*CONTROL OF LASER RADIATION PARAMETERS*

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# Theory of laser array phase locking by Fourier coupling

D.V. Vysotsky, A.P. Napartovich, V.N. Troshchieva

***Abstract*. The type of coupling in a fibre laser array phase locked with the help of an external mirror located at the focal distance from the plane of output ends of individual lasers is studied analytically. The explicit expression is derived for the eigenvalue of the resonator and the restriction on the width of the tuning range in which laser array phase locking is preserved is determined. The influence of the spread in the optical lengths of fibres on the phase-locking efficiency is considered. The phase-locking efficiency is analysed for the spread of optical lengths of fibres considerably exceeding the radiation wavelength.**

***Keywords***: *fibre laser, phase locking, global coupling.*

## 1. Introduction

The phase locking of an array of radiation sources attracts attention of researchers developing high-power coherent radiation sources based on semiconductor and fibre lasers. At present the output power of single-mode semiconductor lasers does not exceed a few watts [1]. Because the ultimate output power of single-mode fibre lasers restricted by nonlinear processes in the active medium is virtually achieved [2], the possibility of further increasing the diffraction-limited output beam power is pinned on the use of spectral and coherent summation of laser beams.

In the case of the spectral summation of laser beams [3], each channel generates at its own frequency, and then the output radiation is extracted outside through a common diffraction grating in the form of a single beam. In systems with coherent summation, the radiation phase locking should be provided in all channels of the system at one collective mode frequency. Unlike the spectral summation of beams, the phase locking of radiation in channels provides narrowband radiation, which is necessary for some applications. In addition, the spectral summation method can be used to further increase the output power of a system consisting of phase-locked subsystems.

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A stable phase locking can be provided by the global (parallel) coupling of each laser in the array with the rest of the lasers [4]. At present the most developed method of global coupling is the selection of the collective modes of an array by losses in some external or intracavity spatial filter. The coherent summation of the radiation fields of an array of diode lasers coupled in pairs by fibre X-couplers is described in [5, 6]. In this case, one of the outputs of the X-coupler is used for summation of fields, while the other remains open (cleaved at an angle), which leads to losses. The losses during lasing can be reduced to zero by selecting the appropriate phase difference for the fields at the open output due to destructive interference. Such architecture can be successively extended on many phase-locked lasers. Its disadvantage is coupling out radiation of the entire array to a single-mode fibre and unstable lasing [7].

Another method for coupling lasers, which was first realised in [8], is based on the Talbot effect [9]. A Talbot filter for fibre lasers with active cores located along a circle is made in the form of an additional circular waveguide of a certain length [10]. It was shown in [11] that the global coupling in the Talbot filter provides the phase locking of radiation channels even in the case of a large spread of their optical lengths. A disadvantage of this configuration is its complexity and rigid requirements to its adjustment. In [12], preliminary results were obtained which demonstrate the possibility of phase locking of a 19-channel fibre by means of the two-dimensional Talbot effect in a passive fibre.

Another phase-locking method, which was proposed and realised in [13], is based on coupling via a common diffraction grating with the same radiation intensity distribution in different diffraction orders. However, the type of coupling in this scheme has not been investigated in detail so far.

The field distribution in the focal plane of a lens or a mirror is the far-field radiation distribution. The production of far-field radiation distributions of the minimum size is one of the problems of laser technology. This size for a wide-aperture solid-state laser was minimised in [14, 15] by placing a limiting diaphragm in the focal plane of one of the mirrors. The diaphragm selected the laser mode with the minimal radius of the far-field distribution, suppressing efficiently all other modes.

It was pointed out in review [16] that the coupling of an array of lasers by means of an external resonator with a diaphragm placed in the focal plane has the global nature. However, such a scheme even for the ideal array has losses because only the central maximum is used in the far-field distribution, while the side orders are lost. Later, the authors of [17, 18] considered the use of apertures of different types in the Fourier plane of an external resonator for various multichannel lasers. In particular, the phase locking of an array of $CO_2$ lasers was achieved in [19, 20] with the help of a system of holes in the diaphragm corresponding to multiple maxima of the far-field distribution for the in-phase mode.

The natural development of the above-mentioned method of far-filed filtration is the use of the output ends of the laser array itself as a selecting aperture in the Fourier plane if the radiation distributions coincide. Such a compound resonator was first applied for phase locking semiconductor lasers with fibre pigtails in [21]. Later, this method was used for phase locking fibre lasers [22]. The aim of our paper is to analyse the global coupling scheme based on the use of an external mirror with the output ends of a system of active fibres located in the focal plane of the mirror [21]. For brevity, we will call this scheme the Fourier-resonator scheme.

## 2. Properties of the field reproduction in the Fourier resonator

Consider a one-dimensional array of lasers with the array period $\Lambda$ and the aperture width $2a$ of each laser (Fig. 1). The external mirror with the radius of curvature is located at a distance of $L = R/2$ from the output apertures of lasers, so that the output ends of the elements of the array are located in the focal plane of the mirror. To decrease diffraction losses in the direction perpendicular to the figure plane, we propose to use a collimating cylindrical lens. We assume for simplicity that the reflection of the radiation field from the output face of each laser can be neglected, i.e. individual fibres play the role of double-pass amplifiers with total reflection from highly reflecting ends. In addition, the consideration is restricted to the odd number of lasers in the array $N = 2N_e + 1$.

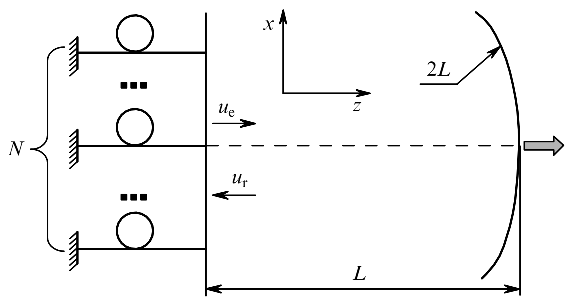


**Figure 1.** Scheme of the array of fibre lasers with a common semi-confocal resonator.

The round-trip transit in the resonator under study, beginning from the plane of the output ends of fibre amplifiers, includes the transit to the mirror and backward, then the injection of an optical image obtained from the mirror to the system of fibre ends and the double transit along the system of fibres. In this paper, we restrict ourselves to analysis of a passive system, by assuming that the reflection from the mirror and highly reflecting ends is ideal. Because the radiation field returned on the fibre ends does not coincide in the general case with the field emitted from the ends, it can be represented in the form of a system of the mode fields of individual fibre cores and the field not captured in the cores. The latter is 'spread' over the entire large aperture of the cladding and can be responsible for the far-field background observed in experiments [23]. In this paper, we treat this field as losses.

It is known that the Dirac comb, i.e. the function $\sum_m \delta(x - m\Lambda)$ is converted after the Fourier transform to a similar comb expressed in the Fourier variables. If the ratio of the focal distance of the mirror to the array period is selected properly, the returning radiation field in the infinite system will be reproduced exactly. This means that a semi-confocal resonator in the paraxial optics approximation has a mode in the form of a comb of delta functions. For two-dimensional periodic arrays, the delta-function effect in the reproduction is preserved. If we now replace the infinite array by a finite periodic structure with the peak amplitudes having a smooth envelope, we can hope that the array reproduction will be preserved at least approximately. Indeed, it was shown in [24] that in the limit of a great number of elements, the one-dimensional comb of Gaussian beams with the common Gaussian envelope is the mode of the construction shown in Fig. 1.

It was shown in [25] that, although the coupling in such an array is inhomogeneous within a large system, but it covers all the elements of the array, so that the resonator with the Fourier self-reproduction has the only transverse mode for which the field phases in all channels are identical. Theoretical studies [24, 26–28] of Fourier-coupled resonators were mainly devoted to the search for functions that are self-reproduced during the Fourier transform, i.e. are the eigenfunctions of the external resonator. Thus, a set of Gaussian beams with a smooth Gaussian envelope was analysed in [27, 28]. In [27], the exact solution was obtained for a one-dimensional infinite system with the Gaussian envelope of fields at the fibre output, the field having a Gaussian profile in each of the fibres. In reality, the transverse mode distribution in a fibre laser is strictly specified by the refractive-index profile of the fibre and vanishes in the space between fibres. It can be shown that in this case, the exactly reproduced distributions for a finite number of elements are absent. However, the approach based on the approximation of real output distributions by Gaussians and the treatment of radiation in the additional images of apertures appearing in a finite fibre array as losses can yield a reasonable accuracy in the case of a great number of lasers.

Let us first analyse qualitatively the properties of a Fourier resonator. The radiation field after the round-trip transit between the output ends of fibres and the external mirror can be expressed in terms of the Fourier transform of the initial distribution

$$\hat{F}[u(x)] = (\mathrm{i}\lambda L)^{-1/2} \int \exp\left(-\frac{\mathrm{i}kxx'}{L}\right) u(x')\mathrm{d}x', \qquad (1)$$

where $\lambda = 2\pi/k$ is the radiation wavelength (integration is performed over the entire axis). If the radiation field emitted by a laser array is

$$u_e = \sum_{n=-N_e}^{N_e} g(n) f(x - \Lambda n) \exp(\mathrm{i}\varphi_n)$$

[where $g(n)$ is the envelope of the field distribution over fibres, $\varphi_n$ is the radiation phase on the $n$th fibre, and $f(x)$ is the mode profile in a fibre), then the radiation field after the round-trip transit is described by the expression

$$u_{\rm r}(x) = \left(-\frac{\rm i}{\lambda L}\right)^{1/2}\left[\int f(x')\exp\left(-\frac{{\rm i}kxx'}{2L}\right){\rm d}x'\right] \times \sum_{n=-N_{\rm e}}^{N_{\rm e}} g(n)\exp({\rm i}\varphi_n)\exp\left(\frac{{\rm i}kx\Lambda n}{L}\right). \quad (2)$$

Expression (2) contains two functions, one of which is the envelope appeared due to the Fourier transform of the field of a beam, and the other is the array sum containing contributions from all fibre lasers. This circumstance demonstrates that the coupling in this system is global. The array sum is an infinite periodic function with the period $\lambda L/\Lambda$ coinciding with the period $\Lambda$ of the laser array under the condition

$$\lambda_0 L = \Lambda^2, \quad (3)$$

which is fulfilled for specified values of $L$ and $\Lambda$ only for one wavelength $\lambda_0$, which we will call the resonance wavelength. When the radiation wavelength is detuned, the period changes, so that the image of the extreme fibres no longer falls on the fibre core. This restricts the admissible interval of radiation wavelengths by the expression

$$\frac{\delta\lambda_{\max}}{\lambda_0} \sim \frac{a}{N_{\rm e}\Lambda}. \quad (4)$$

For the fibre array coupling to be global, the width of the envelope determined by the Fourier transform of the fibre mode should be of the order of the size of the fibre array, i.e. $\lambda L/\pi a \sim N_{\rm e}\Lambda$. This condition and equality (3) impose the restriction

$$a_{\max} \sim \frac{\Lambda}{\pi N_{\rm e}} \quad (5)$$

on the maximum half-width of the aperture of one element. Thus, the filling factor of the emitting aperture should decrease with increasing the size of the laser array. A similar requirement appears in the global coupling scheme in a circular Talbot filter [29]. According to restriction (5), the admissible wavelength range decreases inversely proportional to the square of the number of fibres in the array: $|\delta\lambda_{\max}| \sim \lambda_0/(\pi N_{\rm e}^2)$.

The array sum in (2) for the resonance wavelength $\lambda_0$ determines the profile of the field injected into fibres. The envelope $g(n)$ for a finite laser array has the width of the order of $2N_{\rm e}\Lambda$. In the limit of a great number of fibres and the absence of the phase spread, the profiles of the fields injected into individual fibres will be described approximately by the continuous Fourier transform of the envelope $g(n)$, which has the width of the order of $\lambda_0 L/(\pi N_{\rm e}\Lambda) = \Lambda/(\pi N_{\rm e}) \simeq a$. The required decrease of the filling factor of the emitting aperture with increasing the number of fibres leads to the redistribution of the far-field radiation power from the zero to side orders, which should be corrected by using additional external optics.

The random spread of phase shifts appearing after the double passage of radiation in amplifiers results in a change in the array sum. Below, we analyse the efficiency of the in-phase mode selection in the case of random phase shifts in fibres.

## 3. Effect of the phase spread in fibres on the $Q$ factor of the Fourier resonator

To analyse the field distribution at the end of each fibre, it is convenient to approximate it by a Gaussian beam $u_m(x) = \exp[-(x - \Lambda m)^2/a^2]$, where $m$ is the number of the element. In this case, the field returning at the amplifier input is described by the expression

$$u_{\rm r}(x) = ({\rm i}\lambda L)^{-1/2}\exp\left[-\left(\frac{\pi a x}{\lambda L}\right)^2\right] \times \sum_{n=-N_{\rm e}}^{N_{\rm e}} g(n)\exp\left({\rm i}\varphi_n - 2\pi{\rm i}n\frac{\Lambda x}{\lambda L}\right). \quad (6)$$

The fraction of radiation exciting fibre modes is determined by the overlap integral between the field returned to the laser array and mode field of the $m$th waveguide. The Gaussian envelope in expression (6) weakly varies at the aperture of each of the fibres if the number of fibres is large. We will calculate the overlap integral by neglecting variations of the envelope. In this case, the amplitude $P_m$ of the mode excited in the $m$th fibre has the form

$$P_m = (-2{\rm i}N_a)^{1/2}\exp\left[-(\alpha m N_a)^2\right] \times \sum_{n=-N_{\rm e}}^{N_{\rm e}} g(n)\exp\left[{\rm i}\varphi_n - (n\alpha N_a)^2 - 2{\rm i}nm\alpha^2 N_a\right]. \quad (7)$$

Here, $N_{\rm a} = \pi a^2/(\lambda L)$ is the Fresnel number for a beam from one element and $\alpha = \Lambda/a$ is the inverse filling factor of the array. In fact, $P_m$ is the effective reflection coefficient of the mirror taking into account the condition of the field entry into fibre cores. Condition (3) of the coincidence of the array sum period with the laser array period $\Lambda$ in these variables has the form

$$\alpha^2 N_a = \pi. \quad (8)$$

Under condition (8), expression (7) takes the form

$$P_m = \alpha^{-1}\left(\frac{2}{{\rm i}\pi}\right)^{1/2}\exp\left(-\pi N_a m^2\right) \times \sum_{n=-N_{\rm e}}^{N_{\rm e}} g(n)\exp\left({\rm i}\varphi_n - \pi N_a n^2\right). \quad (9)$$

If the envelope of the fields emitted by fibres is taken in the form $\sim\exp(-m^2\Lambda^2/D^2)$, where $D$ is the half-width if the envelope, then, under the condition $\alpha = \pi D/\Lambda$, is reproduced in the returned field. In the general case, the envelope shape changes after the double passage of radiation in the system of active fibres, for example, due to the gain saturation. However, the condition of the exact reproduction of the envelope is not necessary. It is sufficient only that the array sum would give the field profile close to the mode profile in the fibre.

The factor $P_m$ describes the redistribution of fields in the Fourier resonator. The round-trip transit in the system is closed by the double passage of the returned field over the fibre array. This passage in a passive system results in the multiplication of each of the field amplitudes $P_m$ by the factor $\exp(\mathrm{i}\varphi_m)$, where $\varphi_m$ is the phase shift in a fibre minus an integer number of $2\pi$ because the leakage of the field from fibre core can be neglected. In this case, the eigenvalue of the round-trip transit operator under the condition $\alpha = \pi D/\Lambda$ can have the form

$$\gamma = \left(\frac{2}{\mathrm{i}\pi}\right)^{1/2} \frac{\Lambda}{D} \sum_{n=-N_{\mathrm{e}}}^{N_{\mathrm{e}}} \exp\left(\mathrm{i}\varphi_n - \frac{2n^2\Lambda^2}{D^2}\right). \quad (10)$$

The transformation of the returned field in the case of the phase spread is illustrated in detail in Fig. 2, where the field profiles $|u_{\mathrm{r}}| = (I/I_{\max})^{1/2}$ on the period $\Lambda$ [i.e. the array sum in (6)] are presented for the array of five fibres by approximating the element mode by a Gaussian beam. It was assumed that the condition $\alpha = \pi D/\Lambda$ is fulfilled and the envelope half-width is $D = 2\Lambda$. The latter means that the envelope height for the extreme fibre is $e$ times smaller than that for the central fibre. The distributions are constructed for different values of the mathematical dispersion of phases $\varphi_n$ measured from the phase of the central fibre (for the same random sampling). One can see that for the phase dispersion of the output field $\sim 1$ rad, the returned field distribution strongly differs from the Gaussian profile, and for the random sampling used, the field virtually does not fall in the aperture of the fibre core, which well corresponds to the peak for the zero spread.

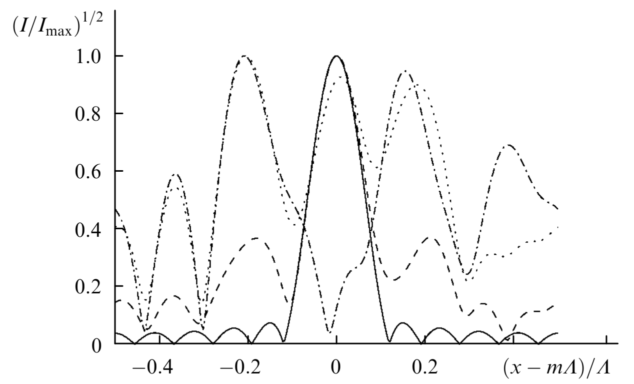


**Figure 2.** Normalised distributions of the field over the period $\Lambda$ returned to the five-fibre array for phase dispersions of beams emerged from fibres equal to zero (solid curve), 0.4 rad (dashed curve), 0.8 rad (dotted curve), and 1.2 rad (dot-and-dash curve).

Experiments [23] have demonstrated the insensitivity of the output radiation of the array of seven fibre lasers in the Fourier resonator to a change in the optical length of one of the fibres. Expression (10) can be used to estimate the dependence of the resonator $Q$ factor on the phase shift in fibres. Figure 3 shows the modulus of the eigenvalue $\gamma$ of the round-trip transit operator normalised to its value in the absence of the phase spread as a function of the phase mismatch in separate elements. The calculation was performed under the same conditions as in Fig. 2. One can see from Fig. 3 and expression (10) that the extreme elements affect the mode stability considerably weaker than the central ones, so that the standard phase dispersion poorly characterises the system behaviour. Expression (10) for the eigenvalue contains the weight function $\exp(-2n^2/N_{\mathrm{e}}^2)$. For this reason, we will characterise the random spread of phases in channels by the phase dispersion with this weight function

$$\sigma_{\mathrm{w}} = \left(\frac{1}{N_{\mathrm{e}}\pi}\right)^{1/2} \left\{ \sum_{n=-N_{\mathrm{e}}}^{N_{\mathrm{e}}} (\varphi_n - \varphi_0)^2 \exp\left(-\frac{4n^2}{N_{\mathrm{e}}^2}\right) - \frac{1}{2N_{\mathrm{e}}} \times \left[ \sum_{n=-N_{\mathrm{e}}}^{N_{\mathrm{e}}} (\varphi_n - \varphi_0) \exp\left(-\frac{2n^2}{N_{\mathrm{e}}^2}\right) \right]^2 \right\}^{1/2}. \quad (11)$$

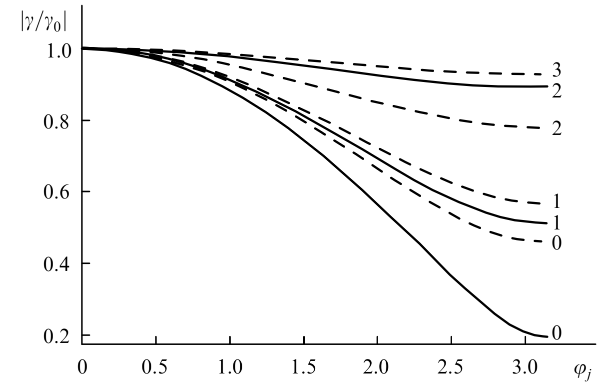


**Figure 3.** Moduli of the eigenvalue of the resonator with five (solid curves) and seven (dashed curves) fibres normalised to unity at zero as functions of the phase detuning in one element. The numbers denote the element number; zero corresponds to the centre.

Figure 4 presents the modulus of the eigenvalue $\gamma$ for the array of five fibres normalised to its value in the absence of the phase spread as a function of the phase dispersion $\sigma_{\mathrm{w}}$ for several sets of random numbers. One can see that the phase spread $\sim 1$ rad can reduce the eigenvalue approximately by half.

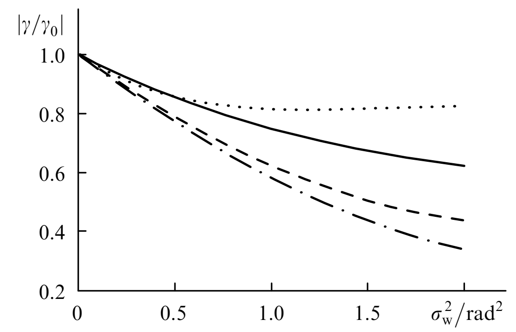


**Figure 4.** Moduli of the eigenvalue of the resonator with five fibres as functions of the phase spread of fields emitted by fibres for four different sets of random numbers.

The spread of the optical lengths of fibres used in lasers can amount to hundreds of wavelengths, which should cause at first glance a complete degradation of the system operation. However, experiments [22, 23] have demonstrated the stability of phase locking seven fibre lasers in the Fourier resonator when the equality of fibre lengths was not controlled. Effects of the same type were observed in the coherent summation schemes with distributed $2 \times 2$ [6] and $4 \times 4$ [7] couplers as a well as upon phase locking with the use of the Talbot coupling in a ring multichannel fibre laser [11].

It seems that the mechanism underlying spontaneous phase locking occurring in the case of the global coupling with a large spread of the optical lengths of channels was first described in [14]. It consists in the self-tuning of the generated radiation frequency to the value providing the maximum $Q$ factor of a compound resonator. Because the array of globally coupled lasers has the only transverse mode, the frequency is tuned due to a high density of the spectrum of longitudinal modes, which is typical for fibre lasers. In this case, the laser emits at the frequency corresponding to the minimal losses in the resonator.

If losses at several frequencies within the amplification band of the active medium are approximately equal, jumps can occur between them, resulting in the spike-mode oscillation [7]. It should be expected that the same mechanism of the laser frequency self-tuning is also possible in the Fourier resonator. However, it should be taken into account that the field reproduction effect in the Fourier resonator depends on the radiation wavelength. As pointed out above, the wavelength interval where this effect is preserved upon detuning from the resonance is determined by the inequality $|\delta\lambda_{\max}|/\lambda_0 \leqslant (\pi N_{\rm e}^2)^{-1}$. Figure 5a shows the dependence of the square of the modulus of the eigenvalue on the detuning from the resonance $\delta\lambda/\lambda_0$ for the ideal system ($\varphi_n = 0$). Because the wavelength dependence of the field reproduction effect is caused by a change in the array period of returned beams, the fraction of radiation returning to the extreme fibres decreases to the greatest degree (Fig. 5b). The narrowing of the wavelength spectrum in which the self-reproduction occurs can restrict the possibility of tuning the radiation frequency to the point of the maximum $Q$ factor of the resonator.

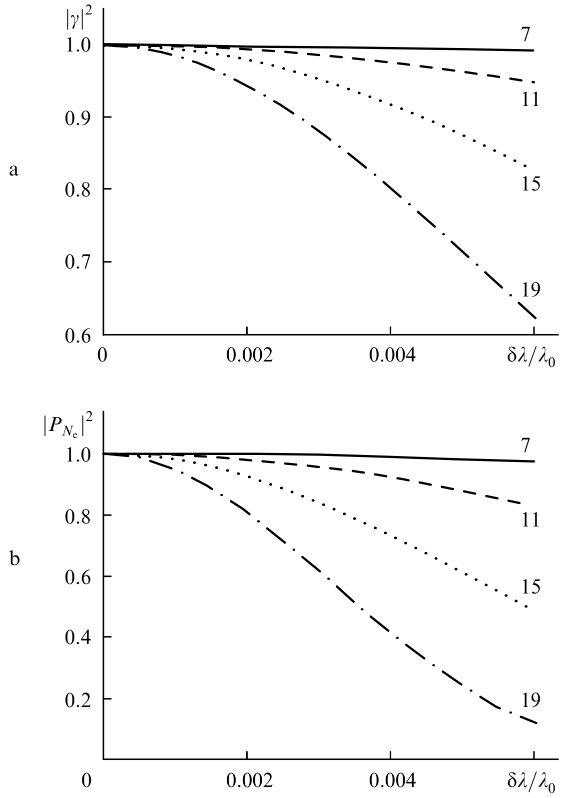


**Figure 5.** Square of the modulus of the eigenvalue (a) and the fraction of radiation injected into the extreme element (b) as functions of the normalised detuning of the radiation wavelength for different numbers of elements in the array (numbers at the right) and the zero phase spread for elements.

We calculated $|\gamma|$ by expression (10) with random phases $\varphi_n$ described by the relation $\varphi_n = 4\pi n_{\rm mod}\delta L_n/\lambda$ ($n_{\rm mod}$ is the mode refractive index and $\delta L_n$ is the difference of lengths of the $n$th and central fibres). Figure 6 presents the values of $|\gamma_{\max}|^2$ calculated for random samplings for arrays of seven and fifteen fibres and $\delta\lambda_{\max}/\lambda_0 = 6.6 \times 10^{-3}$. On the abscissa the dispersion $\sigma_{\rm w}$ of the phase shift after the double passage of radiation in fibres is plotted, which was calculated with the weight function $\exp(-2n^2/N_{\rm e}^2)$ by expression (11). Each point corresponds to the calculated maximum square of the eigenvalue. One can see that the maximum $Q$ factor of the resonator within the amplification band is also a random quantity, which depends on the particular random sampling of the optical lengths of fibres.

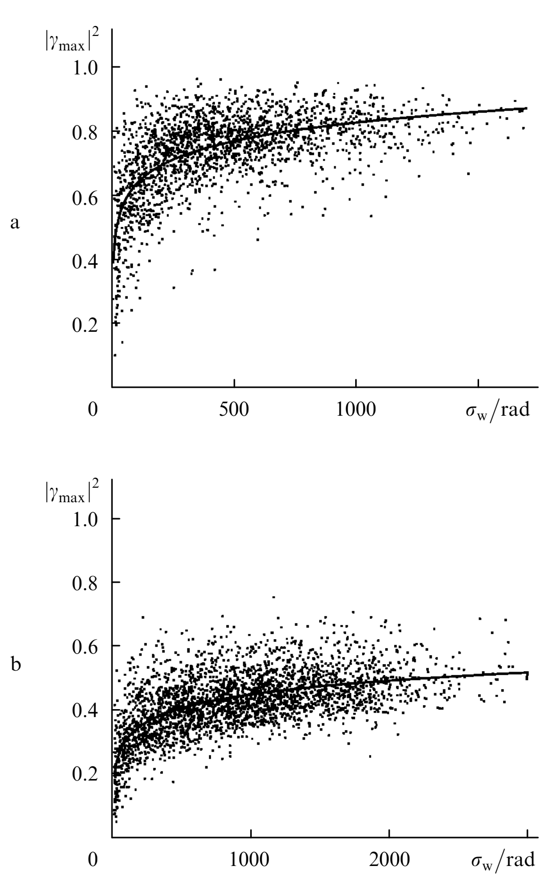


**Figure 6.** Square of the maximum eigenvalue modulus within the spectral band $\delta\lambda_{\max}/\lambda_0 = 0.0066$ as a function of the weighted dispersion $\sigma_{\rm w}$ of the phase difference of fields in fibres appeared after the double passage of radiation in the resonator with seven (a) and fifteen (b) fibres. The solid curves are approximations by the function $(\chi + \theta\ln\sigma_{\rm w})/N$, where $\chi$ and $\theta$ are constants.

It follows from Fig. 6 that the $Q$ factor of the Fourier resonator in the limit of a large spread in optical lengths increases on average with increasing the dispersion of the phase difference and decreases with the number of fibres.

Of interest is the dependence of $|\gamma_{\max}|^2$ averaged over all random realisations on the dispersion of the phase spread. The problem for the homogeneous global coupling is reduced to the search for the maximum of the quantity

$$|\gamma|^2 \propto N^{-2}\left|\sum_{n=1}^{N}\exp(\mathrm{i}\varphi_n)\right|^2,$$

where $N$ is the number of channels. This problem was solved in [14], where it was shown that the average maximum value of $|\gamma|^2$ tends asymptotically in $N$ to the dependence of the type $[\mathrm{const} + \ln(N\sigma_{\varphi}\delta\lambda_{\max}/\lambda)]/N$ ($\delta\lambda_{\max}$ is the amplification bandwidth and $\sigma_{\varphi}$ is the standard phase dispersion). It was pointed out in [29] that the problem of phase locking multichannel fibre lasers with circularly located cores and a circular filter is reduced to the same problem. As pointed out above, the coupling between channels in the Fourier resonator is global but contributions from different channels prove to be different. Therefore, analysis performed in [14] is inapplicable to the Fourier resonator. Not solving here the problem of finding asymptotics in the number of channels, we verified the fulfilment of the approximation of $\langle N|\gamma_{\max}|^2\rangle$ by the function $\chi + \theta\ln\sigma$, where $\chi$ and $\theta$ are the approximation parameters. This approximation proved to be quite satisfactory in the case of a large phase spread. Figure 7 shows the dependence of the coefficient $\theta$ on the number $N$ of elements. For the amplification bandwidth used in calculations, the phase locking of a seven-channel laser is virtually ideal, whereas the coupling efficiency for a fifteen-channel laser decreases approximately by half.

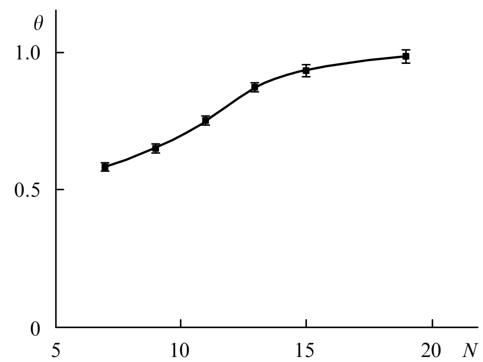


**Figure 7.** Dependence of the approximation parameter $\theta$ on the number $N$ of phase-locked fibres.

## 4. Conclusions

We have studied analytically in the paraxial approximation the phase locking of radiation of a one-dimensional laser array in the Fourier resonator. It has been found that phase locking is violated upon detuning from a certain resonance wavelength inherent in the given system. By approximating optical fibre modes by Gaussian beams, we have derived the explicit expression for the eigenvalue of the round-trip transit operator, which determines the fraction of radiation returned from the external mirror into fibre cores. The spread of the optical lengths of fibres results in the increase in the in-phase mode lasing threshold. The rate of decrease of the effective reflection to the cores of fibres caused by the spread of their optical lengths has been analysed. It has been found that the wavelength self-tuning in the amplification band of the active medium results in the increase in the phase-locking efficiency with increasing the spread of the fibre lengths. Our analysis has shown that a one-dimensional chain containing up to 15 fibre lasers can be efficiently phase-locked.

## References

1. Donnelly J.P., Huang R.K., Walpole J.N., Missaggia L.J., Harris C.T., Plant J.J., Bailey R.J., Mull D.E., Goodhue W.D., Turner G.V. *IEEE J. Quantum. Electron.*, **39**, 289 (2003).
2. Babushkin A., Platonov N.S., Gapontsev V.P. *Proc. SPIE Int. Soc. Opt. Eng*, **5709**, 98 (2005).
3. Daneu V., Sanchez A., Fan T.Y., Choi H.K., Turner G.W., Cook C.C. *Opt. Lett.*, **25** (6), 405 (2000).
4. Fader W.J., Palma G.E. *Opt. Lett.*, **10**, 381 (1985).
5. Lyndin N.M., Sychugov V.A., Tikhomirov A.E., Abramov A.A. *Kvantovaya Elektron.*, **21**, 1141 (1994) [*Quantum Electron.*, **24**, 1058 (1994)].
6. Sabourdy D., Kermene V., Desfarges-Berthelemot A., Lefort L., Barthelemy A., Even P., Pureur D. *Opt. Express*, **11** (2), 87 (2003).
7. Shirakawa A., Matsuo K., Ueda K. *Techn. Dig. CLEO'2004* (San Francisco, USA, 2004) CThGG2.
8. Antyukhov V.V., Glova A.F., Kachurin O.R., Likhansky V.V., Napartovich A.I., Pis'mennyi V.D. *Pis'ma Zh. Eksp. Teor. Fiz.*, **44**, 63 (1986).
9. Withrop J.T., Worthington C.R. *J. Opt. Soc. Am.*, **55**, 373 (1965).
10. Wrage M., Glas P., Fischer D., Leitner M., Elkin N.N., Vysotsky D.V., Napartovich A.P., Troshchieva V.N. *Opt. Commun.*, **205**, 367 (2002).
11. Napartovich A.P., Vysotsky D.V. *J. Mod. Opt.*, **50** (18), 2715 (2003).
12. Li L., Schulzgen A., Chen S., Temyanko V.L., Moloney J.V., Peyghambarian N. *Opt. Lett.*, **31** (17), 2577 (2006).
13. Veldcamp W.B., Leger J.R., Swanson G.J. *Opt. Lett.*, **11** (5), 303 (1986).
14. Gerasimov V.B., Zakharov M.V., Lyubimov V.V., Makarov N.A., Orlov V.K. *Kvantovaya Elektron.*, **13**, 1278 (1986) [*Sov. J. Quantum Electron.*, **16**, 839 (1986)].
15. Gerasimov V.B., Zaika V.M., Ivanov A.E., Lyubimov V.V., Makarov N.A., Pel'tikhin O.A. *Kvantovaya Elektron.*, **14**, 912 (1987) [*Sov. J. Quantum Electron.*, **17**, 579 (1987)].
16. Likhansky V.V., Napartovich A.P. *Usp. Fiz. Nauk*, **160**, 101 (1990) [*Physics-Uspekhi*, **33**, 228 (1990)].
17. Golubentsev A.A., Kachurin O.R., Lebedev F.V., Napartovich A.P. *Kvantovaya Elektron.*, **17**, 1018 (1990) [*Sov. J. Quantum Electron.*, **20**, 934 (1990)].
18. Rediker R.H., Corcoran C.J., Pang L.Y., Liew S.K. *IEEE J. Quantum Electron.*, **25**, 1547 (1989).
19. Aleksandrov A.G., Angeluts A.A., Vasil'tsov V.V., Zelenov E.V., Kurushin E.A. *Kvantovaya Elektron.*, **17**, 1462 (1990) [*Sov. J. Quantum Electron.*, **20**, 1370 (1990)].
20. Vasiltsov V.V., Zelenov Ye.V., Kurushin Ye.A., Filimonov D.Yu. *Proc. SPIE Int. Soc. Opt. Eng.*, **2109**, 107 (1993).
21. Corcoran C.J., Rediker R.H. *Appl. Phys. Lett.*, **59**, 759 (1991).
22. Corcoran C.J., Durville F. *Appl. Phys. Lett.*, **86** (20), 201118 (2005).
23. Corcoran C.J., Durville F., Pasch K. *Phase Locking Array Using a Self-Fourier Cavity,* presented at *Passive Fiber Coupling Workshop* (Stanford University, USA, 20 January, 2006).
24. Liu L. *J. Phys. A: Math. Gen.*, **27**, L285 (1994).
25. Leger R.J. *Fundamentals of Coherent and Incoherent Beam Combining,* presented at *Passive Fiber Coupling Workshop* (Stanford University, USA, 20 January, 2006).
26. Caola M.J. *J. Phys. A: Math. Gen.*, **24**, L1143 (1991).
27. Corcoran C.J., Pasch K.A. *J. Phys. A: Math. Gen.,* **37** (37), L461 (2004).
28. Corcoran C.J., Pasch K.A. *J. Opt. A: Pure Appl. Optics*, **7**, L1 (2005).
29. Vysotsky D.V., Napartovich A.P. *Kvantovaya Elektron.*, **35**, 705 (2005) [*Quantum Electron.*, **35**, 705 (2005)].