

NONLINEAR OPTICAL PHENOMENA

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Effect of optical activity on propagation of two-dimensional spatial solitons in cubic photorefractive crystals

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Abstract. **The self-focusing of two-dimensional Gaussian beams in cubic photorefractive optically active crystals of the $(\bar{1}\bar{1}0)$ cut in the external electric field arbitrarily oriented in the cut plane is theoretically studied. The ranges of orientation of the external electric field are found in which the divergence of the light beam is smaller than the diffraction divergence. The dependences illustrating the existence of two-dimensional spatial solitons in a 15-mm-thick $Bi_{12}SiO_{20}$ crystal are constructed taking into account the optical activity or neglecting it. The shape of a light beam propagating in the crystal is determined.**

Keywords: *two-dimensional spatial soliton, self-focusing, nonlinear interaction, photorefractive crystal, optical activity.*

1. Introduction

An advantage of spatial solitons in photorefractive crystals over solitons of other types (for example, Kerr solitons) is that they can be produced at very low light-beam powers [1]. The study of spatial solitons attracts permanent interest because they can be used for the optical switching of light beams in modern precision optical devices and energy and information transfer over narrow light channels without diffraction losses.

Studies of the propagation and interaction of two-dimensional light beams in photorefractive crystals showed that two-dimensional beams are more interesting for optical applications because their maximal resolvable amount per unit area is proportional to N^2 rather than N, as for one-dimensional beams (N is the number of beams resolvable per unit length). In addition, two-dimensional beams are emitted by most of the lasers and do not require the additional transformation.

Equations describing the propagation and interaction of two-dimensional light beams in photorefractive crystals are more complicated than one-dimensional equations. In particular, the distribution of the internal electric field potential cannot be described by a simple analytic expression [2, 3]. In addition, an external electric field partially screened by a light beam induces the optical anisotropy in a crystal, which violates the initial axial symmetry of the incident light beam. In this connection the soliton propagation of light beams with the axial symmetry in a photorefractive crystal becomes problematic [4].

The propagation and interaction of Gaussian beams in the quasi-soliton regime were studied, as a rule, in uniaxial crystals (for example, SBN [5, 6]). The propagation of two-dimensional light beams in cubic photorefractive crystals was studied in [7, 8]; however, the theoretical interpretation was performed by using a simplified model of the distribution of the spatial charge-field potential in a crystal.

In this paper, we study the propagation of two-dimensional light beams in cubic photorefractive optically active crystals in the quasi-soliton regime by using the formalism [2, 3, 6] based on equations [9] and describing the propagation of two orthogonal components of the envelope of the light-beam electric field in an anisotropic medium with the help of the method proposed in [8, 10].

2. Theory

We will describe the propagation of a two-dimensional light beam in cubic photorefractive optically active crystals by using the system of equations obtained in the paraxial approximation based on Maxwell's equations and basic equations of the photorefractive effect [9]:

$$\mathrm{i}\frac{\partial \boldsymbol{A}}{\partial z}+\frac{1}{2k_0n_0}\left(\frac{\partial^2 \boldsymbol{A}}{\partial x^2}+\frac{\partial^2 \boldsymbol{A}}{\partial y^2}\right)-\frac{k_0n_0^3}{2}(\boldsymbol{A}\hat{r}\boldsymbol{E})+\mathrm{i}\rho[\boldsymbol{e}_z,\boldsymbol{A}]=0, \quad (1)$$

$$\nabla^2\varphi+\nabla\ln(1+I)\nabla\varphi=E_0\frac{\partial}{\partial x}\ln(1+I), \quad (2)$$

$$\boldsymbol{E}=-\nabla\varphi+\boldsymbol{E}_0, \quad (3)$$

where $\boldsymbol{A}=\boldsymbol{A}(x,y,z)$ is the complex vector envelope of the light-beam electric field; $k_0=2\pi/\lambda$ is the wave vector of the light beam in vacuum; n_0 is the unperturbed refractive index; $\hat{r}$ the electrooptical tensor of third rank; ρ is the specific optical rotation; $I=\boldsymbol{A}\cdot\boldsymbol{A}^*/I_{\mathrm{d}}$ is the relative light-beam intensity; I_{d} the dark intensity of the crystal including the background illumination; $\boldsymbol{E}_0$ is the external electric field applied to the crystal along the x axis (Fig. 1); E_0 is the

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projection of the vector $\boldsymbol{E}_0$ on the x axis; $\boldsymbol{E}$ is the internal electric field strength in the crystal in the presence of the light beam; φ is the redefined electric potential related to the potential ϕ of the spatial discharge field by the expression [6]

$$\varphi = \phi + E_0 x; \tag{4}$$

$\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ are the right set of three unit vectors of the Cartesian coordinate system xyz (the z axis coincides with the propagation direction of the light beam).

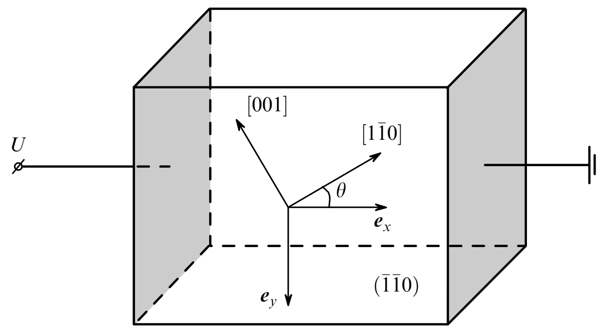

Figure 1. Orientation of the coordinate system with respect to crystallographic axes (U is the voltage applied to the crystal).

Taking into account (3), vector equation (1) for a cubic crystal of class 23 of the ($\bar{1}\bar{1}$0) cut can be written in the form of the equivalent system of scalar differential equations in partial derivatives:

$$\mathrm{i}\frac{\partial A_x}{\partial z} + \frac{1}{2k_0 n_0}\left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2}\right) - \frac{k_0 n_0^3}{2}\left\{r_{41}\left[\left(E_0 - \frac{\partial\varphi}{\partial x}\right)\right.\right.$$

$$\left.\left.\times(\mu_1 A_x + \mu_2 A_y) - \frac{\partial\varphi}{\partial y}(\mu_2 A_x + \mu_3 A_y)\right]\right\} - \mathrm{i}\rho A_y = 0, \tag{5}$$

$$\mathrm{i}\frac{\partial A_y}{\partial z} + \frac{1}{2k_0 n_0}\left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2}\right) - \frac{k_0 n_0^3}{2}\left\{r_{41}\left[\left(E_0 - \frac{\partial\varphi}{\partial x}\right)\right.\right.$$

$$\left.\left.\times(\mu_2 A_x + \mu_3 A_y) - \frac{\partial\varphi}{\partial y}(\mu_3 A_x + \mu_4 A_y)\right]\right\} + \mathrm{i}\rho A_x = 0,$$

where r_{41} is the electrooptical coefficient; functions μ_i ($i = 1$, 2, 3, 4) are determined by the relations

$$\mu_1 = 3\sin\theta\cos^2\theta, \quad \mu_2 = \cos\theta(1 - 3\sin^2\theta),$$

$$\mu_3 = \sin\theta(1 - 3\cos^2\theta), \quad \mu_4 = 3\cos\theta\sin^2\theta;$$

where θ is the orientation angle.

System of equations (5) can be also used to describe the propagation of light beams in crystals of class $\bar{4}$3m by assuming $\rho = 0$.

3. Numerical simulation of the nonlinear interaction of a light beam with a crystal

Consider a 0.6328-μm Gaussian light beam incident on a 15-mm-thick $Bi_{12}SiO_{20}$ (BSO) crystal ($n_0 = 2.54$, $r_{41} = -5\times 10^{-12}$ m V^{-1}, $\rho = 22$ deg mm^{-1}), the beam waist of radius $r_0 = 16.53$ μm being in the ($\bar{1}\bar{1}$0) plane of the crystal. The relative intensity I in the waist is described by the expression $I = (I_0/I_d)\exp[-(x^2 + y^2)/r_0^2]$, and its maximum value is unity ($I_0 = I_d$).

Let us assume first the beam has the x polarisation, i.e. the electric-field vector of the light wave is directed along the external electric field vector $\boldsymbol{E}_0$ (Fig. 1). The orientation dependence of the maximum relative intensity $I_{max}(\theta)$ of the x-polarised light beam at the crystal output is presented in Fig. 2 [curve (*1*)]. The analysis of this curve shows that in intervals $0 < \theta < 187°$ and $347° < \theta < 360°$ the nonlinear interaction of light with the crystal caused by the external electric field reduces the divergence of the light beam compared to the diffraction divergence when $E_0 = 0$ [see straight line (*2*)], i.e. a partial self-focusing of the beam occurs, whereas in the interval $187° < \theta < 347°$ the beam diverges under the action of the external electric field even greater than in the absence of the field. The maximum output intensity I_{max} is achieved for $\theta \approx 25°$ (point A).

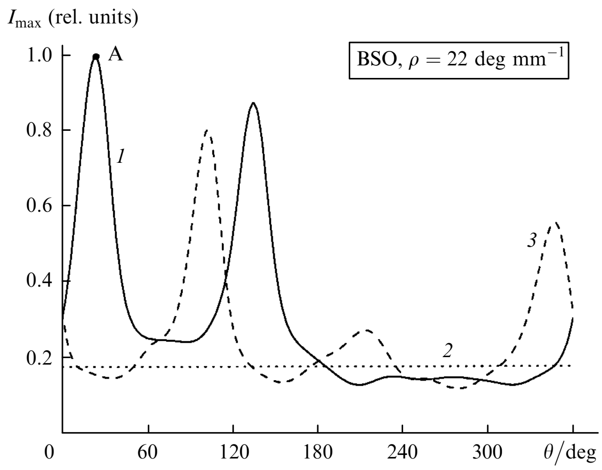

Figure 2. Dependences of the maximum relative intensity of a Gaussian light beam at the output of a 15-mm-thick crystal on the orientation angle θ for the radius of the input-beam waist $r_0 = 16.53$ μm in the cases of the x-polarised input beam, $E_0 = 15$ kV cm^{-1} (*1*), arbitrarily polarised input beam, $E_0 = 0$ (*2*), and y-polarised input beam, $E_0 = 15$ kV cm^{-1} (*3*).

The dependence of I_{max} on the orientation angle θ for the y-polarised light beam incident on the crystal [curve (*3*) in Fig. 2] shows that the input y-polarisation is less advantageous for efficient self-focusing than the x-polarisation. Note that, as in the case of one-dimensional light beams (see, for example, [11]), for $\theta = 90°$ the input y-polarisation is preferable for efficient self-focusing; however, the maximum output value I_{max} achieved for the x-polarised beam exceeds the corresponding value for the y-polarised beam. Therefore, we will further perform the study for the x-polarised input beam at $\theta = 25°$.

Note that the light-beam radius r_0 and the external electric field E_0 were selected in the construction of curve (*1*) in Fig. 2 so that the maximum relative intensity of the output beam (at the point A) would be equal to unity.

In the case of one-dimensional spatial solitons, the constancy of the FWHM (full-width at half maximum) of a light beam during its propagation is considered, as a rule, as the soliton criterion. In the case of two-dimensional solitons, such a criterion is no longer valid because the light-

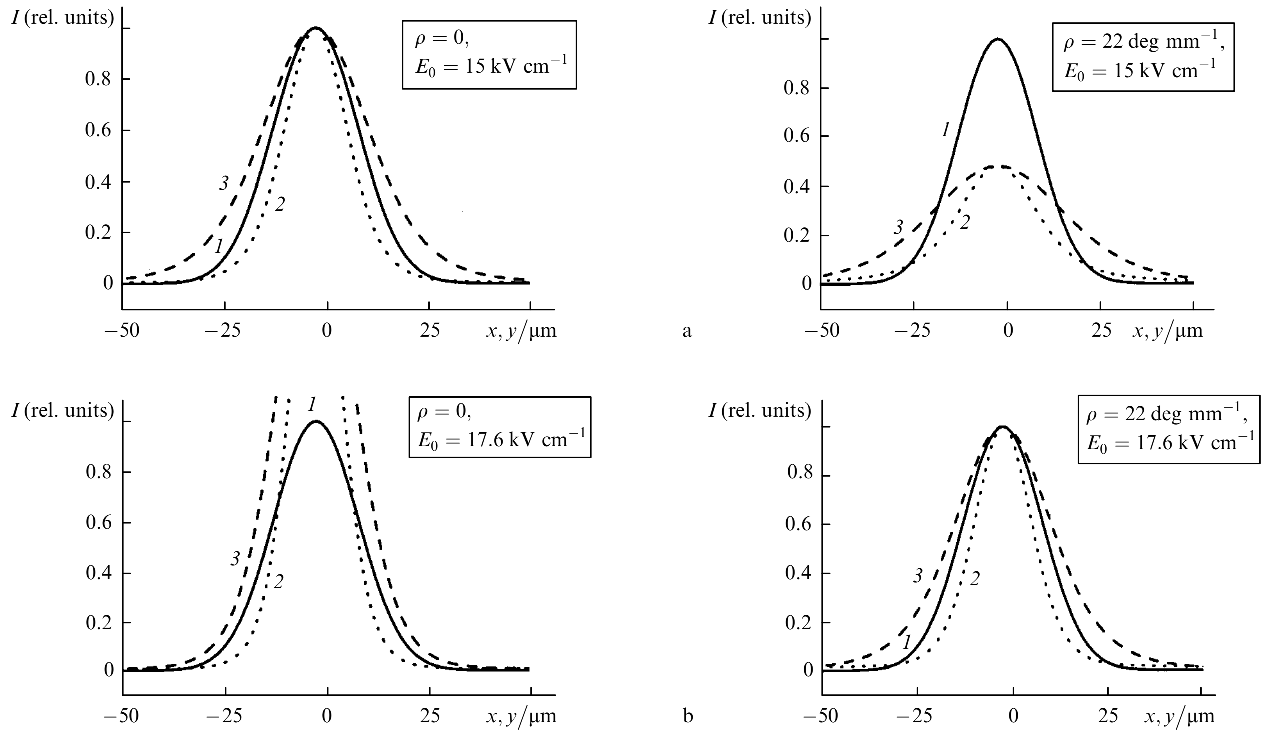

Figure 3. Influence of the optical activity on the condition of obtaining the soliton propagation regime for a Gaussian light beam with the x-polarisation and waist radius $r_0 = 14.78$ μm for the orientation angle $\theta = 25°$ in a crystal with the 'switched off' optical activity ($\rho = 0$), $E_0 = 15$ kV cm^{-1} (a) and in an optically active crystal ($\rho = 22$ deg mm^{-1}), $E_0 = 17.6$ kV cm^{-1} (b): (*1*) the input beam general section; (*2*) the output beam zx section; (*3*) the output beam zy section.

beam FWHMs measured along and perpendicular to the external electric field become substantially different due to the nonlinear interaction of light with the crystal. Therefore, we will use the constancy of the maximum relative intensity $I_{\max}$ of the light beam at the crystal input and output as the criterion for the soliton propagation of the light beam.

It is easy to establish that the optical activity increases, as a rule, the external electric field strength required for achieving the soliton regime because optical rotation during the propagation of the light beam violates the optimal self-focusing of the beam. Obviously, by preserving the external electric field ($E_0 = 15$ kV cm^{-1}), the beam-waist radius in the case of the 'switched off' optical activity should be taken smaller than for the 'switched on' optical activity. Thus, for $\theta = 25°$, the beam radius r_0 required to obtain the maximum relative intensity $I_{\max} = 1$ at the crystal output with the 'switched off' optical activity ($\rho = 0$) decreases from 16.53 to 14.78 μm (Fig. 3a). For such an input-beam radius, the soliton regime cannot be achieved ($I_{\max} < 1$) in an optically active crystal ($\rho = 22$ deg mm^{-1}). To obtain the soliton regime for $r_0 = 14.78$ μm, the external electric field strength should be increased up to 17.6 kV cm^{-1} (Fig. 3b). In this case, the light beam in the crystal with the 'switched off' optical activity experiences the additional self-focusing, i.e. its maximum relative intensity exceeds unity.

The selection of r_0 and E_0 for achieving the quasi-soliton regime can be optimised by using the existence curves (Fig. 4) for two-dimensional spatial solitons for the optically active crystal under study [curve (*1*)] and in the absence of optical activity [curve (*2*)] for the orientation angle $\theta = 25°$, corresponding to the point A in Fig. 2.

Because a two-dimensional Gaussian beam, strictly speaking, is not a soliton beam, we can consider only

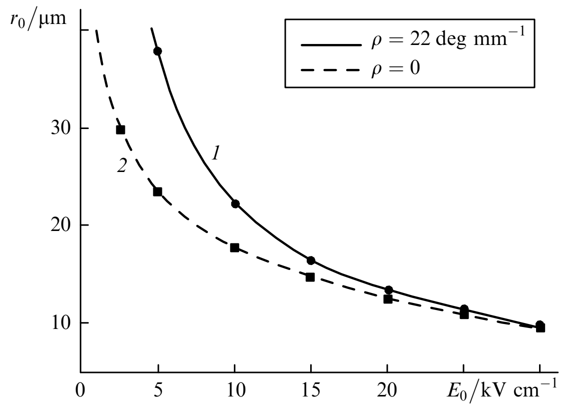

Figure 4. Existence curves for two-dimensional spatial solitons for the x-polarised input light beam in a 15-mm-thick crystal and the orientation angle $\theta = 25°$, taking the optical activity into account (*1*) and neglecting it (*2*).

the quasi-soliton regime of its propagation, i.e. by having equalised the maximum intensities of the beam at the input and output of the crystal due to a proper selection of the external electric field strength, we cannot guarantee their equality for any coordinate z. This is illustrated in Fig. 5, where the dynamics of variation in the maximum relative intensity of the light beam over the crystal length is shown. One can see that, for small values of the coordinate z in the optically active crystal [curve (*1*)], the beam is first focused (the maximum relative intensity becomes greater than unity) and then returns to the initial state (in intensity but not in shape). The additional study of this dependence in a thicker crystal reveals some periodicity of spatial oscillations of the

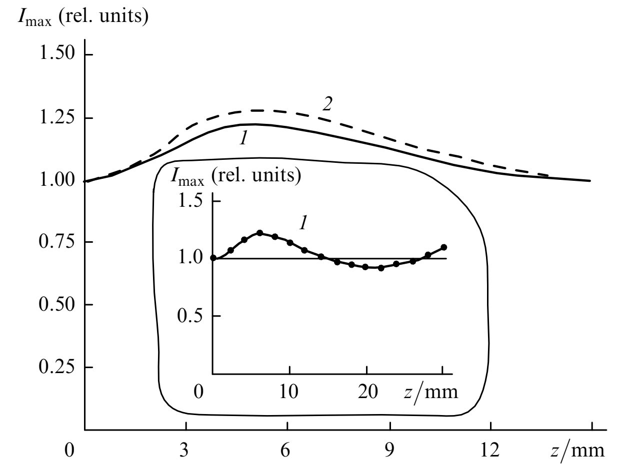

Figure 5. Dynamics of variation in the maximum relative intensity of the x-polarized light beam with the waist radius $r_0 = 14.78$ μm inside a 15-mm-thick crystal for the orientation angle $\theta = 25°$, taking the optical activity into account, $E_0 = 17.6$ kV cm^{-1} (*1*) and neglecting it, $E_0 = 15$ kV cm^{-1} (*2*). The inset shows the same for larger values of z.

beam (the increase in the curve is changed to its decrease and then, vice versa, the passage from the decrease to increase is observed), which was observed earlier in the case

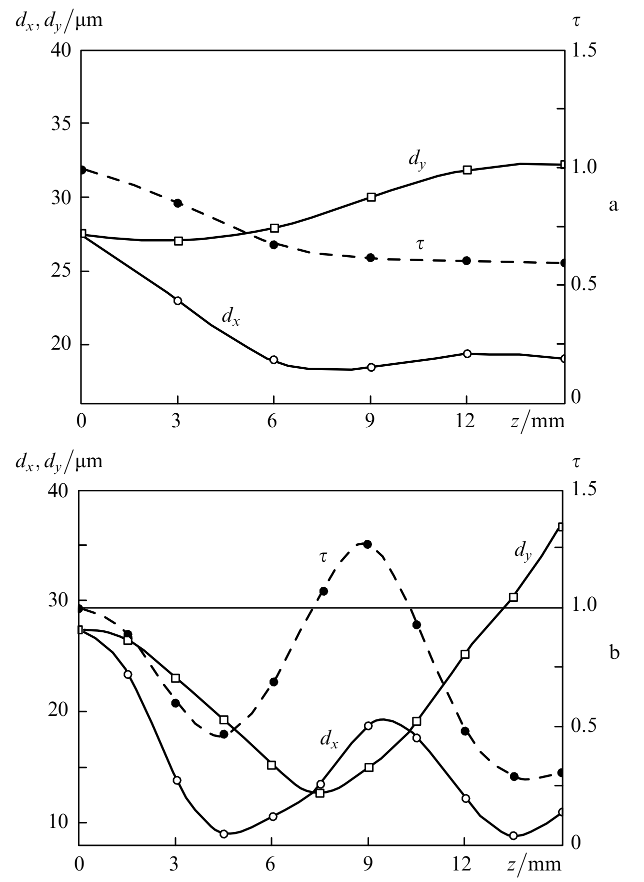

Figure 6. Dependences of diameters d_x, d_y, and the ellipticity τ of the x-polarised light beam with the waist radius $r_0 = 16.53$ μm ($d_x = d_y = 27.52$ μm) on the coordinate z inside a crystal for the orientation angle $\theta = 25°$ in the case of a weak saturation, $I_0/I_d = 1$ (a) and a moderate saturation, $I_0/I_d = 5$ (b).

of one-dimensional not strictly soliton light beams (see, for example, [12]). If the optical activity is 'switched off', the dependence of the maximum relative intensity on the coordinate z [curve (*2*)] is qualitatively similar to the dependence considered above, but to achieve the quasi-soliton regime, the external electric field of a higher strength is used.

Consider a change in the shape of a light beam propagating in a BSO crystal. We will characterise the deviation of the beam shape from the circular shape by the beam ellipticity $\tau = d_x/d$, where d_x is the beam FWHM measured in the beam xz section and d_y is the beam FWHM measured in the beam yz section.

Figure 6 shows the dependences $d_x(z)$, $d_y(z)$, and $\tau(z)$. Let us analyse first the weak-saturation regime [3], when $I_0 = I_d$ (Fig. 6a). We see that in this case, d_y is always greater than d_x and the ellipticity τ does not exceed unity. In the case of moderate saturation ($I_0 = 5I_d$), the dependences d_x and d_y are no longer monotonic (Fig. 6b), and the ellipticity τ is more than unity in the region 7.2 mm $< z <$ 10.3 mm, i.e. the orientation of the ellipse becomes almost orthogonal to the previous orientation. A similar situation was observed earlier in a uniaxial SBN crystal [3]. Note that the beam diameters d_x and d_y coincide ($\tau = 1$) for certain values of the coordinate z (7.3 and 10.3 mm), i.e. the beam almost preserves its initial axial symmetry.

Note that, although the achievement of the true soliton regime required for the propagation of a two-dimensional Gaussian light beam in a photorefractive crystal seems problematic, such a regime can be probably achieved for a hypothetical two-dimensional non-Gaussian beam. The problem of determining the shape of such a beam, its polarisation, and intensity will be considered elsewhere.

4. Conclusions

We have obtained the modified system of scalar equations describing the propagation of two-dimensional light beams in a cubic optically active photorefractive crystal of class 23, which can be also used for crystals of class $\bar{4}3$m. Some changes in the equations of this system compared to a similar system used in [2, 3] take into account the optical activity and the features of the electrooptical effect in cubic crystals, where all the nonzero components of the electro-optical tensor are identical due to a high symmetry of the crystal and, therefore, cannot be neglected, as in other crystals with a lower symmetry (for example, SBN).

The intervals of the orientation angle have been determined in which the nonlinear interaction of light with a BSO crystal in the drift regime reduces the diffraction divergence of the light beam, providing the quasi-soliton propagation of the beam in the crystal. The influence of the optical activity on the formation of the quasi-soliton regime has been investigated. The existence curves for two-dimensional spatial solitons in a cubic photorefractive crystal have been constructed, which facilitate the choice of the external electric field and radius of the input Gaussian beam for obtaining the quasi-soliton regime. Variations in the shape of a light beam propagating in a BSO crystal in the weak and moderate saturation regimes have been determined. It has been shown that in the case of weak saturation ($I_0 = I_d$), the beam diameter d_x measured along the external electric field is smaller than the diameter d_y measured in the direction perpendicular to the field, i.e. the beam ellipticity is less than unity over the entire propagation length of the beam in the crystal. As I_0 is increased ($I_0 = 5I_d$), the dependence $\tau(z)$ becomes nonmonotonic: the ellipticity of the beam propagating in the crystal changes from values smaller than unity to values exceeding unity, and then again becomes smaller than unity. For some values of the coordinate z, the beam preserves the initial axial symmetry.

The results obtained in the paper can be used to control the self-focusing of light beams and change their shape in various devices in quantum electronics and photonics. They can also stimulate experimental studies on the optimisation of the propagation and interaction of light beams in sillenite crystals. In addition, these results can be useful for the development and improvement of light amplifiers based on neodymium-doped optically active BSO crystal generating stimulated emission [13].

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