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Numerical simulation of a *Q*-switched cw chemical HF laser

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Abstract. **A mathematical model of a *Q*-switched cw chemical laser is developed. The model is based on a simplified description of the active medium and the exact wave description of the mode structure of radiation. The model is used to study numerically the spatial – angular parameters of the output beam of a HF laser. We considered an unstable cavity that provided a high stability of the axis of the laser-pulse radiation pattern upon *Q*-switching with the help of a rotating mirror. The results of numerical simulation showed that for typical dimensions of a plane or an unstable cavity of the positive branch in a cw chemical laser, the angular radiation divergence rapidly changes in time and on average exceeds the diffraction limit by an order of magnitude. The problem can be solved by replacing a rotating mirror *Q* switch by a fixed one with time-dependent transmission. We also considered a negative-branch unstable cavity which can provide a stable radiation divergence upon *Q*-switching produced by a rotating mirror.**

Keywords: *chemical laser, cavity Q-switching, repetitively pulsed lasing, unstable cavity.*

1. Introduction

A common feature of theoretical [1, 2] and experimental [3 – 6] studies of the repetitively pulsed regime of continuously pumped *Q*-switched lasers is that they were devoted to the analysis of formation of the required temporal profile of the output laser beam. The spatial – angular parameters of radiation and the influence of *Q*-switching on the angular divergence of the output beam were not considered in these papers. At the same time, this question is quite important for practical applications requiring the transport of radiation to remote objects.

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This paper is devoted to the numerical simulation of a supersonic cw chemical HF laser operating in the *Q*-switching regime. A mathematical model developed in the paper allows us to analyse both the laser-pulse shape and the dynamics of the angular and spatial parameters of the output beam. The numerical model of a chemical laser should include in the general case the equations describing the kinetic and gas-dynamic parameters of the active medium and also the equations for calculating the spatial structure of the light field in the cavity. In the case of pulsed lasing, a mathematical model becomes considerably more complicated compared to the case of cw lasing because the equations of the model become nonstationary. The numerical solution of these equations for high-power lasers with a large volume of the active medium proves to be a complicated problem. Because of this, the relevant physical processes are described in practice in a simplified form. The numerical model presented below is based on the approximate description of the active medium. Such an approach allows us to use a sufficiently accurate wave description of the mode structure of radiation and to analyse the angular parameters of the output beam of a repetitively pulsed HF laser.

2. Description of calculations

2.1 Numerical model of the active medium of an HF laser

We assume that the active medium of a chemical laser is formed by a flat nozzle unit (nozzle grid) placed in the yz plane (Fig. 1a) so that the direction of the gas flow along the x axis is perpendicular to the optical axis of the cavity. The amplification properties of the active medium are described in the approximation commonly used in the numerical models of HF lasers. It is assumed that the kinetic and gas-dynamic properties of the active medium in the cavity volume in the absence of radiation change only in the direction of the x axis (along the flow) and the beam intensity and the gain in the medium is nonlocal also only along the x axis.

By neglecting the details of generation of HF(υ) molecules in excited vibrational states (where υ is the vibrational quantum number) and their relaxation in the gas flow, these physicochemical processes are described by the generalised pump functions $W(x)$ and relaxation losses $\tau(x)$, which are proportional to the relaxation time. Then, the nonstationary equation for the gain $g(x, y, z, t)$ in the active medium can be written in the form

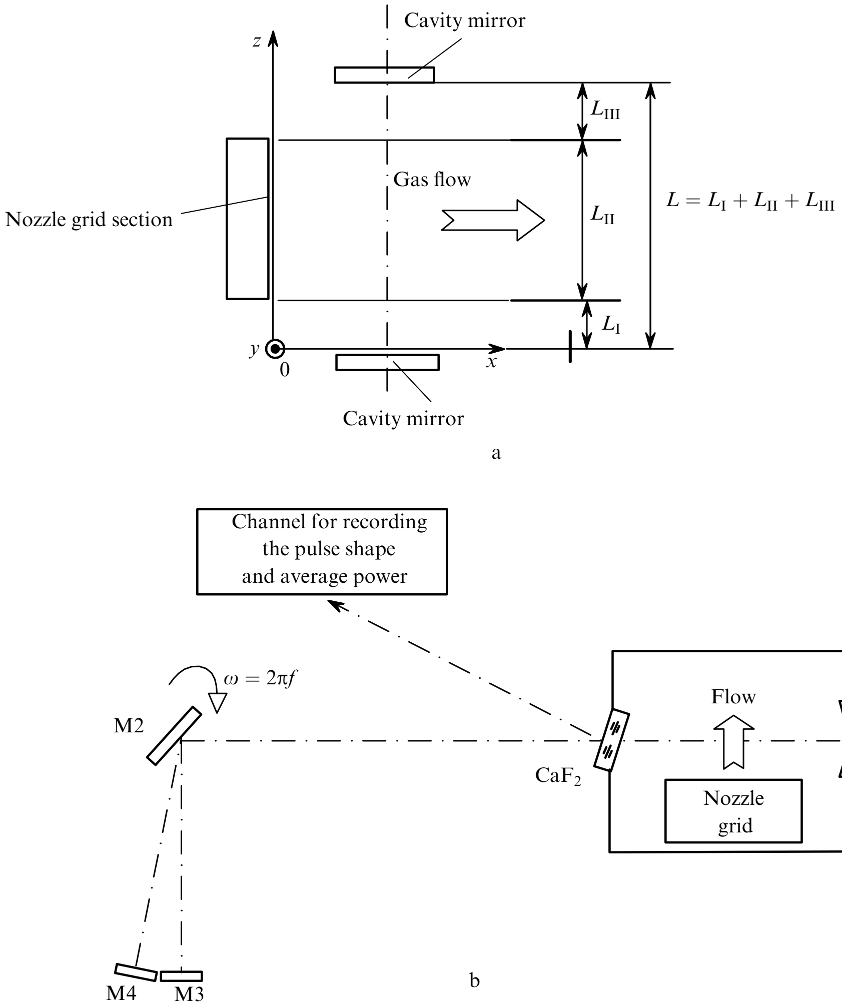

Figure 1. Scheme of a stable optical cavity (a) and a simplified variant of the experimental optical scheme used in calculations (b).

$$\frac{\partial g}{\partial t} + V\frac{\partial g}{\partial x} = W(x) - \frac{g}{\tau(x)} - I(x,y,z,t)g\frac{2\sigma}{h\nu}, \quad (1)$$

where V is the gas flow velocity in the cavity; $h\nu$ is the photon energy at the working wavelength; $I(x,y,z,t)$ is the light intensity in the cavity at the point with coordinates x, y, z at the instant t; and σ is the stimulated emission cross section.

The form of functions $W(x)$ and $\tau(x)$ can be determined from the condition according to which the stationary solution $g_0(x)$ of Eqn (1) in the absence of radiation [for $I(x,y,z,t) \equiv 0$] corresponds to the small-signal-gain distribution $g_0^*(x)$ obtained with the help of a sufficiently accurate model of the active medium of the HF laser based on the system of equations for a boundary layer [7]. Thus, the determination of the functions $W(x)$ and $\tau(x)$ is reduced to the solution of the variation problem

$$g_0^*(x) = g_0\left[W(x), \frac{1}{\tau(x)}\right]. \quad (2)$$

Numerical estimates showed that the functions

$$W(x) = k_1 \exp\left(-\frac{x^2}{8}\right), \quad (3)$$

and

$$\tau(x) = \frac{k_2}{1 + (0.78x)^2}, \quad (4)$$

satisfy accurately enough Eqn (2) for a real stand model of the HF laser [8]. Here, $k_1 = 3.2 \times 10^4$ s^{-1} and $k_2 = 7.2 \times 10^{-6}$ s are numerical coefficients and coordinates x are expressed in centimetres. To use this numerical model, it is necessary to determine the stimulated radiation cross section σ. This parameter (together with the relaxation constant) determines the dependence of the gain saturation in the active medium on the laser radiation intensity. The stimulated radiation cross section was determined by comparing the saturated gain in the HF laser calculated by using our numerical model with the gain calculated in [1]. The saturated gain was calculated with the help of the two-dimensional flow model [8] based on the 'narrow channel' approximation. This model allows one to calculate quite accurately the energy parameters of the laser. The model of the active medium should be supplemented with the equation for the radiation intensity $I(x,t)$ in the cavity. In this case, as in [8], the simplest geometrical approximation

$$\frac{\mathrm{d}I}{\mathrm{d}t} = c\mu g I - \frac{I}{\tau_{\mathrm{loss}}} \tag{5}$$

was used, where c is the speed of light; μ is the degree of cavity filling with the active medium; and τ_{loss} is the photon lifetime in the cavity, which depends on its Q factor.

Equations (1)–(5) comprise a closed system of equations for calculating both the light intensity distribution in the cavity and the profile of the saturated gain along the active-medium flow. Calculations were performed for the parameters of the HF laser used in [8]. It was assumed that the left edges of mirrors (Fig. 1a) are located along the cut line of the nozzle grid. The system of nonstationary equations (1)–(5) was solved by using the iteration procedure. Iterations were continued until the establishment of distributions of the radiation intensity and gain in the medium which did not change from iteration to iteration and corresponded to stationary lasing. The values of the parameter σ were selected during calculations so that to obtain the best agreement with the data presented in [1].

2.2 Numerical model of the optical cavity

The system of equations (1)–(5) describes only temporal and energy parameters of laser radiation. To describe the spatial structure of the output beam, Eqn (5) is replaced by the wave equation for the complex light field $U(x, y, z, t)$ in the cavity, which is represented as a sum of two counter-propagating complex paraxial waves $E_{\pm}(x, y, z, t)$

$$U(x, y, z, t) = E_{+}(x, y, z, t) \exp\left[\mathrm{i}(2\pi\nu t - \beta_z z\right]$$

$$+E_{-}(x, y, z, t) \exp\left[\mathrm{i}(2\pi\nu t + \beta_z z\right], \tag{6}$$

where $\beta_z \equiv \beta_z(\varepsilon)$ is the radiation propagation constant in a medium with the dielectric constant $\varepsilon(x, y, z, t)$; ε_0 and ε_{r} are the normal and relative dielectric constants of the medium, respectively; and ν is the laser frequency. The waves $E_{\pm}(x, y, z, t)$ are normalised so that the square of the modulus should be equal to the radiation intensity. Each of the waves in the small-angle approximation is described by the nonstationary paraxial equation in the form

$$2\mathrm{i}\beta_z(\varepsilon)\left(\frac{1}{c}\frac{\partial E_{\pm}}{\partial t} \pm \frac{\partial E_{\pm}}{\partial z}\right) - \frac{\partial^2 E_{\pm}}{\partial x^2} - \frac{\partial^2 E_{\pm}}{\partial y^2}$$

$$= -\beta_z \mathrm{i}\left[g(x, y, z, t) - \varkappa\right] E_{\pm} \tag{7}$$

for radiation propagating in the active region ($L_{\mathrm{I}} < z < L_{\mathrm{I}} + L_{\mathrm{II}}$) (see Fig. 1a) and

$$\pm 2\mathrm{i}\beta_z(l)\frac{\partial E_{\pm}}{\partial z} - \frac{\partial^2 E_{\pm}}{\partial x^2} - \frac{\partial^2 E_{\pm}}{\partial y^2} = 0 \tag{8}$$

for radiation propagating outside the active medium ($0 < z < L_{\mathrm{I}}$ and $L_{\mathrm{II}} < z < L_{\mathrm{III}}$), where ε, $\varkappa$ and l are the dielectric constant, coefficient of losses, and the active-region length, respectively.

Equations (7) and (8) are supplemented with boundary conditions on the optical elements of the cavity. In particular, for rotating mirror Q switch M2 (Fig. 1b), they have the form

$$E_{+}(x, y, L_2^{+}, t) = r_2(t) A_2(x, y) E_{+}(x, y, L_2^{-}, t), \tag{9}$$

$$E_{-}(x, y, L_2^{-}, t) = r_2(t) A_{\mathrm{CaF_2}}(x, y) E_{+}(x, y, L_2^{+}, t), \tag{10}$$

where $r_2(t)$ is the complex amplitude reflection coefficient of mirror M2 taking into account its tilt with respect to the optical axis of the adjusted cavity; L_2 is the coordinate z of mirror M2; and $A_2(x, y)$ and $A_{\mathrm{CaF_2}}(x, y)$ are the aperture functions of mirror M2 and a CaF_2 output coupler, respectively. These functions are equal to unity at points with coordinates x, y located in the light aperture plane and to zero at points located outside the aperture. Equation (7) was solved by the splitting method, while homogeneous parabolic equation (8) was solved by the spectral method.

2.3 Testing of the numerical algorithm for a *Q*-switched stable cavity

The numerical algorithm was tested by comparing the results of calculations with experimental data obtained in [9], where the parameters of the optical scheme are presented. Figure 1b shows a simplified variant of the optical scheme used in calculations. The geometry of a stable cavity was selected so that the development of lasing should occur independently in cavities formed by mirrors M1, M2, M3 and M1, M2, M4. This circumstance significantly simplifies the numerical model and allows us to perform calculations only for one of the cavities, for example, M1–M2–M3. In this case, the reflection coefficient $r_2(t)$ of rotating mirror Q switch M2 was written in the form

$$r_2(t) = \exp\left[\mathrm{i}\frac{2\pi}{\lambda}\alpha(t)x\right], \tag{11}$$

where λ is the radiation wavelength; $\alpha(t)$ is the time-dependent misalignment angle of the mirror [the dependence $\alpha(t)$ presented in Fig. 2 was used in calculations].

We simulated the real process of formation of the transverse radiation mode during the propagation of light in the cavity. The dynamics of the light field in the cavity was described with the time step Δt equal to the time of light propagation from one end mirror to the other, i.e. $\Delta t = L_{\mathrm{r}}/c$

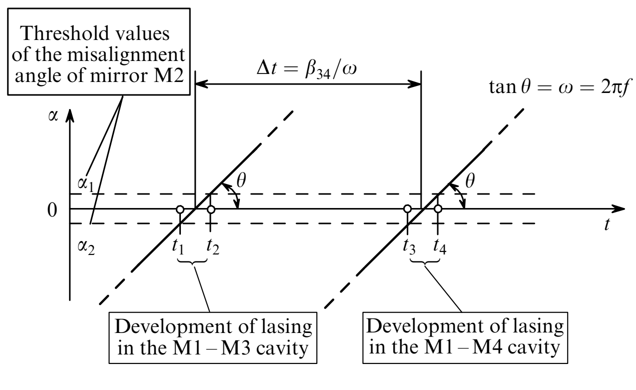

Figure 2. Time dependence of the misalignment angle of rotating mirror M2 used in calculations; β_{34} is the angle between the normals to mirrors M3 and M4 in Fig. 1; ω is the angular rotation velocity of mirror M2.

(in the case under study, $\Delta t = 13$ ns and L_r is the coordinate z of mirrors M3 and M4 equal to the cavity base). The parameters of the active medium and the tilt angle of the rotating mirror Q switch were assumed constant within each time step. First the propagation of two counterpropagating waves E_+ and E_- was calculated at each step by using Eqns (7) and (8). We took into account changes in the light fields caused by (i) nonlinear amplification of light in the active medium; (ii) diffraction from apertures restricting light beams; (iii) refraction of rotating mirror M2; and (iv) radiation losses at the CaF_2 output coupler (Fig. 1b). The spatial structure of the light intensity in the active medium was determined by calculating the propagation of counterpropagating beams. These data were used to calculate the spatial distribution of the saturated gain for the next iteration step in time [from Eqn (1)]. A passage to the next iteration step in time was performed by rotating mirror M2 through the angle $\Delta\psi = 2\pi f \Delta t$ (where f is the rotation frequency of the mirror Q switch).

The radiation field at the initial instant was specified in the form of two plane counterpropagating waves simulating weak noise spontaneous radiation. When the cavity was misaligned, the losses of the light fields exceeded amplification and therefore no lasing occurred and only weak noise radiation of a constant intensity was present in the cavity. When mirror M2 was turned to the position in which the amplification of light exceeded losses (at instant t_1, Fig. 2), the amplification of noise radiation was observed. In this case, the fraction of stimulated radiation in the light fields E_+ and E_- of counterpropagating waves very rapidly increased and the spatial structure of this radiation approached the structure of transverse modes of the cavity. After next iterations in time, the rotation angle $\Delta\psi$ of the mirror Q switch achieved the value at which losses in the resonator again exceeded amplification (at instant t_2). This resulted in the gradual reduction of the power of counterpropagating waves E_+ and E_- during each successive transit of radiation in the cavity down to the spontaneous noise level. This moment corresponded to the quenching of lasing. Then, we calculated the dynamics of the saturated gain in the active medium at each successive step in time in the absence of radiation in the cavity. This calculation regime was used up to the instant t_3 when the cavity again proved to be aligned. Thus, we simulated experimental conditions in the time intervals t_1, t_2 and t_3, t_4. The calculations were performed only for the $\upsilon = 2 \rightarrow \upsilon = 1$ laser transition by assuming that rotational relaxation times are much shorter than typical pulse durations, and therefore the saturation of vibrational lines was neglected. Losses of radiation propagated through the CaF_2 output coupler consisted of four Fresnel reflections after the round trip of radiation in the cavity. It was assumed that the output beam was formed after reflection from only one face of the output coupler.

The energy and kinetic parameters of laser pulses calculated and measured for different rotation frequencies of mirror Q switch M2 were compared in [9]. The comparison demonstrated their good agreement. In particular, calculations predicted correctly the excess of the peak power in the Q-switched cavity over the average power of cw lasing and also the time period during which the active medium has time to recover its amplifying properties.

3. Calculation results

3.1 Q-switching of a stable cavity

Calculations of the intensity distribution of the output radiation in a stable cavity (Fig. 1b) at different instants performed for a mirror Q switch rotating at a frequency of 250 Hz gave the following results. A rather complicated evolution of the transverse structure of radiation is observed which is caused by the rotation of one of the mirrors and competition of the transverse modes of the cavity with the large Fresnel number $N \sim 50$. In this case, the angular divergence of the beam is also modulated in time and exceeds on average the diffraction limit almost by an order of magnitude. It is clear that it is unsuitable to use a Q-switched stable resonator to deliver pulsed radiation to remote objects.

3.2 Q-switching of a positive-branch unstable cavity

It is well known that unstable cavities can provide a high directivity of radiation from wide-aperture lasers. Having a comparatively small length, these cavities allow lasing at the lowest transverse mode. Taking this circumstance into account, we simulated the repetitively pulsed regime of the HF laser with an unstable cavity. The latter represents a confocal positive-branch cavity with the magnification factor $M = 2$ containing concave mirror M1, convex input mirror M3, and rotating plane mirror M2. The distances between mirrors are assumed the same as in the case of the stable cavity (Fig. 1b). The beam aperture was limited only by the output coupler and was equal to that of the stable cavity (24×24 mm). Our calculations showed that the lasing dynamics weakly depends on the cavity type and is determined by the parameters of the active medium and characteristics of the modulator of losses. Therefore, the main attention was devoted to the spatially angular parameters of radiation.

Figure 3 presents the calculated radiation pulse shape $P(t)$ corresponding to the rotation frequency of mirror M2 equal to 250 Hz, and the change in the angular divergence $\varphi_{0.5}(t)$ of the output radiation during lasing. The function $\varphi_{0.5}(t)$ is the time dependence of the cone angle within which 50 % of the instant output power is emitted. For example, $\varphi_{0.5}^{\mathrm{dif}} = \lambda/D$ for a plane wave diffracted from a square

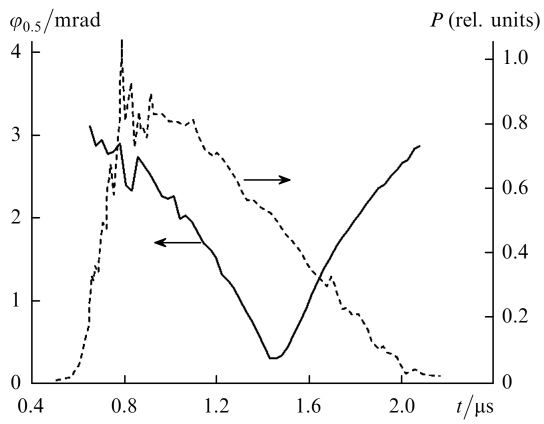

Figure 3. Time dependences of the angular divergence $\varphi_{0.5}$ of radiation at the 50 % power level and the output beam power P for the rotation frequency of mirror M2 equal to 250 Hz (see Fig. 1).

aperture of size $D \times D$. One can see from Fig. 3 that the rotation of mirror M2 results not only in the modulation of the radiation power but also in a considerable change in the output radiation directivity during lasing: the angle within which 50 % of the instant power of the beam is emitted changes during the laser pulse almost by an order of magnitude. Such a change in the angular divergence of radiation is obviously explained by the misalignment of the cavity caused by Q-switching produced by the rotating mirror. The rotating mirror causes both the displacement of the 'centre of gravity' of the beam and broadening of its angular spectrum. In this case, only a very small fraction of the output radiation will be emitted in the direction of a remote object of angular size equal, for example, to the diffraction size of the beam. Therefore, it is necessary to develop a modulator of cavity losses which would not cause the misalignment of the unstable cavity. We simulated a pulse emitted by an unstable cavity laser with a Q switch with variable transmission. In the numerical model, such a Q switch was mirror M2 whose reflectance was varied in calculations so that the modulation of cavity losses reproduced the time dependence of losses taking place upon the rotation of the mirror with a frequency of 250 Hz. In this case, mirror M2 was in the 'aligned' state. The calculations showed that the spatial structure of the transverse mode of the cavity with the diffraction quality of the output beam preserved until the end of the laser pulse was established quite rapidly in this regime. In this case, more than 96 % of the pulse power is emitted.

3.3 *Q*-switching of a negative-branch unstable cavity

A Q switch based on a rotating mirror, which does not cause the misalignment of an unstable cavity, can be constructed by using a negative-branch cavity, which is known to have the minimal sensitivity to the odd-order aberrations and, in particular, to the misalignment of optical elements.

An example of such a scheme is shown in Fig. 4. A telescopic unstable cavity is formed by output convex mirror coupler (*1*), focusing lens (*2*), and end concave mirror (*3*). The cavity has the aperture diaphragm formed by output coupler (*1*) and vignetting diaphragm (*4*). The aperture diaphragm restricts the output beam size and is involved in the formation of the transverse mode of the unstable cavity. Diaphragm (*4*) is used to control cavity losses without changing the spatial structure of the output beam. The cavity is divided into two arms by plane (*5*). The active medium is located in 'hot' arm 1. The arm length L_1 and parameters of output coupler (*1*) determine the main parameters of the cavity: its magnification and equivalent Fresnel number. The parameters of 'cold' arm 2 located at the left of plane (*5*) are selected so that this plane was imaged by optical elements (*2*) and (*3*) to itself with the magnification factor $M_2 = -1$. In this case, the equivalent diffraction length of arm 2 is zero and the sensitivity of the unstable cavity to phase distortions of the optical wedge type located in plane (*5*) is close to zero. Therefore, the turn of a rotating mirror Q switch located in this plane should not change the path of beams in arm 1 and, hence, no angular and translational displacement of the output beam will occur with respect to the normal to output coupler (*1*) passing through its centre. At the same time, the rotation of the mirror Q switch causes the displacement of the beam on mirror (*3*), resulting in the modulation of cavity losses. By changing the rotation velocity of the mirror Q switch and aperture (*4*), the frequency and duration of laser pulses can be controlled.

Figure 5 presents the results of numerical simulation of the operation of the scheme in Fig. 4 demonstrated by the radiation intensity distributions calculated at different instants for the rotation frequency of the mirror Q switch equal to 250 Hz. Calculations were performed in the one-dimensional approximation and the amplifying properties of the active medium were described by the simplest Rigrod formula [10]. One can see from Fig. 5 that the laser pulse of duration ~ 3 μs is formed upon rotation of the mirror Q switch. The light field produced in the plane of mirror (*3*) (Fig. 5a) is displaced approximately for the same time from one edge of diaphragm (*4*) to the other. The near-field output beam has virtually the same spatial profile, changing only in intensity (Fig. b). Note that the position of the axis of the output radiation pattern also remains invariable during lasing (Fig. 5c). The calculations showed that, for parameters of the optical scheme selected here, all intra-

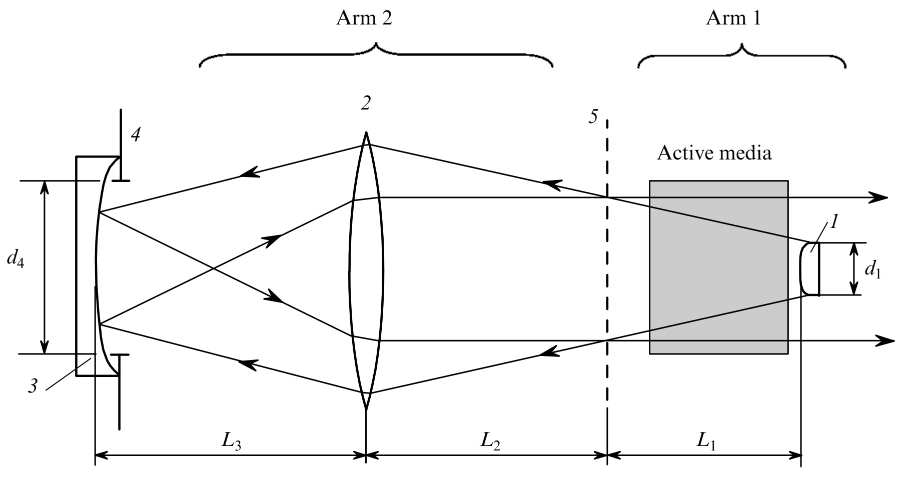

Figure 4. Optical scheme of a negative-branch unstable cavity: (*1*) convex mirror ($R = -300$ cm; $d_1 = 1.2$ cm); (*2*) lens ($f = 225$ cm, $d_2 = 4$ cm); (*3*) concave mirror ($R = 770$ cm); (*4*) diaphragm ($d_4 = 4$ cm); (*5*) plane in which a rotating mirror Q switch is located; $L_1 = 150$ cm, $L_2 = 150$ cm, and $L_3 = 321.4$ cm are optical arm lengths.

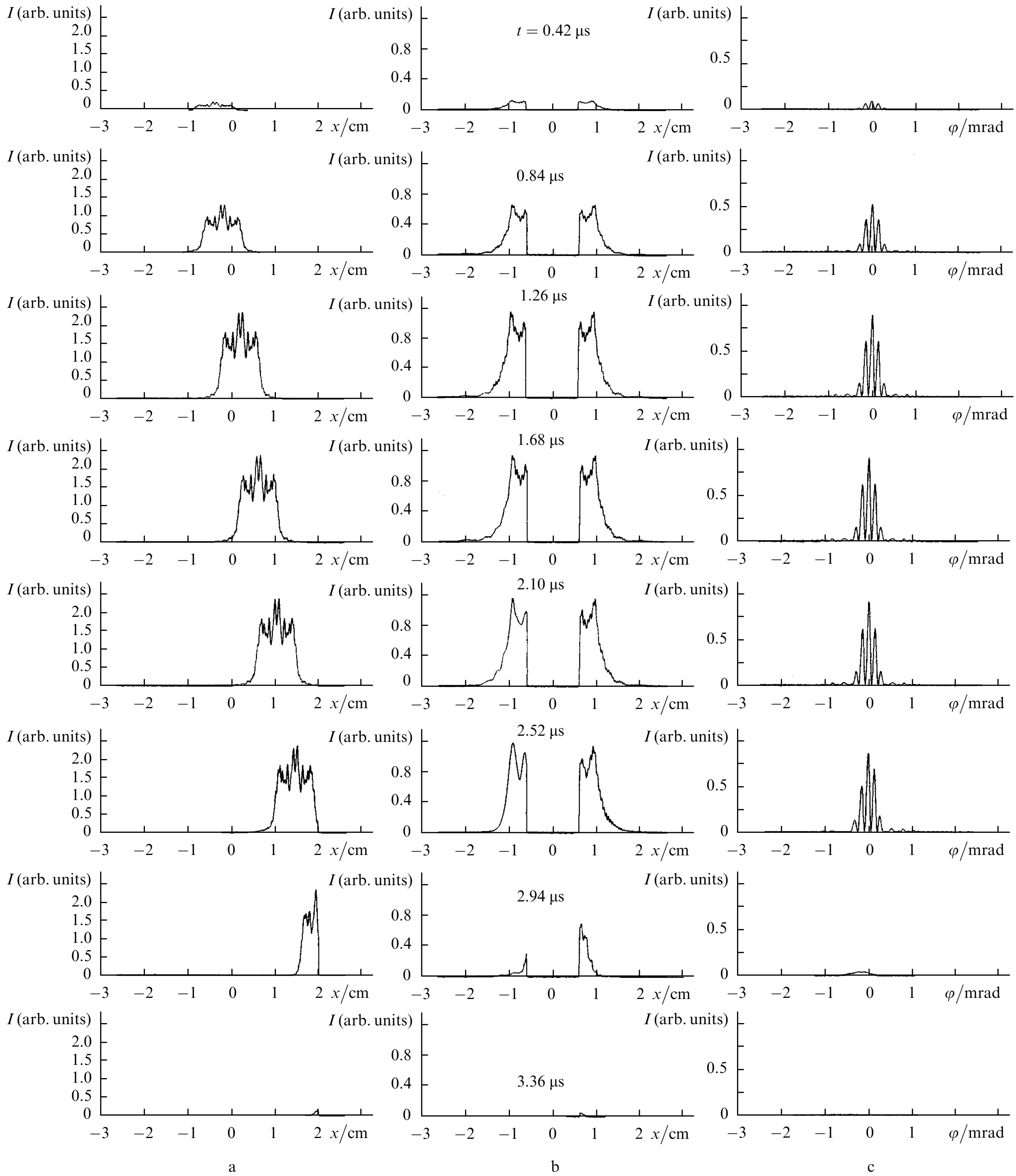

Figure 5. Radiation intensity distributions in the plane of concave mirror (*3*) (a) and near-field (b) and far-field (c) output radiation intensity distributions corresponding to different instants during the rotation of the mirror *Q* switch in Fig. 4.

cavity diffraction losses were concentrated on output coupler (*1*) and limiting diaphragm (*4*). Losses on diaphragm (*4*) have rather steep decay and rise fronts and remain approximately constant during the laser pulse. Because the beam profile in the output coupler plane almost does not change, radiation losses in the cavity also remain constant.

An advantage of the *Q*-switching method proposed in the paper is its simplicity. The laser cavity can contain not one but several 'cold' arms. Each of the arms can have different parameters, so that laser pulses, for example, with different duration or curvature of the wave front will be generated during the total revolution of a rotating mirror *Q* switch. In the latter case, it is possible to reduce considerably the requirements to the accuracy of radiation focusing on a remote object if the criterion for efficiency of its transport is the excess over the light intensity threshold.

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