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Nonunidirectional amplification of a weak light wave upon two-wave interaction in uniaxial photorefractive crystals with different recording mechanisms of the refractive index grating

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***Abstract*. The stationary two-wave interaction in an optically uniaxial photorefractive crystal on a transmission photorefractive grating produced due to the photogalvanic or diffusion mechanism is considered. The possibility of the nonunidirectional amplification of a weak wave, which is consistent with a change in its polarisation state caused by the interaction, is shown. The conditions of the nonunidirectional energy exchange in Fe : $LiNbO_3$ and SBN crystals with the local and photorefractive responses, respectively, are determined.**

Keywords: *photorefractive grating, two-wave interaction, uniaxial crystal.*

1. Introduction

The studies of two-wave o – o and e – e interactions in uniaxial photorefractive crystals such as $LiNbO_3$, $Sr_xBa_{1-x}NbO_6$ (SBN), $BaTiO_3$, etc. demonstrate the unidirectional energy exchange between light waves in the case of the traditional symmetric geometry (the photorefractive grating vector $\boldsymbol{K}$ is directed along the crystallographic axis Z and the normal $\boldsymbol{x}^{\mathrm{o}}$ to the input and output faces of a sample is directed along the crystallographic axis X) [1 – 8]. The o – e interaction can be realised by using circular photogalvanic currents [8 – 13]. In the case of the polarisation o – e interaction ($\boldsymbol{K} \perp Z$; $\boldsymbol{x}^{\mathrm{o}} || X$) on a photorefractive grating produced by a spatially oscillating photogalvanic current, the energy exchange between light waves is nonunidirectional [9].

The coincidence of polarisations of light waves incident on a crystal with the polarisation of one of its optical eigenmodes excludes a continuous change in the polarisation state of the light field at the interaction wavelength. According to [14 – 18], this effect explains the nonunidirectional energy transfer from a strong light wave to a weak one upon two-wave interaction on a photorefractive gating produced due to the diffusion – drift mechanism of charge separation in cubic photorefractive crystals.

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In this paper, we consider the nonunidirectional energy exchange upon stationary two-wave interaction in Fe : $LiNbO_3$ (the $3m$ symmetry group) and SBN ($4mm$) crystals for the symmetric geometry and arbitrary polarisation of incident light waves. It is assumed that a photorefractive grating is formed in Fe : $LiNbO_3$ and SBN crystals due to the linear photogalvanic effect and diffusion, respectively. In the first case, the photorefractive response belongs to the local type, and in the second – to the unlocal type [1 – 3, 5 – 9].

2. Two-wave interaction in a Fe : $LiNbO_3$ crystal

Consider interaction of two arbitrarily polarised plane light waves $\tilde{\boldsymbol{S}}_0 = \boldsymbol{S}_0 \exp[\mathrm{i}(\omega t - \boldsymbol{k}_{\mathrm{S}0}\boldsymbol{r})]$ and $\tilde{\boldsymbol{R}}_0 = \boldsymbol{R}_0 \exp[\mathrm{i}(\omega t - \boldsymbol{k}_{\mathrm{R}0}\boldsymbol{r})]$ on a transmission photorefractive grating in a Fe : $LiNbO_3$ crystal. Due to the birefringence in the crystal, the light field in it is a superposition of four plane waves (Fig. 1):

$$\tilde{\boldsymbol{S}}_{\mathrm{o,e}} = S_{\mathrm{o,e}}\boldsymbol{e}_{\mathrm{o,e}} \exp[\mathrm{i}(\omega t - \boldsymbol{k}_{\mathrm{So,Se}}\boldsymbol{r})],$$
$$\tilde{\boldsymbol{R}}_{\mathrm{o,e}} = R_{\mathrm{o,e}}\boldsymbol{e}_{\mathrm{o,e}} \exp[\mathrm{i}(\omega t - \boldsymbol{k}_{\mathrm{Ro,Re}}\boldsymbol{r})], \quad (1)$$

where $\boldsymbol{e}_{\mathrm{o,e}}$ are the unit polarisation vectors for ordinary and extraordinary waves in the crystal. Note that the interaction

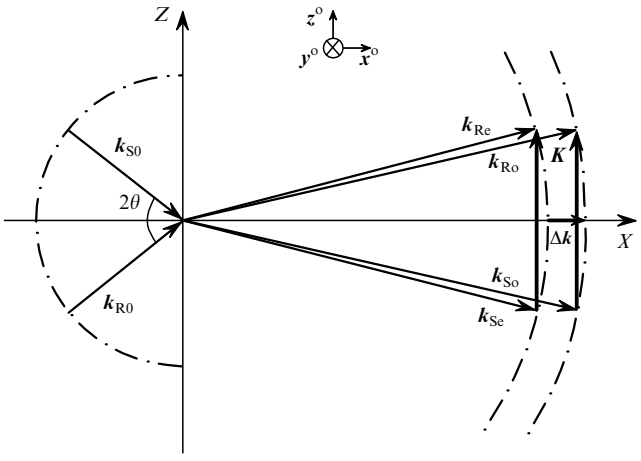

Figure 1. Vector diagram of the two-wave interaction in the mode approach.

of light waves on the photorefractive grating causes the dependence of the scalar amplitudes $S_{o,e}$ and $R_{o,e}$ on the longitudinal coordinate (the interaction length) x. The polarisation states of light waves $\tilde{\boldsymbol{S}} = \tilde{\boldsymbol{S}}_o + \tilde{\boldsymbol{S}}_e$ and $\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{R}}_o + \tilde{\boldsymbol{R}}_e$ change along x both due to the difference in the phase velocities of the natural waves of the medium and directly due to two-wave interaction. If the absorption of light is neglected, the intensities of these waves can change only due to their interaction. Within the framework of accepted approximations, absorption in expressions (1) can be taken into account with the help of the additional factor $\exp(-\alpha x/2)$ (where α is the absorption coefficient) [18].

We will assume that the electric charge separation is mainly determined by the photogalvanic current [9, 10]

$$\boldsymbol{\delta}_{pv} = \hat{\beta} : (\tilde{\boldsymbol{E}} \circ \tilde{\boldsymbol{E}}^*), \tag{2}$$

where $\hat{\beta}$ is the photogalvanic tensor; $\tilde{\boldsymbol{E}} = \tilde{\boldsymbol{S}}_o + \tilde{\boldsymbol{S}}_e + \tilde{\boldsymbol{R}}_o + \tilde{\boldsymbol{R}}_e$ is the electric strength vector of the light field in the crystal; the symbol '$\circ$' means the dyadic product of vectors. The general expression for the photogalvanic current $\boldsymbol{\delta}_{pv}$ taking the light-field mode structure into account can be written in the form

$$\boldsymbol{\delta}_{pv} = \boldsymbol{\delta}_{pv0} + \frac{\boldsymbol{\delta}_{pv1}}{2}\exp(iKz) + \frac{\boldsymbol{\delta}^*_{pv1}}{2}\exp(-iKz), \tag{3}$$

where

$$\boldsymbol{\delta}_{pv0} = \hat{\beta} : \{(\boldsymbol{e}^*_o \circ \boldsymbol{e}_o)(|S_o|^2 + |R_o|^2) + (\boldsymbol{e}^*_e \circ \boldsymbol{e}_e) \times (|S_e|^2 + |R_e|^2) + [(\boldsymbol{e}^*_o \circ \boldsymbol{e}_e)(S^*_o S_e + R^*_o R_e) \times \exp(i\Delta k x) + \text{c.c}]\}; \tag{4}$$

$$\boldsymbol{\delta}_{pv1} = 2\hat{\beta} : \{(\boldsymbol{e}_o \circ \boldsymbol{e}^*_o)S_o R^*_o + (\boldsymbol{e}_e \circ \boldsymbol{e}^*_e)S_e R^*_e + (\boldsymbol{e}^*_o \circ \boldsymbol{e}_e)[S_e R^*_o \exp(i\Delta k x) + S_o R^*_e \exp(-i\Delta k x)]\}; \tag{5}$$

$K = |\boldsymbol{k}_{Ro} - \boldsymbol{k}_{So}| = |\boldsymbol{k}_{Re} - \boldsymbol{k}_{Se}| = 2\pi/\Lambda$; Λ is the spatial period of the grating; $\Delta k = |\boldsymbol{k}_{Ro} - \boldsymbol{k}_{Re}| = |\boldsymbol{k}_{So} - \boldsymbol{k}_{Se}|$ is the distance between the wave surfaces (Fig. 1).

The spatial separation of the electric charge produces a photovoltaic field. By using the results obtained in [9], it is easy to show that in the case of the symmetric interaction geometry, the amplitude of the component of this field oscillating along the coordinate z can be written in the form

$$E_1 = -2\frac{\beta_{31} S_o R^*_o + \beta_{33} S_e R^*_e}{\sigma_{pv}}, \tag{6}$$

where σ_{pv} is the homogeneous component of the photoconductivity of the crystal; $\beta_{31,33}$ are components of the photogalvanic tensor. The photovoltaic field induces through linear electrooptical effect the perturbations of the permittivity of the medium (photorefractive grating), which exert some reverse action on the light field.

Below, we will use the fixed-pump approximation, assuming that the scalar amplitudes satisfy the conditions $R_{o,e} \equiv R_{o0,e0} = \text{const} \gg S_{o,e}$, where R_{o0} and R_{e0} are components of the pump-wave amplitude at the boundary (for $x = 0$). In the case of symmetric geometry, the intermode process is absent. Under these conditions and in the paraxial approximation, the equations for coupled waves can be obtained in the form

$$\frac{dS_o}{dx} = i\frac{\pi n^3 r_{13}}{2\lambda} E_1 R_o, \quad \frac{dS_e}{dx} = i\frac{\pi n^3 r_{33}}{2\lambda} E_1 R_e. \tag{7}$$

where r_{13} and r_{33} are components of the electrooptical tensor and n is the refractive index.

The solution of Eqns (6) and (7) can be obtained in the form

$$S_o = S_{o0} + r_{13}\frac{\beta_{31} S_{o0} R^*_o + \beta_{33} S_{e0} R^*_e}{\beta_{31} r_{13}|R_o|^2 + \beta_{33} r_{33}|R_e|^2} \times R_o\left[\exp\left(-i\frac{\gamma_{pv} x}{2}\right) - 1\right], \tag{8}$$

$$S_e = S_{e0} + r_{33}\frac{\beta_{31} S_{o0} R^*_o + \beta_{33} S_{e0} R^*_e}{\beta_{31} r_{13}|R_o|^2 + \beta_{33} r_{33}|R_e|^2} \times R_e\left[\exp\left(-i\frac{\gamma_{pv} x}{2}\right) - 1\right], \tag{9}$$

where S_{o0} and S_{e0} are components of the weak-wave amplitude at the boundary (for $x = 0$); $\gamma_{pv} = 2\pi n^3 \times (\beta_{31} r_{13}|R_o|^2 + \beta_{33} r_{33}|R_e|^2)(\lambda\sigma_{pv})^{-1}$ is the coupling constant for the symmetric geometry of the two-wave interaction on the photorefractive grating formed due to the photogalvanic effect in the $Fe : LiNbO_3$ crystal.

It follows from relations (8) and (9) that upon the o–o ($S_{e0} = R_e = 0$) or e–e ($S_{o0} = R_o = 0$) interaction, the polarisation of light waves does not change, the energy exchange is absent, and only the phase of the weak signal wave changes. The o–e interaction is impossible. These facts are well known [1–13] and do not require any special discussion.

In the case of arbitrarily and identically polarised waves incident on a crystal ($R_{o,e} \propto S_{o0,e0} \neq 0$), the vector amplitude of the weak signal wave can be written in the form

$$\boldsymbol{S} = \boldsymbol{S}_{||}(x)\left\{1 + \frac{(\beta_{31}|S_{o0}|^2 + \beta_{33}|S_{e0}|^2)(r_{13}|S_{o0}|^2 + r_{33}|S_{e0}|^2)}{I_{S0}(\beta_{31} r_{13}|S_{o0}|^2 + \beta_{33} r_{33}|S_{e0}|^2)} \times \left[\exp\left(-i\frac{\gamma_{pv} x}{2}\right) - 1\right]\right\} + \boldsymbol{S}_{\perp}(x) \times \frac{(r_{13} - r_{33})S_{o0} S_{e0}(\beta_{31}|S_{o0}|^2 + \beta_{33}|S_{e0}|^2)}{I_{S0}(\beta_{31} r_{13}|S_{o0}|^2 + \beta_{33} r_{33}|S_{e0}|^2)} \times \left[\exp\left(-i\frac{\gamma_{pv} x}{2}\right) - 1\right], \tag{10}$$

where $\boldsymbol{S}_{||}(x) = S_{o0}\boldsymbol{e}_o + S_{e0}\boldsymbol{e}_e \exp(i\Delta k x)$ is the vector amplitude of the signal wave in the absence of interaction, and the vector $\boldsymbol{S}_{\perp}(x) = S^*_{e0}\boldsymbol{e}_o - S^*_{o0}\boldsymbol{e}_e \exp(i\Delta k x)$ is orthogonal to the vector $\boldsymbol{S}_{||}(x)$ ($\boldsymbol{S}_{||}\boldsymbol{S}^*_{\perp} = 0$); and I_{S0} is the intensity of the weak wave at the boundary (for $x = 0$).

One can see from (10) that the orthogonal component $\boldsymbol{S}_{\perp}(x)$ of the light field of the weak wave appears even in the

absence of the intermode interaction, which we neglected above. The change in the polarisation state of the weak signal wave in this case is related to the different efficiencies of two intramode processes, which is described by the nonzero factor $(r_{13} - r_{33})$. Note that, when the direction of the polarisation of incident weaves does not coincide with the ordinary or extraordinary axis, the nonunidirectional amplification of the weak light wave with the intensity $I_S = |\boldsymbol{S}(x)|^2$ is possible. It should be noted that not only the intensity I_S changes on the interaction length but also the intensities of the orthogonal components $I_{||} = |\boldsymbol{S}_{||}(x)|^2$ and $I_\perp = |\boldsymbol{S}_\perp(x)|^2$ of the light field, as well as the intensities $I_o = |S_o|^2$ and $I_e = |S_e|^2$ of the weak ordinary and extraordinary waves ($I_S = I_{||} + I_\perp = I_o + I_e$). The value of I_S does not change only if the orthogonal component $\boldsymbol{S}_\perp(x)$ is neglected or if $\beta_{31} = \beta_{33}$.

Note that the amplitude of the photovoltaic field can be obtained from expression (6), taking (8) and (9) into account, in the form

$$E_1 = -2\frac{\beta_{31}S_{o0}R^*_{o0} + \beta_{33}S_{e0}R^*_{e0}}{\sigma_{pv}}\exp\left(-i\frac{\gamma_{pv}x}{2}\right)$$

$$= E_{10}\exp\left(-i\frac{\gamma_{pv}x}{2}\right). \qquad (11)$$

Figure 2a shows the dependences of the intensity gains I_S/I_{S0}, $I_{||}/I_{S0}$, $I_\perp/I_{S0}$, I_o/I_{S0} and I_e/I_{S0} on the interaction length x for incident light waves linearly polarised at an angle of 45° ($S_{o0}/S_{e0} = R_{o0}/R_{e0} = 1$) to the grating vector $\boldsymbol{K}$. We used in calculations the typical electrooptical coefficients $r_{33} = 30.8$ pm V^{-1} and $r_{13} = 8.6$ pm V^{-1} for a Fe : $LiNbO_3$ crystal [8]. According to the data obtained in [9], the components of the photogalvanic tensor at a wavelength of 440 nm are $\beta_{31} = 7.9 \times 10^{-8}$ V^{-1} and $\beta_{33} = 7.3 \times 10^{-8}$ V^{-1}. The photoconductivity σ_{pv} was set equal to 10^{-10} Ω^{-1} m^{-1}, which for the total light-field intensity in the crystal $I_0 \simeq I_R = 5$ mW mm^{-2} (where I_R is the pump wave intensity) corresponded to the typical amplitude of the photovoltaic field $E_{pv} \approx 40$ kV cm^{-1}.

One can see from Fig. 2a that the gain oscillates over x. It follows from expressions (8)–(11) that the spatial period of these oscillations equal to $4\pi/\gamma_{pv} \approx 1$ is twice the oscillation period of the amplitude of the photovoltaic field. The interference pattern of this field within a spatial period Λ is shown in Fig. 2b. The maximum gain is achieved at points $x_{l\max} = 2\pi(2l+1)/\gamma_{pv}$ ($l = 0, 1, 2, ...$). Note that, as the interaction length x increases from 0 to $x_{l\max} \approx 0.5$ mm, the photovoltaic field shifts in the transverse direction by the spatial period Λ. It should be emphasised that the amplification of the weak wave becomes possible due to the amplification of the extraordinary wave. The efficiency of the e–e process is proportional to the maximum electrooptical coefficient r_{33}. The intensity of the ordinary wave for $x > 0$ is smaller than its intensity at the boundary or is equal to the intensity at points $x_{l\min} = 4\pi l/\gamma_{pv}$ ($l = 0, 1, 2, ...$), at which the total amplification is absent. The efficiency of the o–o process is proportional to the electrooptical coefficient r_{31} of the Fe : $LiNbO_3$ crystal ($r_{31} < r_{33}$). A comparison of the dependences $I_{||}/I_{S0}$ and $I_\perp/I_{S0}$ on x shows that the amplification of the weak wave in the case under study is related to a great extent to the transformation of its polarisation state directly due to interaction with the

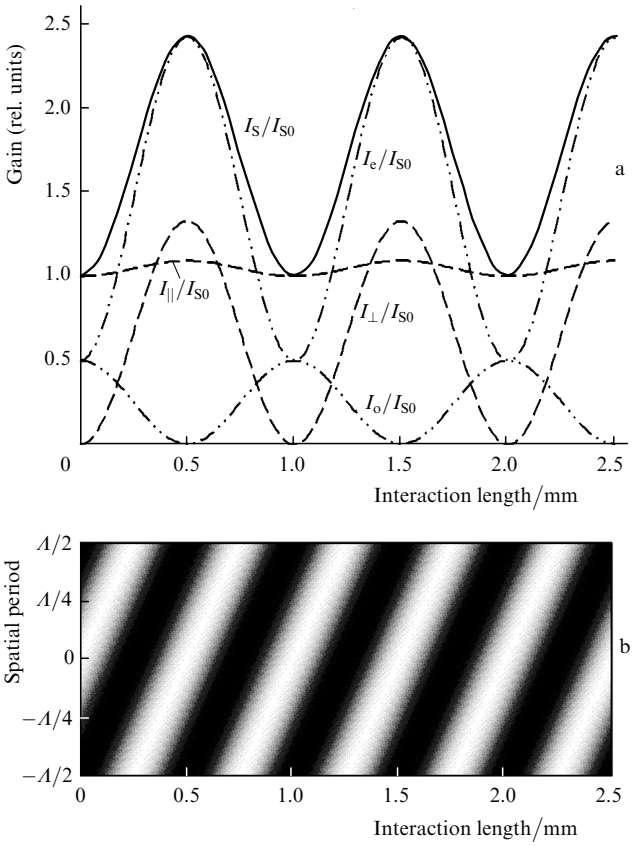

Figure 2. Dependences of the gain I_S/I_{S0} of the weak light wave intensity and the gains $I_{||}/I_{S0}$, $I_\perp/I_{S0}$, I_o/I_{S0}, and I_e/I_{S0} of its components on the interaction length x in the Fe : $LiNbO_3$ crystal for linearly polarised waves incident at an angle of 45° to the vector $\boldsymbol{K}$ of the photorefractive grating (a), and the interference pattern of the spatial charge field with one spatial period Λ (b).

pump wave. A strong amplification of the parallel component of a weak light field with the amplitude $\boldsymbol{S}_{||}(x)$ and intensity $I_{||}$ becomes possible if photogalvanic coefficients β_{31} and β_{33} strongly differ from each other ($\beta_{31} \neq \beta_{33}$). These coefficients for the Fe : $LiNbO_3$ crystal have close values, and therefore the amplification of the weak wave is insignificant.

The weak light wave is amplified in this case because the phases of each of the natural waves [see (8) and (9)] do not coincide with the phase of the amplitude of photovoltaic field (11) for $x > 0$. Therefore, the photorefractive grating proves to be displaced with respect to the partial interference patterns produced by ordinary and extraordinary waves. The phase difference for the e–e process oscillates along x with the amplitude 22°, by changing the sign at points $x_{l\max}$, where amplification changes to attenuation. For the o–o process, the phase difference monotonically increases with increasing x. The phase differences for the o–o and e–e processes in the initial region $0 \leqslant x \leqslant x_{l\max}$ have opposite signs, so that the o-wave is attenuated, while the e-wave is amplified. At the point where the attenuation of the o-wave changes to its amplification, the phase difference for the o–o process is 180°.

The difference of the phases of each of the natural waves from the phase of the photorefractive grating is explained by the fact the grating is formed simultaneously by the o–o and e–e processes, which are matched with each other

through the amplitude of the spatial charge field [see (6) and (7)]. The influence of these processes on each other leads to the difference of phases of the o- and e-waves from that of the photorefractive grating.

3. Two-wave interaction in an SBN crystal

Consider the two-wave interaction on a photorefractive grating in an SBN crystal for arbitrarily polarised incident light waves and symmetric geometry. It can be shown within the framework of approximations used in section 2 that variations in scalar amplitudes over the interaction length are described by equations (7) for coupled waves. However, unlike the previous case, the characteristic photovoltaic field in the SBN crystal is small compared to the diffusion filed $E_d = 2\pi k_B T/(\Lambda e)$ (where e, k_B and T are the elementary electric charge, Boltzmann constant, and absolute temperature, respectively) [2, 5, 8]. Therefore, in the absence of an external electric field we can assume that the amplitude of E_1 of the spatial charge field is determined only by the field E_d:

$$E_1 = -imE_d = -2i(S_o R_o^* + S_e R_e^*)E_d/I_0, \tag{12}$$

where m is the modulation coefficient of the interference pattern and $I_0 = |S_o|^2 + |S_e|^2 + |R_o|^2 + |R_e|^2 \simeq |R_o|^2 + |R_e|^2$ is the total intensity of the light field.

The solution of Eqns (7) and (12) can be obtained in the form

$$S_o = S_{o0} + \frac{m_0}{2} R_o \left[1 \pm \frac{(r_{13} - r_{33})|R_e|^2}{r_{13}|R_o|^2 + r_{33}|R_e|^2}\right] \times \left[\exp\left(\pm\frac{\gamma_d x}{2}\right) - 1\right], \tag{13}$$

$$S_e = S_{e0} + \frac{m_0}{2} R_e \left[1 \mp \frac{(r_{13} - r_{33})|R_o|^2}{r_{13}|R_o|^2 + r_{33}|R_e|^2}\right] \times \left[\exp\left(\pm\frac{\gamma_d x}{2}\right) - 1\right], \tag{14}$$

where m_0 is the modulation coefficient of the interference pattern on the input face of the crystal and $\gamma_d = 2\pi n^3(r_{13} \times |R_o|^2 + r_{33}|R_e|^2)E_d/(\lambda I_0)$ is the coupling constant for the case under study. The signs '+' and '–' correspond to the same and opposite directions of the coordinate axis z and crystallophysic axis Z, respectively (see Fig. 1).

Because the photorefractive grating is formed in the case under study due to the diffusion mechanism of charge separation, which can be realised if $m_0 \neq 0$, the o–e interaction does not appear. As in the case considered in section 2, the polarisation of light waves in the crystal does not change upon the o–o or e–e interaction. The weak wave is monotonically amplified or attenuated with increasing the interaction length x. The two-wave amplification coefficient Γ is equal to $\pm 2\pi n^3 r_{13} E_d/\lambda$ and $\pm 2\pi n^3 r_{33} E_d/\lambda$ for the o–o and e–e interactions, respectively.

For arbitrarily and identically polarised incident waves, the vector amplitude $\boldsymbol{S}$ of the weak signal wave and the two-wave gain Γ can be written in the form

$$\boldsymbol{S} = \boldsymbol{S}_{\parallel}(x)\exp\left(\pm\frac{\gamma_d x}{2}\right) \pm \boldsymbol{S}_{\perp}(x)\frac{(r_{13} - r_{33})R_o R_e}{r_{13}|R_o|^2 + r_{33}|R_e|^2} \times \left[\exp\left(\pm\frac{\gamma_d x}{2}\right) - 1\right], \tag{15}$$

$$\Gamma = \pm\gamma_d + \frac{1}{x}\ln\left\{1 + \left|\frac{(r_{13} - r_{33})R_o R_e}{r_{13}|R_o|^2 + r_{33}|R_e|^2} \times \left[1 - \exp\left(\mp\frac{\gamma_d x}{2}\right)\right]\right|^2\right\} = \Gamma_{\parallel} + \Gamma_{\perp}. \tag{16}$$

Here, the component $\Gamma_{\parallel}$ describes the unidirectional amplification or attenuation of the parallel component $\boldsymbol{S}_{\parallel}$ of the light field [6, 8, 9], while the component $\Gamma_{\perp}$ describes the nonunidirectional contribution to the energy exchange for the component $\boldsymbol{S}_{\perp}$, which, as shown in [16, 18], does not act back on the photorefractive grating in the fixed-pump approximation because it appears due to diffraction of the pump wave accompanied by the transformation of the initial polarisation state to the orthogonal state.

Figure 3 presents the dependences of the total two-wave gain Γ and its unidirectional ($\Gamma_{\parallel}$) and nonunidirectional ($\Gamma_{\perp}$) components on the interaction length x. The dependences were calculated by using the following parameters of

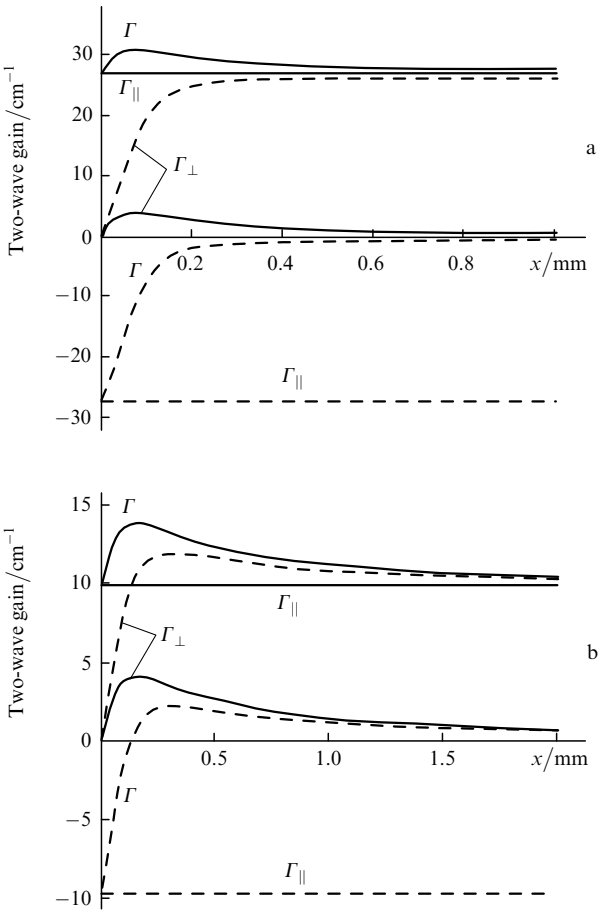

Figure 3. Dependences of the two-wave gain Γ and its unidirectional ($\Gamma_{\parallel}$) and nonunidirectional ($\Gamma_{\perp}$) components in the SBN crystal on the interaction length x for linearly polarised waves incident at angles of 45° (a) and 22.5° (b) to the vector $\boldsymbol{K}$ of the photorefractive grating. The solid and dashed curves correspond to the same and opposite directions of the coordinate axis z and the crystallophysic axis Z, respectively.

an SBN crystal at a wavelength of 633 nm [8]: $n_o = 2.312$, $n_e = 2.299$, $r_{33} = 13.4 \times 10^{-10}$ m V^{-1}, and $r_{13} = 6.6 \times 10^{-11}$ m V^{-1}. The grating period Λ was set equal to 5 μm, and the incident waves were assumed linearly polarised at an angle of $\theta = 45^\circ$ to the grating vector $\boldsymbol{K}$ ($R_o/R_e = S_{o0}/S_{e0} = 1$).

As follows from Fig. 3a and (16), when the z and Z axes have the same direction, the weak wave is amplified with increasing the interaction length x. In this case, the dependence $\Gamma(x)$ is nonmonotonic, which is explained by the nonmonotonic dependence $\Gamma_\perp(x)$, which tends to zero for $x \to \infty$. Because the component $\Gamma_\parallel = \gamma_d$ is independent of x, the total gain upon saturation is $\Gamma_{sat} = \gamma_d$. The dependence $\Gamma_\perp(x)$ has the maximum equal to ~ 3.8 cm^{-1} at $x = 0.78$ mm.

If the z axis is directed oppositely to the crystallophysic axis Z, the weak wave is monotonically attenuated with increasing x. However, the attenuation rate decreases due to the increase in the intensity $I_\perp$ of the orthogonal component of the signal wave. Note that the attenuation of the weak wave due to a decrease in the intensity $I_\parallel$ ($I_\parallel \to 0$ for $x \to \infty$) of its parallel component is almost completely compensated by the amplification of this wave due to an increase in the intensity $I_\perp$. The saturation intensity is $I_{sat} = I_{S0}(r_{13} - r_{33})^2/(r_{13} + r_{33})^2 \simeq 0.82 I_{S0}$. In this case, $\Gamma_{sat} = 0$, which indicates that no energy transfer occurs in the saturation regime.

Thus, although in this case the total energy exchange between the waves is also determined by two oppositely directed fluxes, the nonunidirectional energy transfer does not occur. This, however, does not mean that this regime cannot be realised for differently polarised incident waves. Indeed, we can obtain from (15) the condition

$$\left| \frac{(r_{13} - r_{33}) R_o R_e}{r_{13}|R_o|^2 + r_{33}|R_e|^2} \right| > 1 \tag{17}$$

for nonunidirectional energy transfer for $\gamma_d x \gg 1$. This condition is fulfilled for linearly polarised incident waves when the angle θ between the polarisation vector and the grating vector $\boldsymbol{K}$ satisfies the condition

$$3.1^\circ \approx \frac{r_{33} - r_{13} - [r_{33}^2 + r_{13}^2 - 6 r_{13} r_{33}]^{1/2}}{2 r_{33}} < \theta$$

$$< \frac{r_{33} - r_{13} + [r_{33}^2 + r_{13}^2 - 6 r_{13} r_{33}]^{1/2}}{2 r_{33}} \approx 41.9^\circ. \tag{18}$$

The possibility of nonunidirectional energy transfer, which always amplifies the weak signal wave, is demonstrated by the dependences of the total two-wave gain Γ and its unidirectional ($\Gamma_\parallel$) and nonunidirectional ($\Gamma_\perp$) components on the interaction length x for linearly polarised waves incident at an angle of $\theta = 22.5^\circ$ to the photorefractive grating vector.

4. On the nonlinear self-diffraction problem

As shown above, the transformation of a polarisation state upon two-photon interaction on a photorefractive grating in uniaxial crystals, as upon interaction in cubic photorefractive crystals, gives rise to the nonunidirectional contribution in the energy exchange between light waves [14–18]. This is especially clearly manifested when interactions in crystals with the nonlocal photorefractive response, for example, SBN and GaAs (the symmetry group $\bar{4}3m$) are compared. Indeed, the nonlinear problem of stationary self-diffraction on a nonlocal photorefractive grating in uniaxial crystals can be described by the complete set of equations for coupled waves

$$\frac{dS_o}{dx} = \frac{\gamma}{4} f(m) g_{oo} R_o, \quad \frac{dS_e}{dx} = \frac{\gamma}{4} f(m) g_{ee} R_e,$$

$$\frac{dR_o}{dx} = -\frac{\gamma}{4} f^*(m) g_{oo} S_o, \quad \frac{dR_e}{dx} = -\frac{\gamma}{4} f^*(m) g_{ee} S_e, \tag{19}$$

where $f(m)$ is a function introduced in [18], which depends on the coefficient m; γ is the coupling constant of the photorefractive grating; g_{oo} and g_{ee} are coupling coefficients related to the o- and e-interactions. The structure of these equations does not differ from that of Eqns (11) and (12) from paper [18] devoted to the analysis of two-wave interaction in a crystal of the symmetry group $\bar{4}3m$. The physical meaning of the first integrals of Eqns (19) is similar to that of the integrals in [18]. Therefore, interactions in crystals of the symmetry groups $\bar{4}3m$ and $4mm$ differ only quantitatively and are mainly caused by the different anisotropy of the physical properties of cubic and uniaxial crystals.

5. Conclusions

The stationary two-wave interaction in uniaxial photorefractive $LiNbO_3$ and SBN crystals has been considered in the fixed-pump approximation for the traditional symmetric configuration for arbitrarily polarised incident waves. It has been shown that the nonunidirectional amplification of a weak signal wave can be performed upon interaction on a photorefractive grating in a $LiNbO_3$ crystal produced due to the linear photogalvanic effect and on a photorefractive grating in an SBN crystal produced due to diffusion. It follows from the results obtained in the paper that the presence of the nonunidirectional contribution to the total energy exchange upon two-wave interaction in noncentrally symmetric photorefractive crystals is the general property caused by the self-consistent influence of processes of formation of the spatial-charge field and variations in the polarisation state of light fields during their interaction.

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