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Self-diffraction of light waves by a local photorefractive grating in a crystal of the symmetry group $\bar{4}3m$ in the transverse configuration

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***Abstract*.** **The exact solution of nonlinear equations describing the symmetric stationary interaction of two light waves of an arbitrary polarisation on a local photorefractive grating in a crystal of the symmetry group $\bar{4}3m$ in the transverse configuration of crystal faces is obtained in the paraxial limit. The polarisation of light waves and efficiency of energy transfer between them are studied as functions of the reduced length of a sample and the intensity ratio of incident waves.**

Keywords: *self-diffraction, photorefractive grating, interaction of waves.*

The exact analytic solutions of nonlinear equations describing the matched interaction of two light waves on a photorefractive nonlinearity in non-centrally symmetric crystals give detailed information on the influence of various interaction parameters on the specific features of experimental studies and photorefractive devices [1 – 8].

The analytic solution of two nonlinear equations of coupled waves describing the scalar two-wave interaction on a transmission photorefractive grating in the stationary state was obtained in [1 – 3]. The interaction of two light waves with specified orthogonal polarisations, which is also described by two nonlinear equations, was considered in [4, 5]. The stationary vector two-wave interaction can be described in the paraxial limit by four nonlinear equations for the scalar amplitudes of the orthogonal polarisation components of the light waves. The exact solution of four equations was found for the transmission two-wave interaction on a nonlocal photorefractive grating produced in cubic photorefractive crystals of the symmetry group $\bar{4}3m$ such as GaAs, InP, and CdTe [6, 7]. In the absence of gyrotropy, the analytic results describing the interaction of light waves in these crystals are also valid for interaction in crystals of the symmetry group 23.

In this paper, the exact solution is obtained for nonlinear equations describing the stationary self-diffraction of two light waves of an arbitrary polarisation by a local photorefractive grating produced in a crystal of the symmetry group $\bar{4}3m$ in the so-called transverse configuration of crystal faces [7, 8] (Fig. 1). The polarisation of the interacting light waves and efficiency of energy exchange between them are studied as functions of the reduced interaction length and the intensity ratio of incident waves.

Figure 1 shows the scheme of two-wave interaction in a cubic photorefractive crystal in the external permanent electric field $\boldsymbol{E}_0$ applied along the vector $\boldsymbol{K}$ of the interference light grating with the intensity $I = I_0[1 + m\cos(\boldsymbol{Kr})]$. In this case, the field $\boldsymbol{E}_0 = E_0\boldsymbol{p}$ and the space-charge field $\boldsymbol{E}_{sc} = E_{sc}\boldsymbol{p} = [-mE_{eff}\exp(i\boldsymbol{Kr})/2 + \text{c.c.}]\boldsymbol{p}$ induce due to the linear electrooptical effect [9] the perturbations of the dielectric constant $\Delta\varepsilon_{ij}^{(0)} = n^4 r_{41} E_0 g_{ij}$ and $\Delta\varepsilon_{ij}^{sc} = n^4 r_{41} \times E_{sc} g_{ij}$, of the same structure, where n is the refractive index of a medium and r_{41} is the electrooptical coefficient. For the transverse configuration, the components of the unit vector $\boldsymbol{p}$ and of the second-rank tensor g_{ij} in the crystal-physics coordinate system are determined by the relations $p_3 = g_{ii} = g_{12} = g_{21} = 0, -p_1 = p_2 = g_{13} = g_{31} = -g_{23} = -g_{32} = -1/\sqrt{2}$ [7].

The external field $\boldsymbol{E}_0$ is produced by the electric voltage applied to the crystal sides (usually, with metal coatings). Let us neglect the blocking effect in the electrode region [8, 10] and consider the case when the amplitude E_0 of the external electric field satisfies the condition $E_d \ll E_0 \ll E_q$, where E_d and E_q are the diffusion and trap-saturation fields, respectively [8]. In the case of interaction on a transmission photorefractive grating with the grating spacing $\Lambda = 2\pi/|K| > 20$ μm in crystals with the trap concentration $N_a > 10^{22}$ m^{-3}, the typical fields are $E_d < 80$ V cm^{-1} and $E_q > 100$ kV cm^{-1}. Therefore, the condition mentioned above is satisfied for $E_0 \approx 10$ kV cm^{-1}. In this case, the local component makes the main contribution to the effective amplitude E_{eff} of the field E_{sc}. In the approximation linear in the modulation coefficient m, this amplitude is equal with good accuracy to the external field: $E_{eff} = E_0$ [8].

For convenience of analysis, we will use a special coordinate system with axes z and y coinciding with the axes of the coupling matrix $\boldsymbol{H}$ [6, 7]. For the transverse configuration under study, the components of the matrix $\boldsymbol{H}$ in the traditional coordinate system with axes directed along the TE and TM components of the light-field polarisation are $H_{MM} = H_{EE} = 0$, and $H_{ME} = -1$. In this case, the axes z and y are located at angles of $\pm 45°$ to the grating vector $\boldsymbol{K}$ [7], as shown in Fig. 1. For the grating spacing Λ presented above, the paraxial limit is valid, in which only the components $y(S_y, R_y)$ and $z(S_z, R_z)$ of the vector amplitudes of the two interacting light waves

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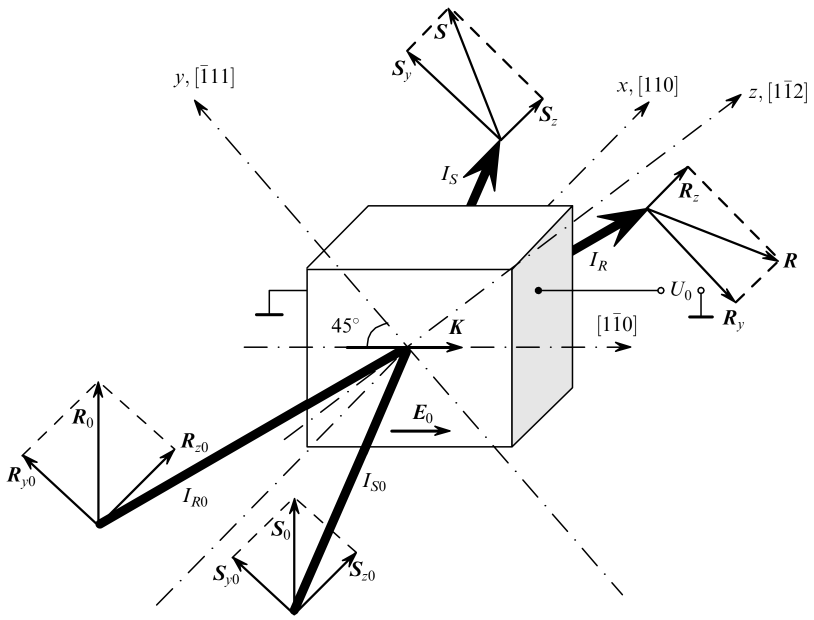

Figure 1. Two-wave interaction in a photorefractive crystal of the symmetry group $43m$ in an external electric field in the transverse configuration.

$$\tilde{\boldsymbol{S}} = \boldsymbol{S}\exp[\mathrm{i}(\omega t - \boldsymbol{k}_S\boldsymbol{r})],\ \tilde{\boldsymbol{R}} = \boldsymbol{R}\exp[\mathrm{i}(\omega t - \boldsymbol{k}_R\boldsymbol{r})]$$

can be considered nonzero. In the approximations adopted above, the equations for coupled waves can be written in the form

$$\frac{\mathrm{d}S_{y,z}}{\mathrm{d}x} = \mp\mathrm{i}\frac{\gamma}{2}\frac{S_yR_y^* + S_zR_z^*}{I_0}R_{y,z},$$
$$\frac{\mathrm{d}R_{y,z}}{\mathrm{d}x} = \mp\mathrm{i}\frac{\gamma}{2}\frac{S_y^*R_y + S_z^*R_z}{I_0}S_{y,z}, \tag{1}$$

where $\gamma = 2\pi n^3 r_{41}E_0/\lambda$ is the coupling constant of the photorefractive grating; λ is the light wavelength;

$$m = 2\frac{S_yR_y^* + S_zR_z^*}{I_0} \tag{2}$$

is the modulation coefficient of the interference pattern. Equations (1) and (2) differ from equations (11) and (12) obtained in [7] by the right-hand side linear in m and the presence of the imaginary unit, which appears due to the local photorefractive response of the crystal.

The system of equations for coupled waves (1) has the obvious integral $I_0 = |S_y|^2 + |S_z|^2 + |R_y|^2 + |R_z|^2 = \mathrm{const}$ corresponding to the law of conservation of the light-field energy during its distribution between interacting waves in a nonabsorbing photorefractive crystal. In the case under study, as for a nonlocal response, the laws of conservation of energy $I_y = |S_y|^2 + |R_y|^2$ and $I_z = |S_z|^2 + |R_z|^2$ are fulfilled. The integrals I_y and I_z describe the conservation of the parts of light energy contained in orthogonal polarisation components whose orientation coincides with that of the eigenvectors of the coupling matrix. However, unlike a crystal with a nonlinear response [7], the energy of the light-field components with phases shifted by $\pi/2$ with respect to each other is not conserved in the general case. Therefore, the polarisation of linearly polarised waves incident on a crystal becomes elliptical at the crystal output.

The two additional laws of conversion of energy can be written in the form of integrals

$$|S_y|^2 - |S_z|^2 = I_{\Delta S}, \quad |R_y|^2 - |R_z|^2 = I_{\Delta R}. \tag{3}$$

Taking these laws into account, the equation for the coefficient m can be obtained in the form

$$\frac{\mathrm{d}m}{\mathrm{d}x} = \mathrm{i}\frac{\gamma I_\Delta}{2I_0}m, \tag{4}$$

where $I_\Delta = I_{\Delta S} - I_{\Delta R}$. It follows from (4) that the modulus of the modulation coefficient $m = m_0\exp[\mathrm{i}xI_\Delta/(2I_0)]$ is conserved after the interaction: $|m| = m_0$ (the contrast of the interference pattern does not change). The linear (in x) phase shift for the coefficient m results in a periodic bending of the interference fringes with the period $\Lambda_x = 2\pi_0/(\gamma I_\Delta)$.

If the spatial dependence of the modulation coefficient is known, equations (1) can be decoupled. For example, the equation for the y component of the signal-wave polarisation can be written in the form

$$\frac{\mathrm{d}^2S_y}{\mathrm{d}x^2} - \mathrm{i}\frac{\gamma I_\Delta}{2I_0}\frac{\mathrm{d}S_y}{\mathrm{d}x} + \frac{\gamma^2m_0^2}{16}S_y = 0. \tag{5}$$

The solutions of this equation and similar equations for other components of the light field have the form

$$S_{y,z} = \exp\left(\mathrm{i}\frac{\gamma x}{4}\frac{I_\Delta}{I_0}\right)\left[S_{y0,z0}\cos\left(\frac{\gamma x}{4}\frac{I_\mathrm{d}}{I_0}\right)\right.$$
$$\left.\pm\mathrm{i}\frac{m_0I_0R_{y0,z0} \mp I_\Delta S_{y0,z0}}{I_\Delta}\sin\left(\frac{\gamma x}{4}\frac{I_\mathrm{d}}{I_0}\right)\right], \tag{6}$$

$$R_{y,z} = \exp\left(-\mathrm{i}\frac{\gamma x}{4}\frac{I_\Delta}{I_0}\right)\left[R_{y0,z0}\cos\left(\frac{\gamma x}{4}\frac{I_\mathrm{d}}{I_0}\right)\pm\right.$$

$$\pm\,\mathrm{i}\frac{m_0 I_0 S_{y0,z0} \mp I_\Delta R_{y0,z0}}{I_\mathrm{d}}\sin\left(\frac{\gamma x}{4}\frac{I_\mathrm{d}}{I_0}\right)\Bigg], \quad (7)$$

where $I_\mathrm{d} = (I_\Delta^2 + m_0^2 I_0^2)$.

It follows from the exact analytic solutions (1) and (2) of the nonlinear system of equations of coupled waves (1) that the polarisation and intensity of the interacting waves change periodically over the interaction length with the period $\Lambda_{x\mathrm{d}} = 2\pi I_0/(\gamma I_\mathrm{d})$. The period of spatial oscillations depends on the intensity ratio $\beta = I_{R0}/I_{S0}$ of the incident waves. In the case of small ($\beta \ll 1$) or large ($\beta \gg 1$) intensity ratios, the values of I_Δ/I_0 and I_d/I_0 are independent of β, and variations in the scalar amplitudes over the interaction length x are determined only by the difference of the intensities of the y and z polarisation components of the light field of a strong wave ($I_{\Delta S}$ or $I_{\Delta R}$). Note that the integrals $I_{\Delta S}$ and $I_{\Delta R}$, as the integral I_d, are independent of the phase relations between y and z components of the light field. Therefore, the period of spatial oscillations of the polarisation of light waves in the case of the same linear polarisation of incident light waves coincides with that in the case of the same elliptical polarisation of incident waves if the intensities of the y and z polarisation components of these waves in the first case are equal to the corresponding y and z polarisation components in the second case. A similar note is also valid for the period of spatial oscillations of the light-wave intensity. It is obvious that the same period can be selected for different intensity ratios of the incident waves by varying their polarisation parameters.

Consider the vector amplitudes $\boldsymbol{S}$ and $\boldsymbol{R}$ and intensities I_S and I_R of the interacting light waves when waves with the TE or TM polarisation are incident on a crystal at an angle of $\pm 45°$ to the y axis. In this case, the integral I_Δ is zero, and fringes of the photorefractive grating do not bend over their interaction length x. By using relations (6) and (7), we can obtain the expressions

$$\boldsymbol{S}_{\mathrm{TE,TM}} = \left(\frac{I_0}{1+\beta}\right)^{1/2}\left[\cos\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\frac{\boldsymbol{z}^{(0)} \pm \boldsymbol{y}^{(0)}}{\sqrt{2}}\right.$$
$$\left. -\,\mathrm{i}\sqrt{\beta}\sin\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\frac{\boldsymbol{z}^{(0)} \mp \boldsymbol{y}^{(0)}}{\sqrt{2}}\right], \quad (8)$$

$$\boldsymbol{R}_{\mathrm{TE,TM}} = \left(\frac{I_0}{1+\beta}\right)^{1/2}\left[\sqrt{\beta}\cos\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\frac{\boldsymbol{z}^{(0)} \pm \boldsymbol{y}^{(0)}}{\sqrt{2}}\right.$$
$$\left. -\,\mathrm{i}\sin\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\frac{\boldsymbol{z}^{(0)} \mp \boldsymbol{y}^{(0)}}{\sqrt{2}}\right], \quad (9)$$

$$I_S = \frac{I_0}{1+\beta}\left[\cos^2\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right) + \beta\sin^2\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\right], \quad (10)$$

$$I_R = \frac{I_0}{1+\beta}\left[\beta\cos^2\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right) + \sin^2\left(\frac{\gamma x}{2}\frac{\sqrt{\beta}}{1+\beta}\right)\right], \quad (11)$$

where $\boldsymbol{y}^{(0)}$ and $\boldsymbol{z}^{(0)}$ are the unit vectors of coordinate axes, and the insignificant phase factor $\exp[\mathrm{i}\gamma x I_\Delta/(2I_0)]$ is omitted in (8) and (9). One can see from (10) and (11) that for $\beta = 1$, the intensities of light waves do not change upon self-diffraction. The waves have elliptical polarisation at an arbitrary interaction length. The principal polarisation axes coincide with the coordinate axes y and z, and the ratio of their lengths, characterising the ellipticity of the light wave, is equal to $\tan(\gamma x/4)$. For the reduced length $\gamma x = \pm\pi$, the light waves become circularly polarised, and the crystal serves as a quarter-wave plate. For $\gamma x = 2\pi$, the polarisation of light waves is linear and orthogonal to the polarisation of incident waves. For $\gamma x = 4\pi$, polarisations of the output and input waves coincide. The fixed-field approximation for a strong wave, for example, the reference wave with $I_R \simeq I_0$ is valid if $\gamma x \ll 2(1+\beta)/\sqrt{\beta}$. In this case, the signal-wave intensity increases quadratically: $I_S \simeq I_{S0}[1 + (\gamma^2 x^2/4)]$.

Figure 2 shows the dependences of the intensities I_S and I_R, normalised to the total intensity I_0, on the reduced length γx for different intensity ratios β of the incident waves. It follows from Fig. 2 and expressions (10) and (11) that in the case under study, despite the local nature of the photorefractive response, the efficient energy exchange can occur between the interacting waves. The energy transfer efficiency oscillates depending on γx. The oscillation period depends on the ratio β, which is the characteristic nonlinearity parameter of the given problem. The dependence of the oscillation period on the nonlinearity parameter is typical for nonlinear oscillation processes of different types (see, for example, [11]). In the case considered here, the period of spatial oscillations increases for $\beta \to 1$, as in the classical example of oscillations of a mathematical pendulum whose period increases with increasing the nonlinearity of oscillations. A specific feature is that nonlinearity does not cause the broadening of the spectral band of the oscillation process, which is typical for most nonlinear oscillatory systems.

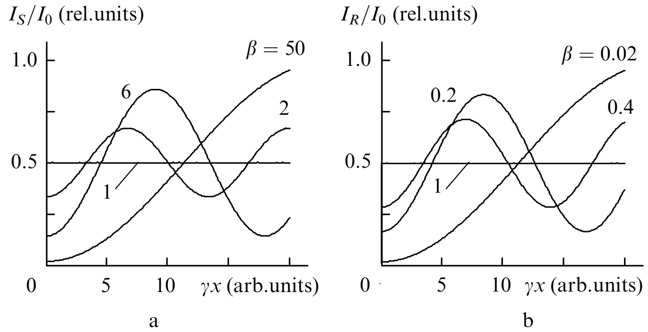

Figure 2. Dependences of the normalised intensities of light waves on the reduced interaction length for different intensity ratios of the waves incident on a crystal.

Figure 3 presents the dependences of the two-wave gain $\Gamma = \ln[\beta I_S(\beta)/I_R(\beta)]/x$ on the intensity ratio β of the incident waves for different reduced lengths γx. A specific feature of the dependence $\Gamma(\beta)$ is its nonmonotonic type. This is explained by the fact that not only amplitudes but also the spatial period of the oscillating intensity distributions of light waves change depending on β. For $\gamma x > 3.73$, this leads to the nonmonotonic dependences $I_S(\beta)$ and $I_R(\beta)$, resulting in the nonmonotonic dependence $\Gamma(\beta)$. The nonmonotonic dependence of the two-wave gain on the intensity ratio of light beams at the input face of a crystal is not typical for most experimental and theoretical studies of two-wave interaction in photorefractive crystals (see, for example, [2, 3, 12–18]). The exception is experiments on the two-wave interaction on a nonlocal photorefractive grating in a $Bi_{12}SiO_{20}$ crystal in the external meander field performed in [19]. However, the presence of a point of inflection in the dependence $\Gamma(\beta)$ is explained in [19] by the influence of corrections to the amplitude of the first harmonic of the space-charge field, which depend nonlinearly on the modulation coefficient m of the interference radiation pattern.

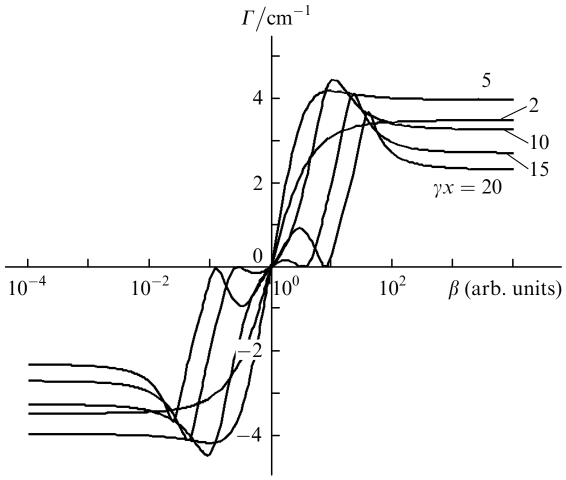

Figure 3. Dependences of the two-wave gain on the intensity ratio of the waves incident on a crystal for different reduced interaction waves.

Thus, we have obtained exact solutions of nonlinear coupled equations for the amplitudes of two light waves of an arbitrary polarisation interacting on a local photorefractive grating produced in a crystal of the symmetry group $\bar{4}3m$ in the case of the transverse configuration of crystal faces. The solutions show that the polarisation of the interacting waves and the efficiency of energy exchange between them depend periodically on the reduced interaction length. The period of spatial oscillations depends on the intensity ration β of the incident waves. The periodic dependence of the energy-exchange efficiency on the reduced interaction length results in the nonmonotonic dependence of the two-wave gain on the ratio β. When the intensities of the incident waves are equal, they are conserved. In this case, linearly polarised light waves can be transformed, depending on the reduced length, to the waves with elliptic (in particular, circular) and orthogonal linear polarisations.

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