PACS numbers: 42.55.Rz; 42.60.Mi; 42.65.Ky; 42.25.Ja
DOI: 10.1070/QE2007v037n06ABEH013347

Control of the radiation parameters of an intracavity frequency-doubled solid-state laser by means of optoelectronic feedback

P.A. Khandokhin, V.G. Zhislina

***Abstract*. A method for chaotic oscillation suppression in intracavity frequency-doubled lasers based on the rate-equation model of a bipolarised solid-state laser is proposed. It is shown that the use of optoelectronic feedback either stabilises the system in the stationary state or leads to quasi-sinusoidal oscillations with a constant amplitude.**

Keywords: *feedback, control of laser radiation parameters, intracavity frequency doubling.*

1. Introduction

The development of the technical basis for quantum electronics – the improvement of old and creation of new lasers, such as diode-pumped solid-state lasers and intracavity frequency-doubled lasers, and demands for these lasers in various fields of technology and medicine [1 – 4] require the further development of the theory of nonlinear dynamics to obtain recommendations for improving the parameters of the lasers. The most important from the practical point of view is the problem of stabilisation of laser radiation parameters, providing the maximum output power, for example, in the second harmonic upon intracavity frequency doubling. There exist two types of phase matching of the waves in nonlinear frequency conversion [5]. In the case of type I phase matching, the waves of the same polarisation are involved in the second harmonic generation. In the case of type II phase matching considered in our paper, the waves with orthogonal polarisations are involved in frequency conversion. In the case of multimode lasing, both frequency doubling (radiation of each of the modes is converted to the second harmonic) and frequency summation (the interaction of the modes in pairs resulting in radiation at the sum frequency) are possible.

It is well known [6, 7] that intracavity frequency conversion is accompanied, as a rule, by the chaotic behaviour of the radiation intensity with the 100 % intensity modulation depth both at the fundamental and converted harmonics. This occurs because the intracavity frequency conversion of radiation to the second harmonic acts as a powerful factor perturbing the stationary state of the system. At the same time, a specific feature of solid-state lasers is that the relaxation time of inverse population in them greatly exceeds the decay time of the field in the cavity. As a result, relaxation oscillations appear which determine a high resonance sensitivity of the laser to various perturbations of the stationary state [8].

The dynamics of such lasers and, hence, the spectrum of relaxation oscillations depend substantially on the type of intermode interaction. There exist the two types of nonlinear interaction between modes in the active medium of a laser: purely energetic (through the saturation of the active medium by the field of individual modes) and phase-sensitive (through the scattering of mode fields by inversion oscillations induced by them) [9, 10]. The rate-equation models taking into account only the first type of mode interaction well describe the behaviour of multimode lasers with a Fabry – Perot resonator. Due to the absence of anisotropy of losses for modes with orthogonal polarisations, simultaneous lasing at these modes can occur. In this case, the interaction of orthogonally polarised modes with the active medium leads to the polarisation burning of the inverse population, resulting in the appearance, along with well-known relaxation oscillations in multimode lasers [9], of low-frequency relaxation oscillations of a new type responsible for the out-of-phase dynamics of orthogonally polarised modes (which we will call polarisation relaxation oscillations). It was shown [6 – 8] that the decisive factor of the dynamic behaviour of bipolarisation lasers is the collective interaction of all the longitudinal modes of the same polarisation with a total ensemble of orthogonally polarised modes. Therefore, all the models of polarisation interaction proposed so far were two-mode models in which the field was represented by two modes with identical or different longitudinal indices and orthogonal polarisations.

The experimental study [8] of an intracavity frequency-doubled solid-state laser with type II phase matching has demonstrated that, as the efficiency of nonlinear frequency conversion increased, the stationary regime became unstable through the Hopf bifurcation at the polarisation relaxation oscillation frequency.

Among papers devoted to the stabilisation of the stationary state of lasers, we can point out a number of theoretical and experimental papers, for example, [9 – 11] in which optoelectronic feedback controlling the laser diode current is proposed, which is proportional either to the deviation of the mode intensity from the stationary value or to derivatives of the mode intensities with respect to time.

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Received 25 August 2006; revision received 5 February 2007
Kvantovaya Elektronika **37** (6) 527 – 533 (2007)
Translated by M.N. Sapozhnikov

The main advantage of such a feedback is that it does not affect the stationary state of the system. In addition, feedback can be easily realised in practice.

In this paper, we studied theoretically the possible ways for expanding the region of stationary operation of a bipolarised laser by using optoelectronic feedback to control either the pump laser diode current or intracavity losses. We propose to modulate pump radiation by using feedback, which is inversely proportional to the derivative of the total radiation intensity or the derivative of the radiation intensity of a weak polarisation mode, while losses can be modulated by means of feedback proportional to the radiation intensity of one of the polarisation modes.

2. Model of a bipolarised laser

The dynamic behaviour of a laser can be described by a system of dimensional rate equations taking into account the angular (polarisation) burning of the population inversion:

$$\frac{dI_1}{d\tau} = \dot{I}_1 = G(N_0 + N_c - l_1 - \varepsilon I_2)I_1, \quad (1a)$$

$$\frac{dI_2}{d\tau} = \dot{I}_2 = G(N_0 - N_c - l_2 - \varepsilon I_1)I_2, \quad (1b)$$

$$\frac{dN_0}{d\tau} = \dot{N}_0 = A_0 - (1 + I_1 + I_2)N_0 - (I_1 - I_2)N_c, \quad (1c)$$

$$\frac{dN_c}{d\tau} = \dot{N}_c = A_c - (1 + I_1 + I_2)N_c - \frac{1}{2}(I_1 - I_2)N_0, \quad (1d)$$

where $\tau = t/T_1$ is the dimensionless time normalised to the relaxation time T_1 of the population inversion; $I_{1,2}$ are the intensities of polarisation modes normalised to the saturation-field intensity; N_0 and A_0 are the components of the population inversion and pump, respectively, homogeneous over the angle and longitudinal coordinate (N_0 is normalised to the population inversion in the absence of lasing and A_0 is normalised to the threshold pump intensity);

$$N_c = \frac{1}{2\pi L}\int_0^L \int_0^{2\pi} N(\psi)\cos 2\psi d\psi dz,$$

$$A_c = \frac{1}{2\pi L}\int_0^L \int_0^{2\pi} A(\psi)\cos 2\psi d\psi dz$$

are the spatially homogeneous angular cosine harmonics of the population inversion and pump; L is the cavity length; $G = T_1/T_c$; T_c is the field relaxation time in the cavity; $l_{1,2}$ are anisotropy parameters of losses in the cavity (in the isotropic case, $l_{1,2} = 1$); ε is the coefficient of nonlinear conversion of the fundamental radiation to the second harmonic radiation, which is proportional to the ratio of the second-harmonic intensity to the product of intensities of the interacting modes.

The system of equations (1) can be easily obtained from the model of a single-mode bipolarised laser [10] by neglecting in it all the terms containing the phases of polarisation modes and introducing a term characterising nonlinear losses upon frequency doubling. It was shown in [10] that in the case of linearly polarised pump radiation, the expression for A_c has the form

$$A_c = A_0 \frac{(1+b)^{1/2} - 1}{b} \cos 2\psi_p,$$

where $b = E_p^2 T_p \tau_2 \mu_p^2/h^2$ is the parameter of saturation of the active medium by the pump radiation; E_p^2 is the pump radiation intensity; τ_2^{-1} is the rate of nonradiative relaxation from the upper absorption level; μ_p is the dipole moment of the absorption line; T_p is the half-width of the absorption line; and h is Planck's constant. The parameter ψ_p determines the orientation of pump polarisation with respect to one of the polarisation modes of the laser. By varying ψ_p, the intensity ratio of polarisation modes can be controlled.

The linear analysis of the system of equations (1) gives the fourth-order characteristic equation having two pairs of the complex conjugated roots $\lambda_{1,2} = \delta_1 \pm i\Omega_1$ and $\lambda_{3,4} = \delta_2 \pm i\Omega_2$. The imaginary parts of the complex conjugate roots are the frequencies Ω_1 and Ω_2 of relaxation oscillations with the decay decrements equal to the real parts δ_1 and δ_2. The first pair of the roots describes in-phase relaxation oscillations, which are well-known in the dynamics of solid-state lasers, while the second pair describes low-frequency relaxation oscillations responsible for the out-of-phase dynamics of the polarisation modes of the laser.

Figure 1 shows the relaxation oscillation frequencies Ω_1, Ω_2 and decrements δ_1, δ_2, as well as the intensities of individual polarisation modes and total radiation as functions of the coefficient ε of nonlinear conversion of radiation to the second harmonic. The solid curves correspond to dependences in the absence of feedback. One can see the appearance of instability through the Hopf bifurcation at the polarisation relaxation oscillation frequency Ω_2: the sign of δ_2 changes at the point $\varepsilon = \varepsilon_{cr}^0$, where ε_{cr}^0 is the critical value of the parameter ε in the absence of feedback. In the region $\varepsilon > \varepsilon_{cr}^0$ near the instability boundary (Fig. 1a), the behaviour of the self-modulation oscillation frequency Ω_{mod} is shown. Figure 1c demonstrates the behaviour of the maximum and minimum values of the total radiation intensity I_{tot} and the intensities I_1 and I_2 of individual modes in the instability region. It follows from Fig. 1 that the undamped oscillations of the polarisation mode intensity are excited near the instability boundary.

Inside the instability region, chaotic oscillations of the mode intensities develop. This process, in the absence of feedback, is illustrated in Fig. 2, where the transfer functions of individual mode intensities and of their total intensity are presented for $\varepsilon = 0$ (the absence of second harmonic generation and the absence of instability) (Fig. 2a) and $\varepsilon = \varepsilon_{cr}^0 + \xi$, where $\xi \ll \varepsilon_{cr}^0$ (a small excess over the critical value of the nonlinear conversion coefficient and excitation of a low-frequency oscillation) (Fig. 2b). Figure 2c shows, for $\varepsilon \gg \varepsilon_{cr}^0$ (a considerable excess over the critical value of ε, chaos), only the transfer function for the total intensity because this function coincides with transfer functions for individual modes in the region of chaos.

To extend the region of stable lasing upon intracavity frequency conversion and suppress chaotic lasing, we introduced into the system of equations (1) the optoelectronic feedback of one of the two types: modulation of the pump parameter A_0 or modulation of losses $l_{1,2}$ of each of the modes. The system of equations (1) with introduced

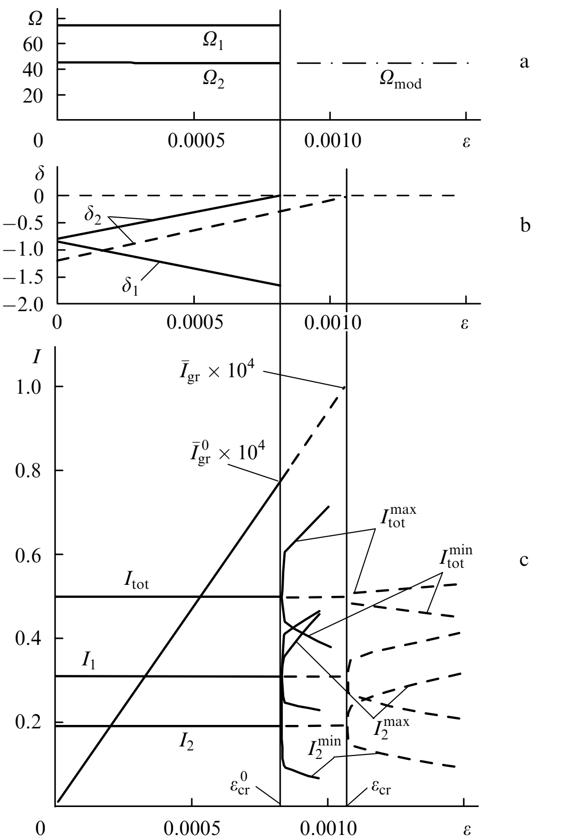

Figure 1. Dependences of the frequencies Ω_1, Ω_2 (a) and decrements δ_1, δ_2 (b) of relaxation oscillations and the intensities of longitudinal modes and their sum (c) on the second harmonic generation efficiency parameter ε in the absence (solid curves) and presence (dashed curves) of feedback. The system parameters: $f_2 = 0.01$, $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 25°$. The unlabeled curves in Fig. 1c belong to the mode of intensity I_1.

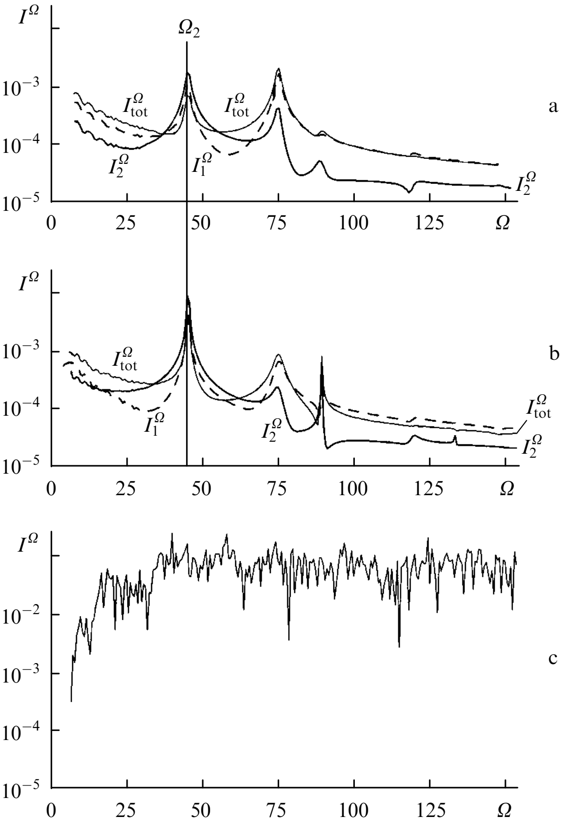

Figure 2. Transfer functions of intensities of individual modes I_1^{Ω} и I_2^{Ω} and of their total intensity I_{tot} in the absence of feedback for the second-harmonic generation efficiency parameter $\varepsilon = 0$ (a), 0.00085 (b), and 0.009 (c). The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 0$, $\varepsilon_{cr}^0 = 0.0082$.

feedback was integrated numerically by the Runge–Kutta method.

2.1 Control of the pump parameter

Optoelectronic feedback providing the control of the pump parameter (laser diode current) and proportional to derivatives of the intensity of polarisation modes is introduced in the system of equations (1) as

$$A_0 = A(1 - f_1\dot{I}_1 - f_2\dot{I}_2).$$

Here, $f_{1,2}$ is the feedback coefficient. If one of these coefficients is zero, selective feedback takes place either over the strong polarisation mode for $f_2 = 0$ or the weak polarisation mode for $f_1 = 0$. If these coefficients are equal, feedback over the total intensity $f_1 = f_2 \equiv f_{tot}$ takes place. Finally, if both coefficients are nonzero and change independently of each other, we are dealing with combined feedback [9, 12]. Studies have shown that the introduction of feedback over the total intensity ($f_{tot} > 0$) or over the weak mode ($f_1 = 0, f_2 > 0$) results in the expansion of the stationary lasing region compared to this region in the absence of feedback. The use of the strong mode of any sign for feedback does not produce stabilisation but, on the contrary, causes the development of instability.

The dashed curves in Figures 1b, c show the behaviour of the decrements of relaxation oscillations and the total radiation intensity in the case of feedback over the total intensity. One can see that feedback causes the displacement of the instability region to the right – to the side of increasing nonlinear conversion coefficient. In addition, feedback in the instability region considerably reduces the modulation depth, making this regime regular.

Figure 3 presents total radiation intensities for small (Fig. 3a) and considerable (Fig. 3b) excesses ε over the instability threshold (i.e. ε_{cr}) before and after the introduction of feedback over the weak mode. The behaviour of the system for the same parameters as in Fig. 2 but with feedback over the total intensity is similar as a whole to its behaviour in the case of feedback over the weak mode. However, feedback over the total intensity proves to be less efficient than that over the weak mode, the stationary lasing region (the maximum nonlinear conversion coefficient ε) being smaller in the case of total-intensity feedback; in addition, the self-modulation regime appears at higher feedback coefficients.

Figure 1c also shows the dependence of the radiation intensity $I_{gr} = \varepsilon I_2 I_1$ at the double frequency during stationary lasing. This dependence clearly demonstrates the increase in the stationary intensity I_{gr} after the introduction of optoelectronic feedback: $\bar{I}_{gr}^0 < \bar{I}_{gr}$, where $\bar{I}_{gr}^0$ and $\bar{I}_{gr}$ are the maximum values of the intensity I_{gr} for the given set of system parameters in the absence and presence of feedback, respectively. The study shows that, other conditions being the same, the second harmonic intensity in the stationary regime in the case of weak-mode feedback is higher than

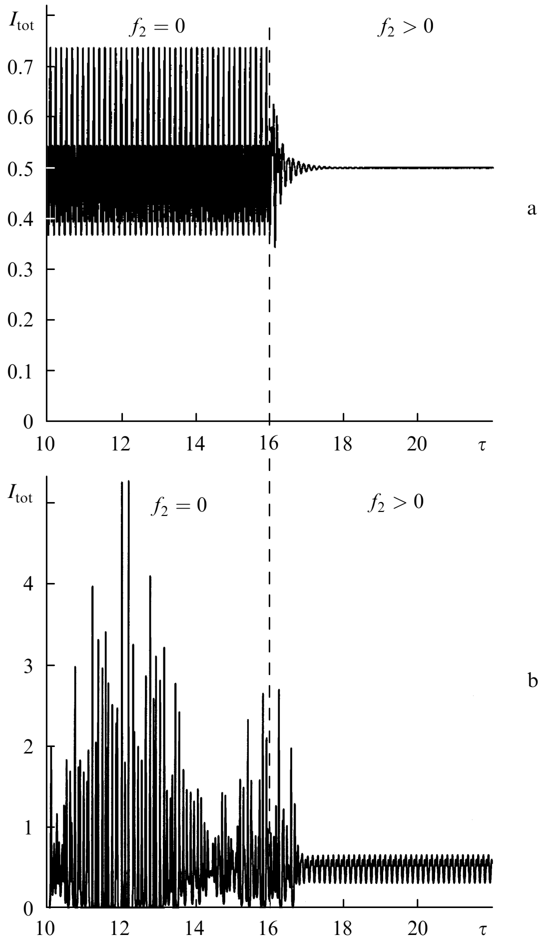

Figure 3. Total radiation intensity for $\varepsilon = 2\varepsilon_{cr}$ (a) and $8\varepsilon_{cr}$ (b) before and after the introduction of weak-mode feedback. The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 0$.

that for total-intensity feedback. This can be explained by the fact that the total radiation intensity is determined not only by the weak mode but also by the strong one, the selective use of the latter not suppressing the low-frequency oscillation responsible for instability but, on the contrary, increasing it. This effect is caused by the coincidence of phases of weak oscillations in the frequency region of both relaxation oscillations in the strong mode, unlike the weak mode where low-frequency and high-frequency oscillations are out-of-phase [13].

The results described above agree with the influence of pump feedback in a two-mode laser without frequency doubling ($\varepsilon = 0$) studied earlier [14]. In this case, suppression is also achieved for feedback proportional to the derivative of the total mode intensity, a weaker suppression being possible for feedback proportional to the derivative of the weak mode intensity and being impossible when the strong mode is used.

Figure 4 shows the dependences of the modulation depth of the total radiation intensity

$$\mu_{tot} = \frac{I_{tot}^{max} - I_{tot}^{min}}{I_{tot}^{max} + I_{tot}^{min}}$$

on the nonlinear conversion coefficient ε in the absence of feedback [Fig. 4, curve (*1*)] and in the presence of total-intensity feedback [Fig. 4, curve (*2*)] and weak-mode

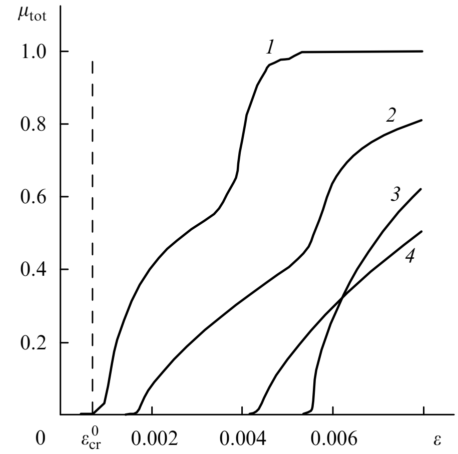

Figure 4. Dependences of the modulation depth of the total intensity μ_{tot} on ε in the absence of feedback ($f_1 = f_2 = 0$) (*1*) and in the presence of total-intensity feedback ($f_1 = f_2 = 0.005$) (*2*) and weak-mode feedback for $f_1 = 0, f_2 = 0.005$ (*3*) and $f_1 = 0, f_2 = 0.02$ (*4*). The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 0$.

feedback [Fig. 4, curves (*3*) and (*4*)]. These dependences clearly demonstrate a higher efficiency of weak-mode feedback. Therefore, we will consider below only weak-mode feedback ($f_1 = 0$, $f_2 > 0$).

It is known from earlier papers [7, 8, 10] that a change in the orientation ψ_p of pump polarisation leads to a change in the ratio of intensities I_1^0 and I_2^0 of polarisation modes in the absence of feedback. In this case, the instability boundaries (ε_{cr}, f_2^{cr}), dividing the regions of stationary lasing and instability, change upon intracavity frequency doubling. Curve (*3*) in Fig. 5 described by the expression

$$\bar{I}_{gr}(\varepsilon_{cr}, f_2^{cr}, \psi_p) = \varepsilon_{cr}(f_2^{cr}, \psi_p) I_1(\psi_p) I_2(\psi_p),$$

shows the dependence of the maximum achievable second-harmonic radiation intensity (at the stationary-regime stability boundary) on the parameter ψ_p in the presence

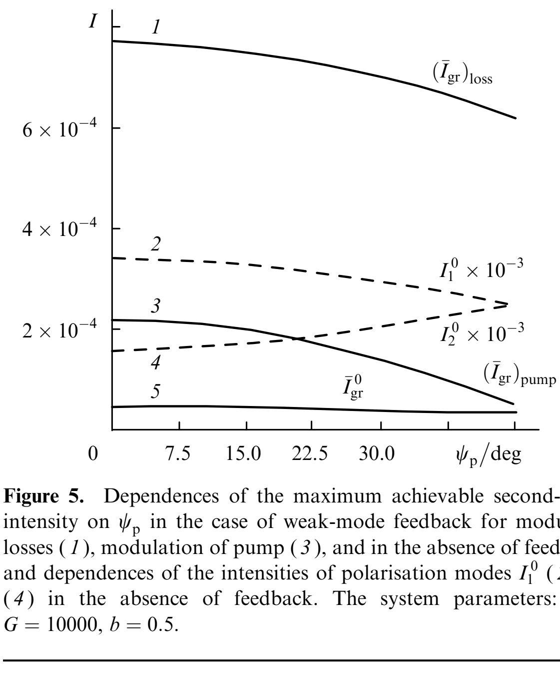

Figure 5. Dependences of the maximum achievable second-harmonic intensity on ψ_p in the case of weak-mode feedback for modulation of losses (*1*), modulation of pump (*3*), and in the absence of feedback (*5*); and dependences of the intensities of polarisation modes I_1^0 (*2*) and I_2^0 (*4*) in the absence of feedback. The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$.

of pump feedback. One can see that the maximum value $(\bar{I}_{gr})_{pump}$ is achieved for $\psi_p = 0$, which corresponds to the maximum difference of mode intensities, while the minimum value is achieved for $\psi_p = 45°$, which corresponds to the modes of equal intensities. For comparison, Fig. 5 shows the dependence of the maximum achievable second-harmonic intensity $\bar{I}^0_{gr}$ in the absence of feedback.

The bifurcation diagram in Fig. 6 summs up the study of the influence of the parameters ε and f_2 on the behaviour of the system. The plane εf_2 is divided into three regions: stationary lasing (I), self-modulation (II), and chaotic (III) oscillations. The stability region boundary for a fixed ratio of mode intensities (determined by the angle ψ_p) depends both on the nonlinear conversion coefficient ε and the feedback coefficient f_2. The boundary between regions II and III is somewhat conditional because we judged the appearance of chaos in the system only from time realisations and power spectra: the appearance of an irregularity and a broad noise component in the power spectrum indicated the passage to the region of chaotic oscillations.

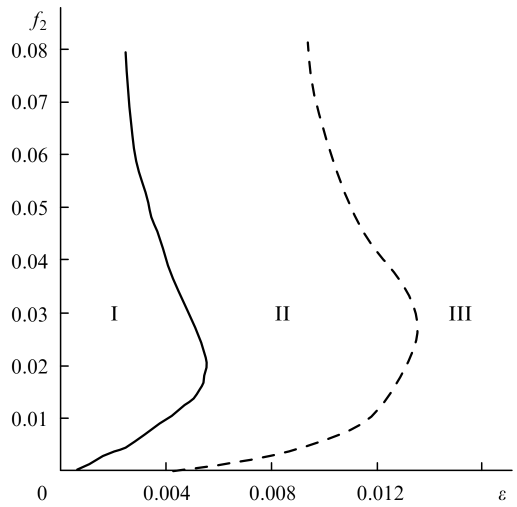

Figure 6. Bifurcation diagram: regions of stability (I), self-modulation oscillations (II), and chaos (III) in the plane εf_2 for weak-mode feedback. The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 0$.

2.2 Modulation of losses

Our study of the action of optoelectronic pump feedback on the system (Fig. 5) shows that the stationary second-harmonic intensity can be increased approximately by a factor of four. However, the instability can be suppressed more efficiently by using feedback of another type by modulating losses. The stabilisation of the process in the stationary state is achieved by introducing out-of-phase modulation of losses in both modes, which is proportional to the intensity of one of the modes. In the system of equations (1), this is described as

$$l_1 = 1 + F, \quad l_2 = 1 - F,$$

where

$$F = -f_1 I_1 \quad \text{or} \quad F = f_2 I_2, \quad f_{1,2} \geqslant \varepsilon - \varepsilon^0_{cr}, \qquad (2)$$

where ε^0_{cr} corresponds to the instability boundary in the absence of feedback.

Such a form of feedback is caused by the simplest theoretical calculations: due to nonlinear intracavity second harmonic generation (described by the parameter ε), the instability appears first of all in the strong mode, whose behaviour is described by Eqn (1a) containing the term $-\varepsilon I_2$ in the parentheses, which is responsible for nonlinearity resulting in the appearance of instability. Therefore, it is necessary to compensate the action of this term with the help of feedback in the form $+\varepsilon I_2$, not producing instability in the weak mode by introducing feedback in the form $-\varepsilon I_2$ in Eqn (1b).

Figures 7 and 8 present the results of introducing the modulation of losses described by expression (2) into the system of equations (1). Figure 7 shows the total radiation intensities for a small (Fig. 7a) and considerable (Fig. 7b) excesses of ε over the critical value before and after the introduction of loss-modulation feedback. The bifurcation diagram in the plane εf_2 is shown in Fig. 8. One can see from Figs 7 and 8 that this method is more efficient than pump modulation not only due a decrease in the feedback coefficient required for stabilisation but also because it provides stable lasing for large nonlinear conversion coefficients.

Note that such a feedback, proportional to the weak-mode intensity (rather than to the derivative of the intensity,

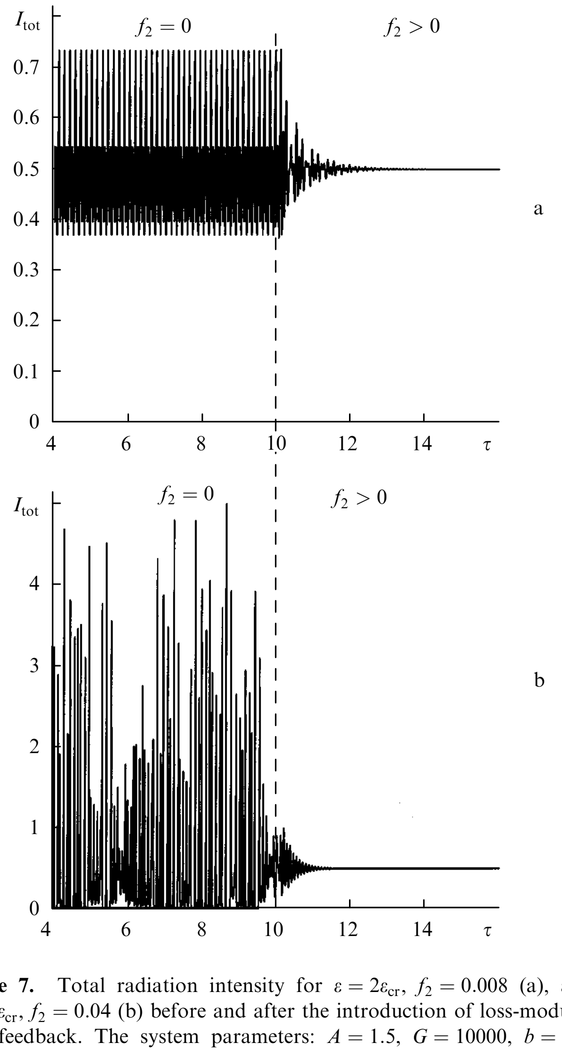

Figure 7. Total radiation intensity for $\varepsilon = 2\varepsilon_{cr}$, $f_2 = 0.008$ (a), and $\varepsilon = 8\varepsilon_{cr}$, $f_2 = 0.04$ (b) before and after the introduction of loss-modulation feedback. The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_p = 0$.

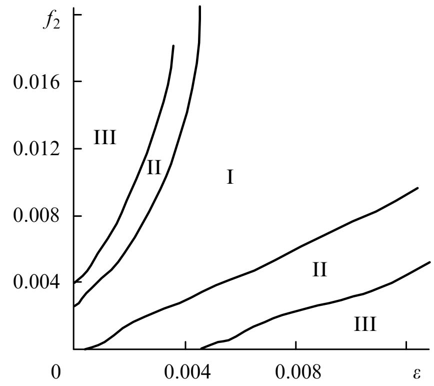

Figure 8. Bifurcation diagram: regions of stability (I), self-modulation oscillations (II), and chaos (III) in the plane εf_2 for loss-modulation feedback. The system parameters: $A = 1.5$, $G = 10000$, $b = 0.5$, $\psi_{\rm p} = 0$.

as in the case of pump feedback), causes a small change in the stationary state of the system, but allows the stabilisation of the system in this new stationary state at higher nonlinear conversion coefficients than in the case of pump feedback.

Curve (*1*) in Fig. 5 shows the behaviour of the maximum achievable second-harmonic radiation intensity $(\bar{I}_{\rm gr})_{\rm loss}$ (at the stability boundary of the stationary regime) in the presence of loss feedback. One can see that, as in the case of pump modulation, the maximum value $(\bar{I}_{\rm gr})_{\rm loss}$ is achieved for $\psi_{\rm p} = 0$, which corresponds to the maximum difference of mode intensities (i.e. to the maximum of the polarisation mode of intensity I_1^0 and the mode minimum of intensity I_2^0), and the minimum value is achieved for $\psi_{\rm p} = 45°$, which corresponds to modes of equal intensities. The behaviour of the system is completely similar to its behaviour in the case of pump modulation, but the maximum achievable value $(\bar{I}_{\rm gr})_{\rm loss}$ for all $\psi_{\rm p}$ is considerably higher.

Upon modulation of losses, as in the case of pump feedback, the bifurcation plane εf_2 is divided into three regions of stability (I), self-modulation oscillations (II), and chaos (III). The boundary of the stability region for a fixed ratio of mode intensities (determined by the angle $\psi_{\rm p}$) depends both on the nonlinear conversion coefficient and the feedback coefficient; however, the shape of stability regions proves to be considerably more complex. Thus, for a fixed value of ε, the increase in $f_{1,2}$ ($f_{1,2} > \varepsilon - \varepsilon_{\rm cr}^0$) up to some limit facilitates stabilisation, but as $f_{1,2}$ is further increased, the instability appears. Note that stability region I is not infinite – as ε is further increased (beyond the range presented in Fig. 8), the system becomes unstable due to the properties of the numerical simulation. In experiments, this will correspond to the appearance of instability due to the amplification of random fluctuations at a high nonlinear conversion coefficient.

Another difference between the action of the pump and loss feedbacks is the behaviour of the frequency of the established self-modulation regime. One can see from Fig. 1a that the self-modulation oscillation frequency is close to the relaxation oscillation frequency, but as the nonlinear conversion coefficient increases [curve (*1*) in Fig. 9], this frequency slowly decreases. Numerical calculations show that this decrease occurs most rapidly in the case of total intensity feedback and most slowly in the case of weak-mode feedback [curves (*1*) and (*3*) in Fig. 9].

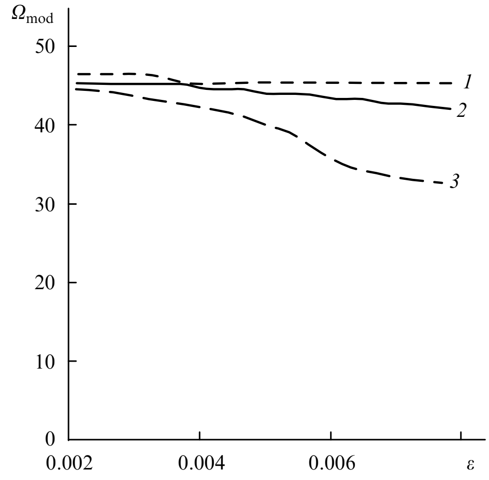

Figure 9. Dependences of the frequency $\Omega_{\rm mod}$ of established self-modulation oscillations on ε for systems with weak-mode (*1*), loss (*2*), and total-intensity (*3*) feedbacks.

3. Conclusions

We have proposed methods to stabilise chaos in a bipolarised frequency-doubled solid-state laser which provide either the passage of chaotic oscillations to the self-modulation regime (quasi-harmonic oscillations with a constant amplitude) or a complete passage of the system to the stationary state. The most efficient stabilisation method is optoelectronic pump feedback, which is proportional to the derivative of the weak-mode intensity. The method for suppressing the chaotic radiation dynamics of a laser with the help of optoelectronic feedback controlling the pump parameter is simple in practical realisation.

The complete stabilisation of the system (transition to the stationary state) even when the nonlinear conversion parameter considerably exceeds the critical value is achieved in the case of feedback performed by the intracavity modulation of losses, which is proportional to the intensity of one of the modes. In this case, the modulation should affect both modes out of phase. Although we are not aware of the experimental realisation of loss feedback in frequency-doubled lasers, we can assume that such an electrooptical Q-switching can be also achieved, for example, by using a Pockels cell.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant No. 06-02-16632) and the grant of the President of the Russian Federation for the Support of Leading Scientific Schools (No. NSh-7738.2006.2).

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