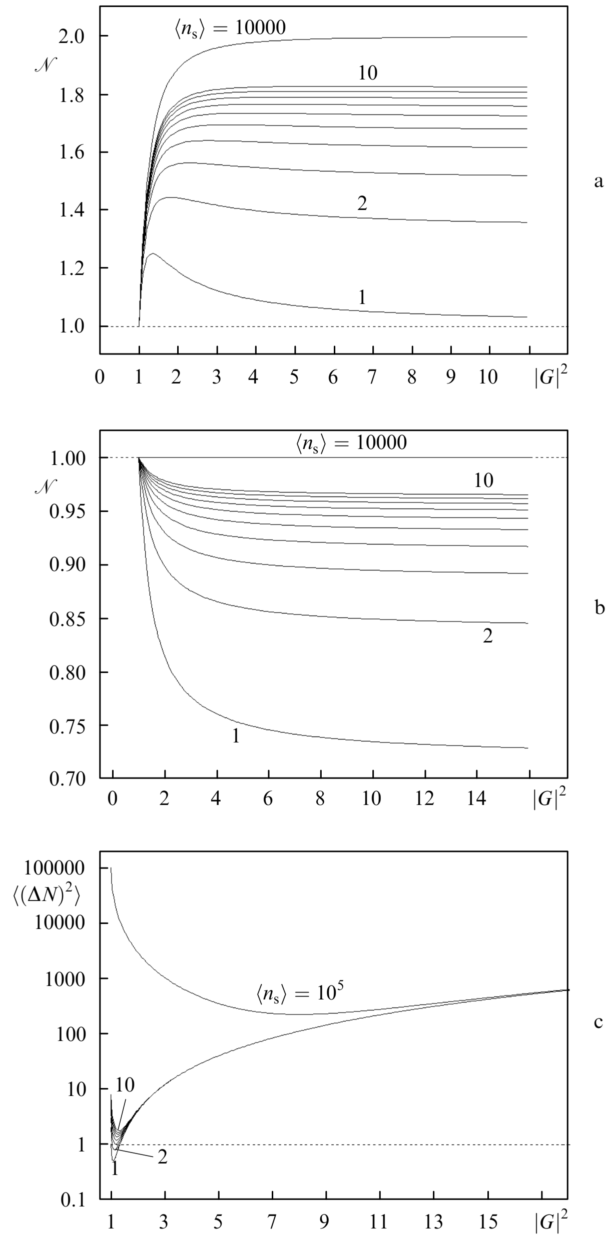

Figure 4. Dependences of the noise parameter $\mathcal{N} = \mathcal{R}_{in}/\mathcal{R}_{out}$ of the field incident on a photodetector ($\langle N \rangle = \langle A_1^+ A_1 \rangle$) on the gain $|G|^2$ for different average numbers $\langle n_s \rangle$ of photons in the input (measured) signal a_s when the input signal is in the chaotic (thermal) quantum state (a) and coherent quantum state (b) (the relative phase is $\Phi = \pi$, $\langle n_s \rangle = 1, 2, 3, \ldots, 10, 10000$), and the dependences of the dispersion of the number of photons $\langle (\Delta N)^2 \rangle$ on $|G|^2$ for the input signal in the coherent quantum state ($\langle n_s \rangle = 1, 2, \ldots, 10^5$) (c).

is ideal (noiseless). An increase in $\mathcal{R}$ to $1.36\langle n_s \rangle$ is also possible for $\langle n_s \rangle = 1$ and $|G|^2 \gg 1$.

The dependences of the noise parameter $\mathcal{N} \equiv \mathcal{R}_{in}/\mathcal{R}_{out}$ of the amplifier on $|G|^2$ for different $\langle n_s \rangle$ are presented in Fig. 4. For the chaotic input signal, it follows from (20) and (21) that for $|G|^2 \gg 1$,

$$\mathcal{N} = 2\,\frac{\langle n_s \rangle}{\langle n_s \rangle + 1}. \tag{35}$$

For $\langle n_s \rangle = 1$, amplification becomes noiseless, i.e. $\mathcal{N} \to 1$ for $|G|^2 \gg 1$.

The coherent signal for $\Phi = 0$, as follows from Fig. 4b, is amplified for $\langle n_s \rangle \gg 1$ almost without increasing the noise parameter $\mathcal{N}$, while for $\langle n_s \rangle = 1$, the ratio of $\mathcal{R}_{in}$ to $\mathcal{R}_{out}$ even increases: $\mathcal{N} \approx 0.72$ for $|G|^2 \to \infty$. If $\Phi = \pi$, amplification introduces the additional noise to the output signal for $\langle n_s \rangle \sim 1$ and is almost noiseless for $\langle n_s \rangle \to \infty$. Figure 4c shows the dependence of fluctuations of the number $\langle (\Delta N)^2 \rangle$ of photons on the gain for different values of the coherent input signal. One can see that, as was mentioned above, the Fock state of the amplified light in which fluctuations of the number of photons are zero, is not achieved in our four-wave mixing scheme even in the case of strong squeezing ($F \ll 1$). Small fluctuations of the number of photons $\langle (\Delta N)^2 \rangle < 1$ are present only for weak input signals ($\langle n_s \rangle < 1$) upon weak amplification. As $\langle n_s \rangle$ increases, fluctuations for the same amplification also increase. As the gain is increased for fixed $\langle n_s \rangle$, fluctuations increase, and for $|G|^2 > 10$, fluctuations of the number of photons are independent of $\langle n_s \rangle$, and in this case $\langle (\Delta N)^2 \rangle \gg 1$.

The dependence of the Fano factor on the gain for different $\langle n_s \rangle$ is shown in Fig. 5. It follows from Fig. 5a that the Fano factor for the chaotic quantum state of the initial signal always exceeds unity and increases with the gain, i.e. the output radiation of the amplifier in this case is always super-Poisson.

The squeezed state of the amplified light can be obtained in the case of the coherent state of the input signal. One can

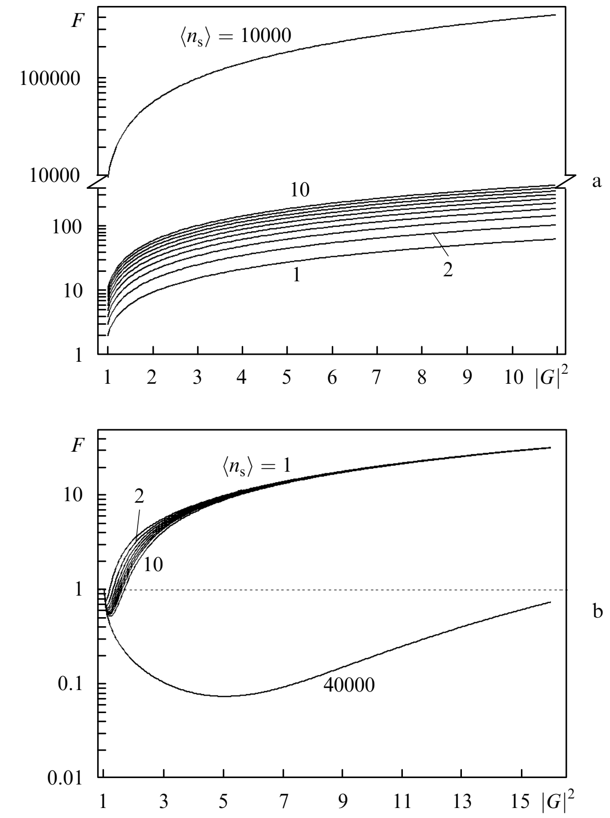

Figure 5. Dependences of the Fano factor $F = \langle (\Delta N)^2 \rangle / \langle N \rangle$ for the field incident on a photodetector ($\langle N \rangle = \langle A_1^+ A_1 \rangle$) on the gain $|G|^2$ for different average numbers $\langle n_s \rangle$ of photons in the input (measured) signal a_s when the input signal is in the chaotic (thermal) quantum state (a) ($\langle n_s \rangle = 1, 2, 3, \ldots, 10, 10000$) (a) and in coherent quantum state ($\Phi = \pi$, $\langle n_s \rangle = 1, 2, 3, \ldots, 10, 40000$) (b).

see from Fig. 5b that slightly squeezed light ($F < 1$) is present at the output in the case of a weak amplification of a small input signal ($\langle n_s \rangle \sim 1$) and $\Phi = \pi$. Strong squeezing ($F \ll 1$) is achieved at large $\langle n_s \rangle$ and the optimal value of the gain $|G|^2$ depending on $\langle n_s \rangle$ [see expressions (32) and (33)]. For gains exceeding this optimal value and $|G|^2 \to \infty$, the output field becomes super-Poisson.

The value of the relative phase $\Phi \approx \pi$ is optimal for the preparation of the squeezed (sub-Poisson) state; for $\Phi = 0$, squeezing is impossible. It follows from (2) that the average number of output photons for $\Phi \approx \pi$ decreases with increasing $|G|^2$, and takes the value $|T|^2|R|^2 = |G|^2 \times (|G|^2 - 1)$ when relation (24) is fulfilled. Thus, for small $\langle n_s \rangle$ and $\Phi \approx \pi$, a considerable amplification of a signal ($\langle N \rangle \gg \langle n_s \rangle$) becomes impossible. The transformation of a weak coherent signal to a squeezed amplified output signal involves difficulties in this case. In the case of a weak coherent signal ($|\alpha|^2 \approx 1$) at the amplifier input, the intense sub-Poisson output signal can be also obtained; however, it is necessary to use a cascade of amplifiers considered here with different gains. At each amplification stage, a field close to a coherent field is produced, but with the increasing average number of photons. The gain $|G_i|^2 \sim 1$ in each of the amplifiers in the cascade ($i = 1, \ldots, M - 1$, where M is the number of amplifiers in the cascade) increases with i and allows the preparation of the field in the coherent state with a large average number of photons at the output of the $M - 1$ amplifier. At the last (Mth) transformation stage, an amplifier with the gain $|G_M|^2 \sim 1$ is used which produces amplified light in the squeezed state.

The dependence of the quantum-statistical parameters $\mathcal{R}$, F, and $\mathcal{N}$ of the amplifier signal on the relative phase $\Phi \in [0, 2\pi]$ are shown in Fig. 6 for different average numbers of photons of the coherent input signal. One can see from Fig. 6a that the value of $\mathcal{R}$ drastically increases at $\Phi \approx \pi$ and a large number of input photons; for $\langle n_s \rangle \to \infty$, we have $\mathcal{R}(\Phi = \pi) \to \langle n_s \rangle \approx \mathcal{R}(\Phi = 0)$. For $\langle n_s \rangle \sim 1$, the relation $\mathcal{R}(\Phi = \pi) \ll \mathcal{R}(\Phi = 0) = \langle n_s \rangle$ takes place; this means that the best value of $\mathcal{R}_{\text{out}}$ for a weak coherent input signal is achieved for $\Phi = 0$.

One can see from Fig. 6b that the squeezed state of the output field ($F \ll 1$) is achieved only for $\Phi \approx \pi$ and $\langle n_s \rangle \gg 1$; in this case, a small deviation of the relative phase Φ from π results in a drastic increase in fluctuations of the output field.

The noise parameter $\mathcal{N}$ also drastically depends on the relative phase Φ (Fig. 6c). One can see that the best value $\mathcal{N} < 1$ is realised at $\langle n_s \rangle \sim 1$ for $\Phi = 0$. When the output field is in the squeezed state ($\Phi = \pi$, $\langle n_s \rangle \gg 1$), the signal is amplified without deterioration of its noise parameter, i.e. $\mathcal{N} = 1$. Note that small errors in the relative phase (near $\Phi = \pi$) resulting in a drastic increase in the noise parameter $\mathcal{N}$ are possible due to fluctuations of the phase of pump fields. For this reason, the values $\Phi \approx 0$ can be used in experiments, because amplification in this case also does not introduce any additional noise ($\mathcal{N} < 1$), and the effect of the strong dependence of the quality on the relative phase Φ of the amplified signal is absent.

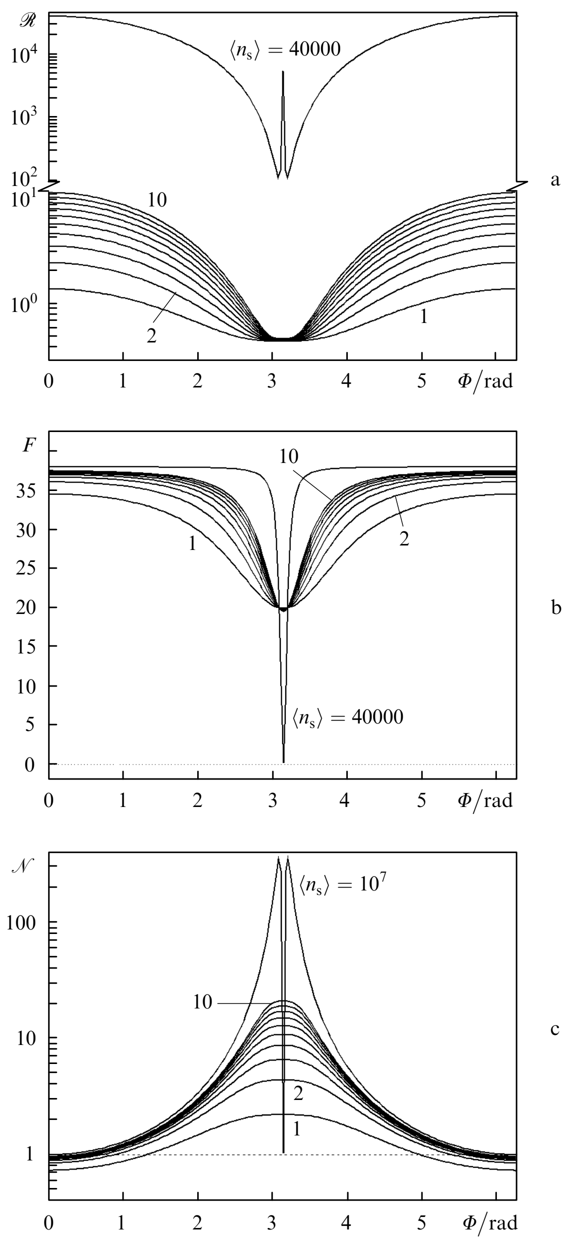

Figure 6. Deopendences of the signal-to-noise ratio $\mathcal{R}$ (a), Fano factor (b), and the noise parameter $\mathcal{N}$ (c) of the amplifier for the field incident on a photodetector ($\langle N \rangle = \langle A_1^+ A_1 \rangle$) on the relative phase for different average numbers of photons in the input (measured) signal a_s and $|G|^2 = 10$.

4. Conclusions

We have shown that the active four-wave mixing interferometer proposed in the paper can be used to amplify considerably weak optical signals in different quantum states without deterioration of their quantum noise parameters. The quantum-statistical properties of the transformed light have been studied for coherent, chaotic (thermal), and Fock quantum states of the input field.

In the case of the coherent input signal, our interferometer is a phase-sensitive amplifier. By providing the optimal matching of the input field and phases of coherent pumping in the four-wave scheme, it is possible to produce the amplified output field that have non-classical statistical properties (squeezed in the number of photons). The intense sub-Poisson light can be produced at large gains $G|^2 \gg 1$ only for sufficiently intense input signals ($\langle n_s \rangle \gg 1$). Under these conditions, the best value $\mathcal{N} \equiv \mathcal{R}_{\text{in}}/\mathcal{R}_{\text{out}} = 1$ is realised. For ultraweak signals ($\langle n_s \rangle \sim 1$), the amplifier improves the quality of the signal and $\mathcal{N} < 1$ for

$|G|^2 \gg 1$. An ultraweak coherent signal also can be transformed to the intense squeezed (sub-Poisson) light by using a cascade of such amplifiers with specially selected increasing gains.

Our analysis has shown that, by using nonlinear four-wave mixing in our scheme, the coherent input signal can be transformed to the intense squeezed (sub-Poisson) signal with the Fano factor $F \ll 1$; however, it is impossible to prepare the purely Fock state with a certain number of photons.

Fluctuations in the number of photons in the case of a chaotic field at the input of our interferometer considerably increase upon strong amplification and $\mathcal{N} \gg 1$. However, in the case $\langle n_s \rangle = 1$, when the inverted output beamsplitter and large gains are used, it is possible to obtain $\mathcal{N} \approx 1$, i.e. amplification occurs without deterioration of the $\mathcal{R}$ value, which allows the measurement of ultraweak chaotic signals. As $\langle n_s \rangle$ increases, the detection quality is impaired under these conditions.

Thus, the transformation of light performed by the amplifying interferometer considered in the paper can improve, under certain conditions, the resolution upon detecting weak optical signals to one photon only.

References

1. Kozlovskii A.V. *Zh. Eksp. Teor. Fiz.*, **129**, 30 (2006).
2. Kozlovskii A.V. *Kvantovaya Elektron.*, **36**, 334 (2006) [*Quantum Electron.*, **36**, 334 (2006)].
3. *Squeezed Light (Special Issue). J. Mod. Opt.*, **34** (6/7) (1987).
4. *Squeezed States of the Electromagnetic Field (Special Issue). J. Opt. Soc. Am. B*, **4** (10) (1987).
5. Yuen H.P., Shapiro J.H. *Opt. Lett.*, **4**, 334 (1979).
6. Slusher R.E., Hollberg L.W., Yurke B., Mertz J.C., Valley J.F. *Phys. Rev. Lett.*, **55**, 2409 (1985).
7. Slusher R.E., Yurke B., Grangier P., Laporta F., Walls D.F. *J. Opt. Soc. Am. B*, **4**, 1453 (1987).
8. McKinstrie C.J., Radic S., Raymer M.G. *Opt. Express*, **12**, 5037 (October, 2004).
9. McKinstrie C.J., Yu M., Raymer M.G., Radic S. *Opt. Express*, **13**, 4986 (June, 2005).
10. McKinstrie C.J., Raymer M.G., Radic S., Vasilyev M.V. *Opt. Commun.*, **257** (1), 146 (2006).
11. Glauber R.J. in *Quantum Optics and Electronics* (New York: Gordon and Breach, 1965; Moscow: Mir, 1966).
12. Mandel L., Wolf E. *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press, 1995; Moscow: Fizmatlit, 2000).