PACS numbers: 42.55.Xi; 42.60.Da
DOI: 10.1070/QE2007v037n03ABEH013429

# Modes of a plano – spherical laser resonator with the Gaussian gain distribution of the active medium

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***Abstract*. Modes of a laser with plano – spherical degenerate and nondegenerate resonators are calculated upon diode pumping producing the Gaussian gain distribution in the active medium. Axially symmetric and off-axis pumpings are considered. It is shown that in the first case the lowest Hermite – Gaussian mode is excited with the largest weight both in the degenerate and nondegenerate resonator if the pump level is sufficiently high or the characteristic size $w_g$ of the amplifying region greatly exceeds the mode radius $w_0$. The high-order Ince – Gaussian modes are excited upon weak off-axis pumping in the nondegenerate resonator both in the absence and presence of the symmetry of the gain distribution with respect to the resonator axis. It is found that when the level of off-axis symmetric pumping of the resonator is high enough, modes with the parameters of the $TEM_{00}$ mode periodically propagating over a closed path in the resonator can exist. The explanation of this effect is given.**

***Keywords***: *modes of a plano – spherical resonator, diode pumping.*

## 1. Introduction

The use of longitudinal diode pumping leads to a considerable increase in the energy efficiency of lasers due to good matching of the emission wavelengths of laser diodes with the absorption bands of active media and the optimisation of the relation between sizes of the pumped region and resonator eigenmode. At the same time, it became possible to control the mode composition of a laser beam by using the spatial formation of amplification in the active medium instead of introducing spatially inhomogeneous intracavity losses (with the help of masks, mirrors with reflection variable along the radial coordinate [1] or Gaussian apertures). This method was employed for selective excitation of the high-order Hermite – Gaussian (HG) modes [2, 3], Laguerre – Gaussian (LG) modes with the angular intensity dependence $\sim\cos^2 l\varphi$ (or $\sin^2 l\varphi$) [4, 5], Ince – Gaussian (IG) modes [6] and the so-called geometric [7], in particular, V modes [8] in a plano – spherical resonator.

The method for generating each of the above-mentioned modes is quite simple. For example, the HG modes can be generated by focusing radiation from a laser diode to the active medium to produce the amplification region at a distance of $x_{\rm g}^{\max}$ from the resonator axis (Fig. 1), which corresponds to the intensity maximum $u_{m0}^{\max}$ of the mode of the selected order. Similarly (in some cases, by introducing additional screens) the IG modes can be obtained. The LG modes were generated by forming a circular region in the amplification region. Geometric modes were excited by using the degeneracy of modes in a semi-confocal resonator. Theoretically, a mode having the maximum overlap of the field distribution with the excited active region should be excited in each case.

It is obvious that, along with the spatial distribution of pumping, a number of other factors affect the excitation efficiency of these modes. Thus, if a three-level active medium is used, the aperture effect dominates [9]. Because such media have strong absorption outside the pump region, the types of modes appear in the resonator as in the case when profiled mirrors or apertures [10] with reflection (transmission) varying from 1 to 0 are used. For four-level media, especially with a low gain, the spatial burning of inversion, gain saturation, birefringence, thermal deformations and various diffraction losses of modes play a key role. The latter depends on the quality of the optical elements of the resonator and its adjustment, which cannot be always controlled in experiments. In particular, this probably explains the fact that under similar conditions some authors interpret modes as the HG modes [2, 3], while the others treat the same modes as the IG modes [6] representing the

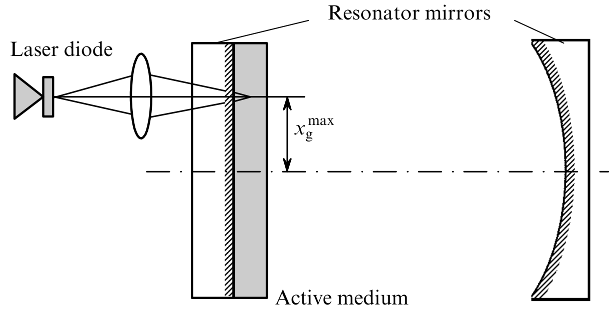


**Figure 1.** Model of the plano – spherical resonator of a diode-pumped laser used in calculations.

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Received 18 September 2006; revision received 24 November 2006
*Kvantovaya Elektronika* **37** (3) 299 – 306 (2007)
Translated by M.N. Sapozhnikov

solution of the wave equation of an empty resonator in elliptic coordinates [11].

The use of diode pumping of solid-state lasers is attractive for selective generation of the high-order HG modes and their subsequent conversion to the LG modes [with the phase of type $\exp(\mathrm{i}l\varphi)$], which have been used in recent years in a variety of scientific fields – from atomic physics to biology (see references [6–9] in paper [12]). In this connection the correct identification and the degree of purity of modes established in diode-pumped lasers are quite important.

In this paper, the results of numerical experiments are presented which were obtained by assuming that diode pumping provides the Gaussian distribution of amplification in the active medium, whereas other effects (gain saturation, nonuniform inversion extraction, etc.) that can distort the field phase and amplitude distributions [13], thereby complicating the excitation of pure modes, are negligible.

## 2. Numerical simulation of the establishment of radiating modes in a plano–spherical resonator

The establishment of modes in diode-pumped lasers was numerically simulated by using the Fresnel program [14] by the Fox–Li method [15]. The four-level active medium of the laser was considered and, therefore, the gain distribution was represented in the form

$$G(r) = 1 + (G_0 - 1)\exp\left[-2\frac{(x - x_{\mathrm{g}}^{\max})^2 + y^2}{w_{\mathrm{g}}^2}\right], \qquad (1)$$

where $w_{\mathrm{g}}$ is the gain distribution radius (at the $\mathrm{e}^{-2}$ level); $G_0$ is the gain maximum*; and $x_{\mathrm{g}}^{\max}$ is the position of the gain maximum with respect to the optical axis of the resonator (Fig. 1). The resonator length $L$ was set equal to 10 cm and the radiation wavelength was 1.055 μm. The active-medium thickness was assumed negligible compared to the resonator length, which is typical for longitudinally diode-pumped lasers. In most cases, two types of plano–spherical resonators were considered: a degenerate semi-confocal resonator with the radius of curvature of a spherical mirror $R = 20$ cm and a nondegenerate resonator with $R = 22$ cm.

### 2.1 Pumping of the active medium of the laser on the resonator axis

Figures 2a, b present the radiation intensity distributions on the plane and spherical mirrors of a semi-confocal laser resonator upon axial pumping for a small gain ($G_0 = 1.2$) and different ratios of the radius $w_{\mathrm{g}}$ to the size $w_0 = (\lambda L/\pi)^{1/2}$ of the resonator eigenmode. Here, unlike the use of a Gaussian aperture (GA) in the resonator, when the beam intensity distribution is Gaussian for any size of the aperture, for the active-medium gain distribution (1) the distribution of the field intensity (amplitude) depends on the ratio $w_{\mathrm{g}}/w_0$. For $w_{\mathrm{g}}/w_0 < 1.5$, the intensity distribution is substantially non-Gaussian. For $w_{\mathrm{g}}/w_0 = 0.9$, the intensity in the wing of the distribution is well described by the function $\sim\exp(-r/w_0)$, while for $w_{\mathrm{g}}/w_0 < 1.5$, this intensity is $\sim\exp(-3r/w_0)$.

For $G_0 \gg 1$ (Fig. 2c), the radiation intensity distribution in the resonator approaches to that obtained by using an equivalent GA located near a plane mirror. Thus, for $G_0 = 64$, the sizes of beams for a resonator with the gain distribution of type (1) and a resonator with the GA differ by no more than 0.16 %.

Thus, in the semi-confocal resonator with the active medium having the gain distribution (1), the parameters of the beam tend with increasing $w_{\mathrm{g}}/w_0$ to the parameters of the beam of an empty resonator, and when $G_0$ increases, they tend to the parameters determined by the size of the equivalent GA.

The dependence of the beam size on the ratio $w_{\mathrm{g}}/w_0$ in a plano–spherical nondegenerate resonator ($R = 22$ cm) is substantially different (Fig. 2d). Even for $w_{\mathrm{g}}/w_0 = 0.75$, the intensity distribution is nearly Gaussian ($M^2 = 1.026$), and for $w_{\mathrm{g}}/w_0 = 3$ it virtually coincides with the distribution for the eigenmode of an empty resonator.

The expansion of radiation in modes of the corresponding empty resonator characterises the types of modes established upon axial pumping of the active medium of a plano–spherical resonator. Such expansions (the moduli of amplitudes of the mode spectrum) are presented for the semi-confocal (degenerate) and plano–spherical nondegenerate resonators for $w_{\mathrm{g}}/w_0 = 1.5$ in Figs 3a, b. Note that in both cases, the energy is mainly concentrated in the fundamental $u_{00}^{\mathrm{HG}}$ mode and modes with the sum of transverse indices $m + n = sN$ ($s = 1, 2, 3, ...$) proportional to the degeneracy multiplicity $N$ of the semi-confocal resonator ($N = 4$), although the relative weight of these modes for the nondegenerate resonator (Fig. 3b) is rather small. For the resonator with the degeneracy multiplicity $N = 6$ ($R =$ 40 cm) and the nondegenerate resonator with close parameters ($R = 42.48$ cm), the dependences of the amplitudes of the expansion modes on the degeneracy multiplicity are similar (Figs 3c, d).

### 2.2 Off-axis pumping of the active medium of the laser

The modes of an off-axially pumped laser with a semi-spherical resonator have a number of features depending not only on the degeneracy parameter $2\pi/\arccos(1 - 2L/R)$ but also on the gain, as well as on whether one excitation region with the gain maximum at the point $x_{\mathrm{g}}^{\max}$ is produced in the active medium (asymmetric off-axis pumping) or two regions with the gain maxima at points $\pm x_{\mathrm{g}}^{\max}$ (symmetric off-axis pumping).

#### 2.2.1 Degenerate resonator

If the pump region is displaced from the axis of a semi-confocal resonator by a distance that is much smaller than $w_0$ ($x_{\mathrm{g}}^{\max} \ll w_0$), the type of modes established in the resonator proves to be quite complex for analysis. For $x_{\mathrm{g}}^{\max} \gg w_0$, the problem is considerably simplified. Figure 4 presents the results of calculations performed for $x_{\mathrm{g}}^{\max} = 2.74 w_0$, $w_{\mathrm{g}}/w_0 = 1.5$, and $G_0 = 1.2$. The intensity distributions on the plane and spherical mirrors of the resonator and in the $xz$ plane inside the resonator are shown ($z$ is the resonator axis). The structure of the beams on the plane mirror and at centre of the spherical mirror is caused by the interference of the forward and backward radiation waves in the resonator. For $w_{\mathrm{g}}/w_0 = 1.5$, the off-axis maxima on the spherical mirror correspond quite

*For a three-level medium, the gain distribution $G(r) = G_0\exp\{-2[(x - x_{\mathrm{g}}^{\max})^2 + y^2]/w_{\mathrm{g}}^2\}$ should be used, which is equivalent to a combination of a uniformly pumped active medium and a Gaussian aperture displaced from the axis.

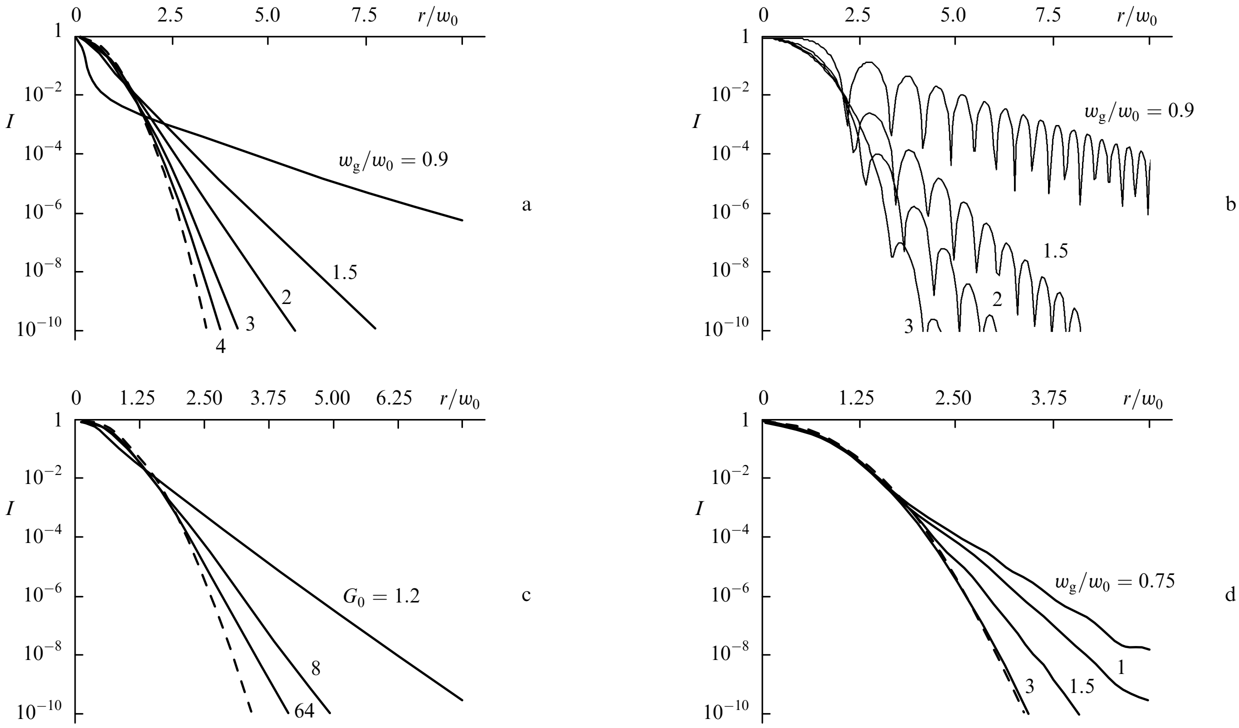


**Figure 2.** Radiation intensity distributions for a laser with degenerate ($L/R = 1/2$) (a–c) and nondegenerate ($L/R = 5/11$) (d) plano–spherical resonators on the plane (a, c, d) and spherical (b) resonator mirrors as functions of the pump region size $w_g/w_0$ and the gain maximum $G_0$ on the resonator axis (c). The dashed curves are the intensity distributions for the $u_{00}^{HG}$ mode.

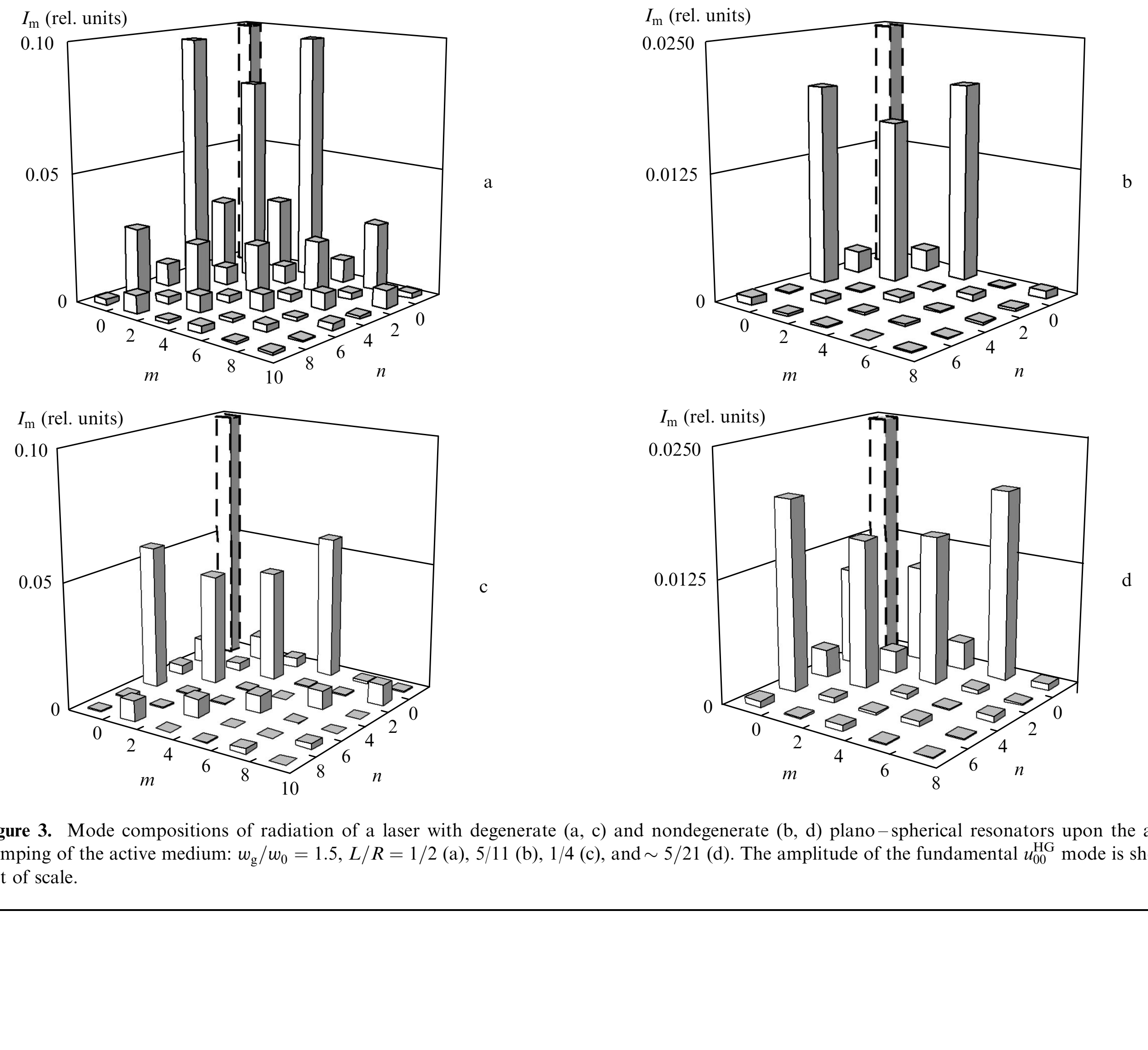


**Figure 3.** Mode compositions of radiation of a laser with degenerate (a, c) and nondegenerate (b, d) plano–spherical resonators upon the axial pumping of the active medium: $w_g/w_0 = 1.5$, $L/R = 1/2$ (a), 5/11 (b), 1/4 (c), and ∼ 5/21 (d). The amplitude of the fundamental $u_{00}^{HG}$ mode is shown out of scale.

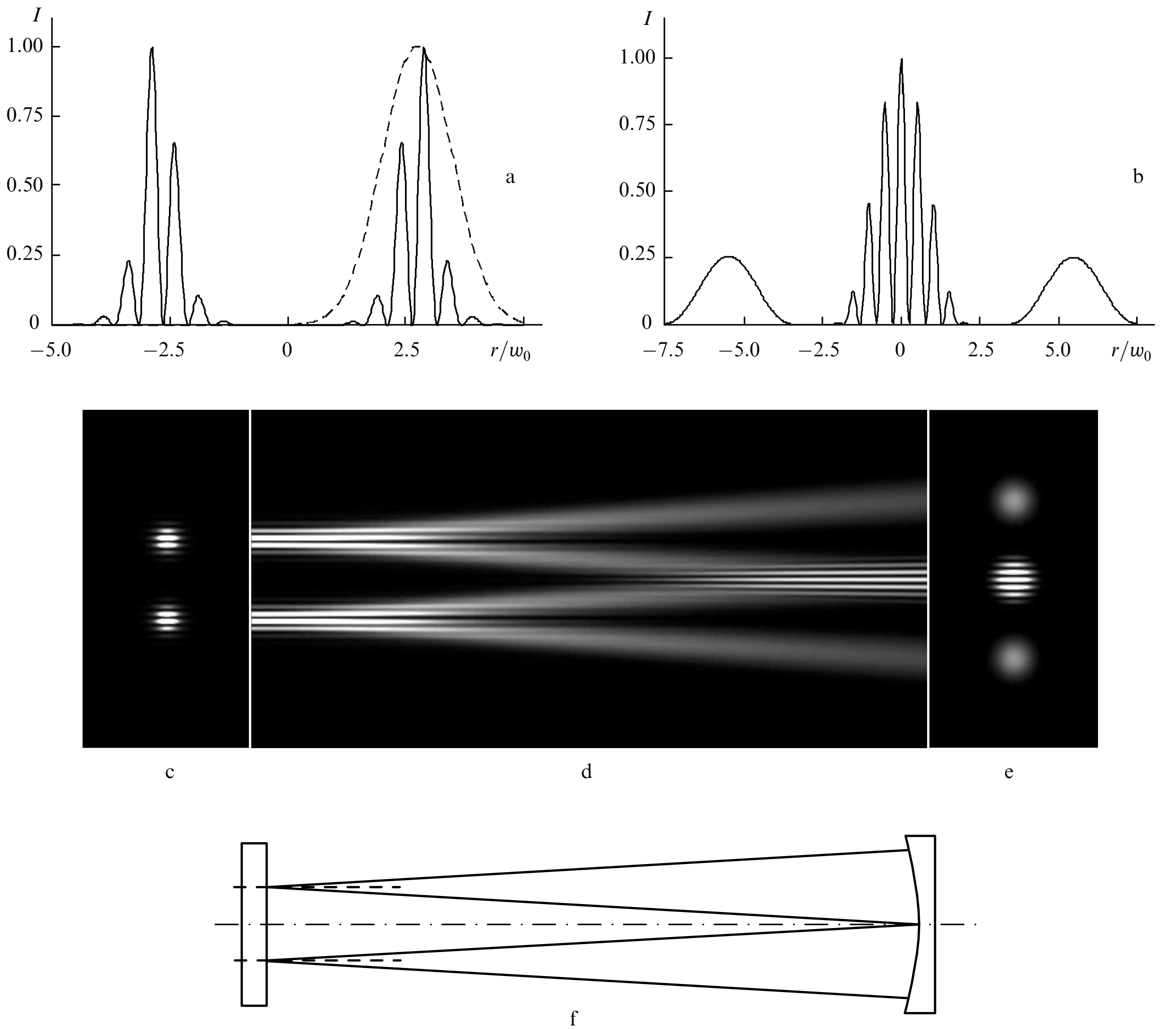


**Figure 4.** Off-axis asymmetric pumping of the active medium of a laser with a semi-confocal resonator ($G_0 = 1.2$, $x_g^{max} = 2.74w_0$, $w_g/w_0 = 1.5$): radiation intensity distributions on the plane (a, c) and spherical (b, e) mirrors and in the resonator volume (d); (f) laser-beam path in the resonator. The dashed curve in Fig. 4a is the pump intensity distribution; the dashed line in Fig. 4f is the position of the gain maximum.

accurately to the size of the $u_{00}^{HG}$ mode of the empty semiconfocal resonator (the deviation from $w_0$ was less than 2 %). As the ratio $w_g/w_0$ decreases, diffraction effects appear, which are similar to those observed upon axial pumping (see Fig. 2). Thus, the field formed in the semiconfocal (degenerate) resonator is an analogue of the beams of a self-closed optical delay line described in [16]. Such beams have the maximum overlap with the off-axis pump region and experience maximum amplification (Fig. 4a). The abode-described type of the field in the degenerate resonator is preserved with increasing the gain $G_0$.

The difference between symmetric and asymmetric off-axis pumpings is manifested as the suppression in the latter case of the intensity of one of the counterpropagating waves in the resonator if a field of the odd symmetry coinciding with the amplification symmetry is used as the 'seeding' field in calculations. In this case, the established field is reproduced in the plane of each of the mirrors after four round trips in the resonator. The intensity distributions presented in Figs 4a–e correspond to the seeding field of the even symmetry; they are observed both for symmetric and asymmetric off-axis pumping and are reproduced on mirrors (Figs 4a–c, e) after each round trip in the resonator.

### 2.2.2 Nondegenerate resonator

Upon off-axial pumping of the active medium, the principle of the maximum gain is also valid when the degeneracy of the resonator modes is removed, i.e. when $2\pi/\arccos(1 - 2L/R) \neq r/s$, where $r$ and $s$ are integers.

For $x_g^{max} = 1.37w_0$, $w_g/w_0 = 1.5$, and $G_0 = 1.2$, a beam with the intensity distribution close to that of the $u_{20}^{HG}$ mode is excited in a plano–spherical resonator with the mirror radius $R = 22$ cm (Fig. 5a). However, the expansion of the field of this beam in the modes of the resonator demonstrates (Fig. 5b) a rather complex structure in which the main components are the $u_{20}^{HG}$ and $u_{02}^{HG}$ modes. The contribution of the $u_{02}^{HG}$ mode at the linear scale is almost unnoticeable due to its low intensity. However, a comparison of this intensity distribution at the logarithmic scale (Fig. 5c) with the distribution of the $u_{20}^{HG}$ mode (Fig. 5d) reveals the presence of two hyperbolic zero-field lines, which are typical for the IG modes [11]. The distribution in Fig. 5c is almost exactly equal to the sum of the $0.9966u_{20}^{HG}$ $-0.0034u_{02}^{HG}$ modes, i.e. of the out-of-phase second-order HG modes (Fig. 5e). The total intensity of modes of other orders does not exceed 0.1 %.

As $x_g^{max}$ further increases for $G_0 = 1.2$, the increasingly

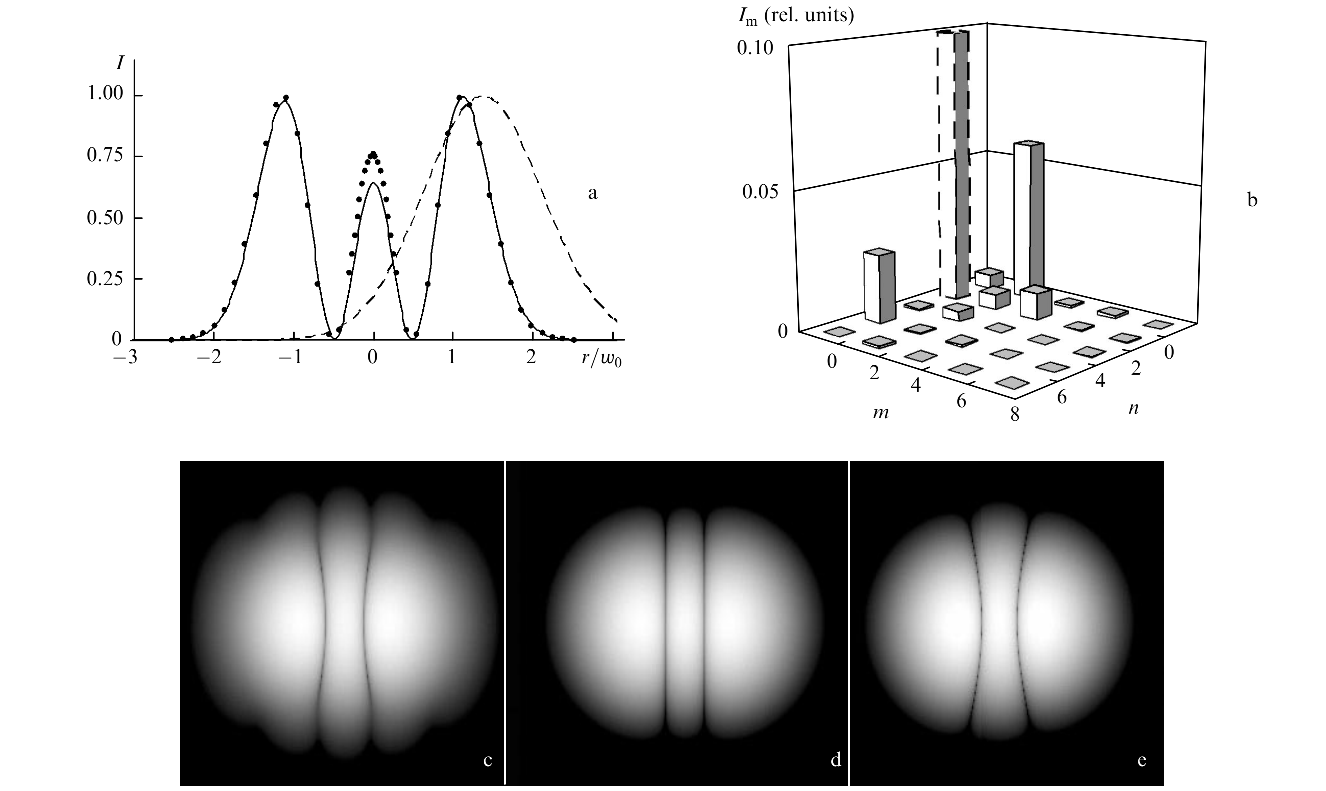


**Figure 5.** Off-axis pumping of the active medium of a laser in the nondegenerate resonator ($L = 10$ cm, $R = 22$ cm, $G_0 = 1.2$, $x_{\rm g}^{\max} = 1.37w_0$, $w_{\rm g}/w_0 = 1.5$): radiation intensity distributions on the plane laser mirror (a, c), the mode composition of radiation (b), and intensity distributions for the $u_{20}^{\rm HG}$ mode (d) and the sum of modes $0.9966u_{20}^{\rm HG} - 0.034u_{02}^{\rm HG}$ (e). The dashed curve in Fig. 5a is the pump intensity distribution; circles are the $u_{20}^{\rm HG}$ mode intensity; (c, d, e) logarithmic intensity scale.

higher-order IG modes are excited in the plano – spherical nondegenerate resonator (the question of whether or not the observed patterns are pure IG states was not studied). Thus, for $x_{\rm g}^{\max} = 2.74w_0$, the field mainly represented by the tenth-order IG mode was established in the resonator (Figs 6a, c). As the ratio $w_{\rm g}/w_0$ decreased from 1.5 to 0.75 (Fig. 6d), the considerable diffraction distortions of the intensity distribution appeared.

It may appear that the type of radiation close to that of the IG modes is explained by the asymmetry of the pump geometry with respect to the resonator axis. However, this is not the case. The addition of another gain region with the maximum at the point $x_{\rm g}^{\max} = -2.74w_0$, which is symmetric to the first one ($x_{\rm g}^{\max} = +2.74w_0$) does not cause any substantial changes (Figs 6b, e). Only the asymmetry of the main maxima in the radiation distribution is eliminated, while the zero-field lines remain hyperbolic, whereas they represent straight lines for the $u_{100}^{\rm HG}$ mode.

As $G_0$ increases upon off-axis asymmetric pumping, the radiation distribution in a nondegenerate resonator represents a strongly distorted distribution of the IG (or HG) mode type in which the intensity maximum shifts from the point $x_{\rm g}^{max}$ with increasing $G_0$ (Fig. 7). Note that the absence of the reduction of the field over the entire aperture of the active medium for the gain distribution of type (1) upon off-axis asymmetric pumping is principally important. Even for $G_0 \geqslant 32$, i.e. for the gains that can be achieved only in dye lasers, the field distribution strongly differs from the type of modes establishing in the nondegenerate resonator with an off-axis GA*.

Upon off-axis symmetric pumping, the field pattern already for $G_0 \geqslant 4$ (Fig. 8) proves to be quite close to that observed in the degenerate resonator. The main difference is that in the degenerate resonator the beam axis in the active-medium plane coincides with the amplification maximum (Fig. 4a), whereas such a coincidence is absent in the nondegenerate resonator (Figs 8a, f). The calculation shows that similar field distributions (in the form of the $u_{00}^{\rm HG}$ mode) propagating over a closed path can also exist in the nondegenerate resonator with two symmetrically located GAs. This effect is explained in the next section.

## 3. Discussion of results

The axial pumping of the active medium in a plano – spherical resonator was studied in several theoretical papers. A disadvantage of these papers is, as a rule, the use of the gain profiles admitting the analytic solution of the problem (see [17, 18]) and corresponding to the pumping conditions for a three-level active medium, when aperture effects dominate (for example, for a Yb : YAG laser [9]). In this case, the influence of the resonator degeneracy on the type of the established field, which was observed experimentally for a four-level

*The lowest eigenmode of both stable and unstable resonators with a GA is the $u_{00}^{\rm HG}$ mode. For a plano – spherical resonator with one off-axis GA, the eigenmode size is independent of the displacement of the GA from the resonator axis; only losses increase, exceeding 99.99 % for $w_{\rm gd} \approx 1.5w_0$ and $x_{\rm gd}^{\max} = 2.5w_0$.

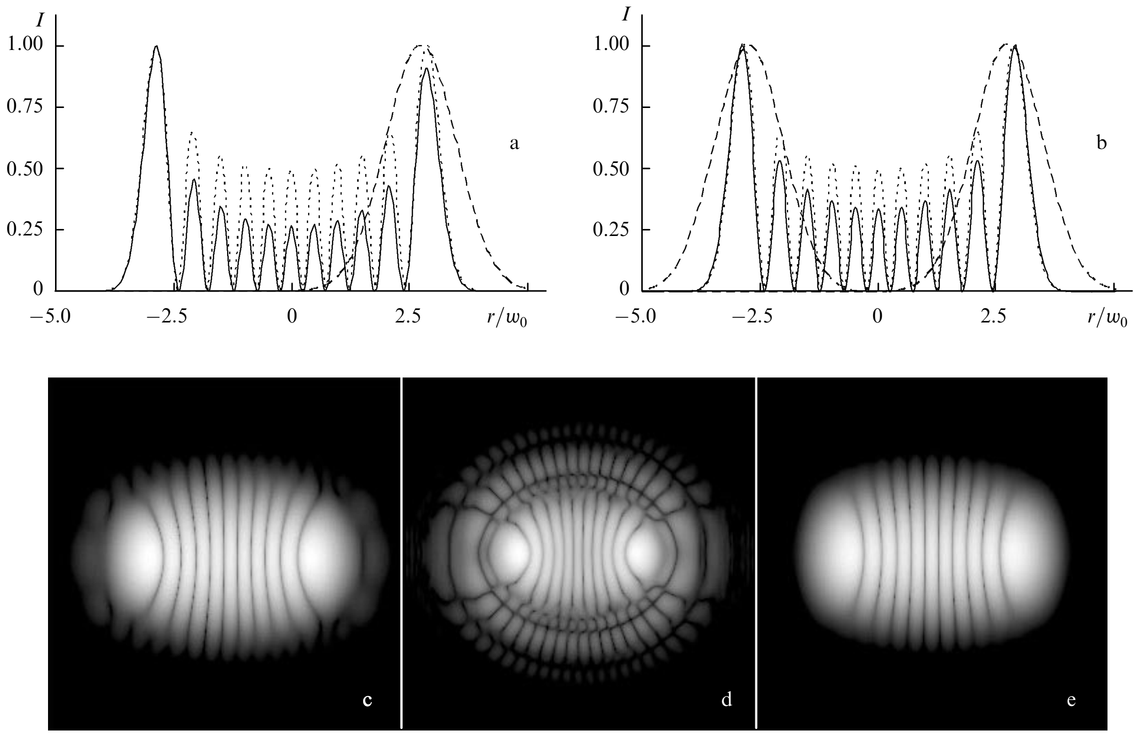


**Figure 6.** Off-axis pumping of the active medium of a laser with the nondegenerate resonator with one (a, c, d) and two (b, e) excitation regions ($G_0 = 1.2$, $L = 10$ cm, $R = 22$ cm, $|x_g^{max}| = 2.74w_0$) for $w_g/w_0 = 1.5$ (a, b, c, d) and $w_g/w_0 = 0.75$ (d). The solid curves in Figs 6a, b are the radiation intensity distributions on the plane resonator mirror; dotted curves are distributions for the $u_{10,0}^{HG}$ mode; dashed curves are pump radiation distributions; (c, d, e) logarithmic intensity scale.

$Nd:YVO_4$ laser [19], was not considered. Our numerical calculations confirm this observation: the dependence of the radius of the modes established in the nondegenerate resonator on the size of the pumped active medium is much weaker (see Figs 4a, d). This effect was theoretically explained in recent paper [20]. On passing from the degenerate to nondegenerate resonator, the interaction between modes changes from a strong resonance coupling to a weak nonresonance coupling, which occurs, in particular, upon strictly axial pumping of the active medium.

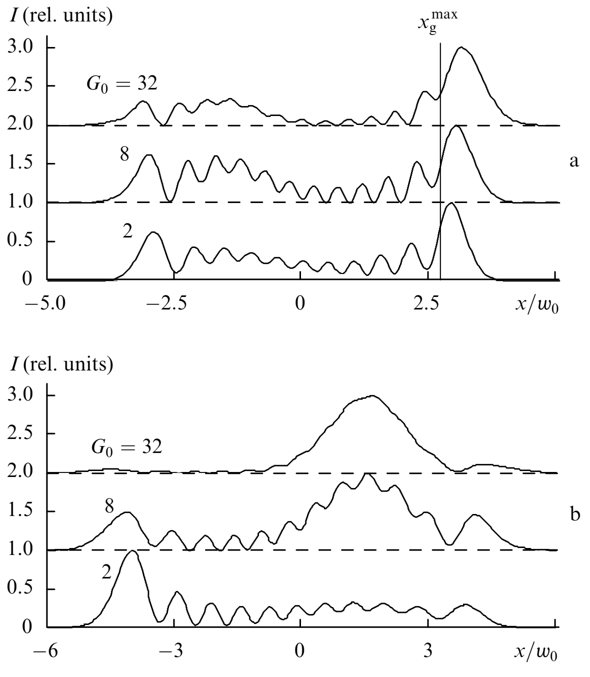


**Figure 7.** Radiation intensity distributions on the plane (a) and spherical (b) mirrors of the nondegenerate resonator ($L = 10$ cm, $R = 22$ cm) as functions of the gain $G_0$ upon asymmetric off-axis pumping ($x_g^{max} = 2.74w_0$). The curves are shifted along the vertical.

The type of the active medium of a laser and the resonator degeneracy are most strongly manifested upon off-axis pumping. In the case of the three-level active medium, pumping compensates ground-state absorption only in a restricted region, which is equivalent to the use of an off-axis GA in a plano – spherical resonator. In this case, the remark made above is valid (see the previous footnote). Upon off-axis pumping (1) of the four-level medium with a low gain, the IG modes are excited in the nondegenerate resonator, which are almost indistinguishable from the HG modes in the case of the small ellipticity parameter $\epsilon$ [11] and a limited dynamic range of the detection equipment. This probably explains the discrepancy between the results obtained in papers [2, 3] and [6] mentioned in Introduction.

The off-axis pumping in the degenerate resonator leads to the generation of the so-called geometric modes [7], which are also called the M and W modes [21]. Such modes were also observed in the study of a $CO_2$ laser [22]. Their origin was discussed in detail by Anan'ev [23] who pointed out that 'the approach based on the introduction of the 'V modes' gives nothing new compared to the traditional method. ...it is more convenient to use the model of a Gaussian beam propagating along a complex closed path than to seek the appropriate superposition of the frequency-degenerate basis

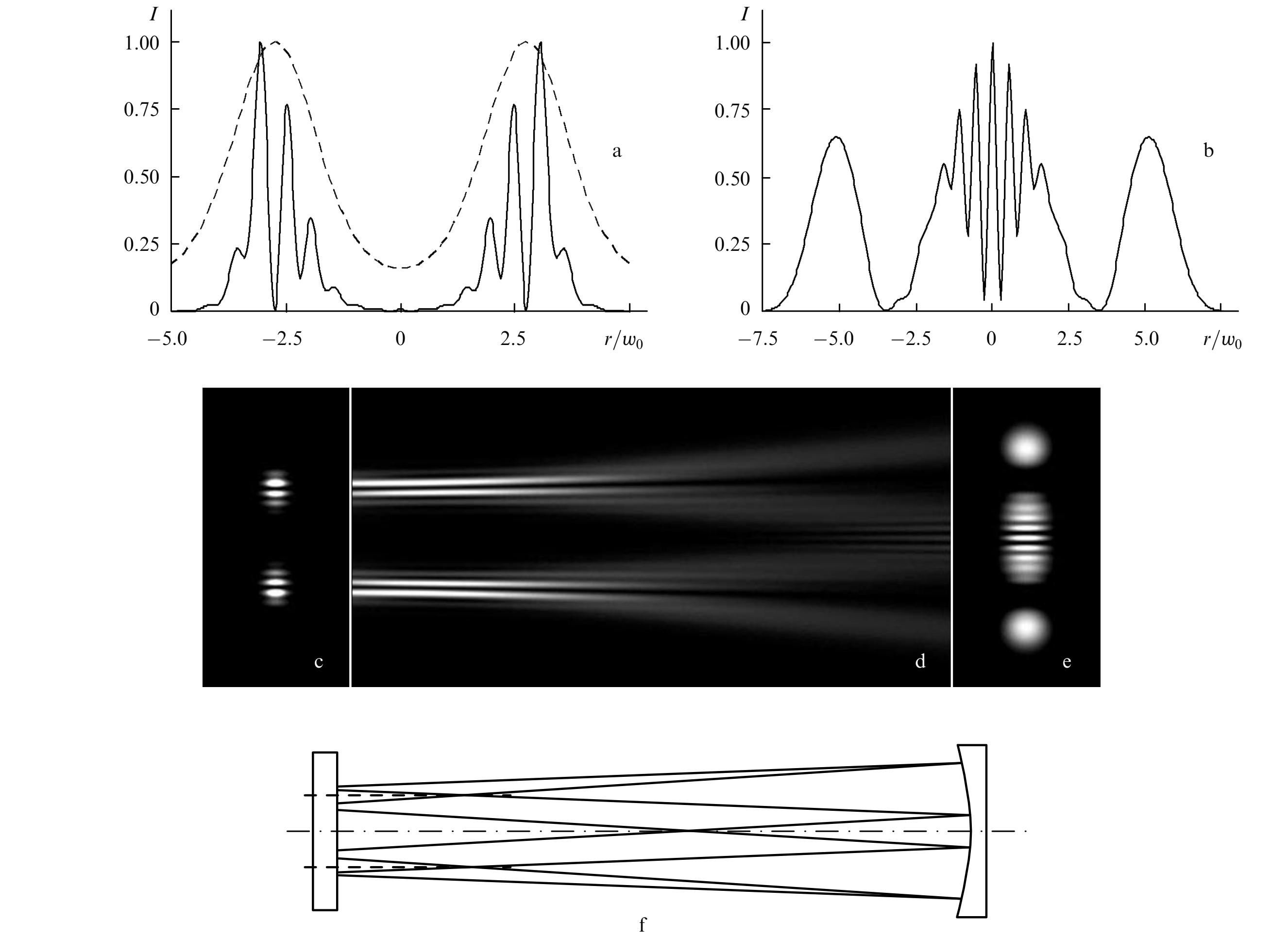


**Figure 8.** Off-axis symmetric pumping ($x_{\rm g}^{\max} = \pm 2.74 w_0$, $w_{\rm g}/w_0 = 1.5$, $G_0 = 8$) of the active medium of a laser with the nondegenerate resonator: radiation intensity distributions on the plane (a, c) and spherical (b, e) mirror and in the resonator volume (d); (f) laser-beam path in the resonator. The dashed curve in Fig. 8a is the pump intensity distribution; the dashed line in Fig. 8f is the position of the gain maximum.

modes. ...such a selection is always possible'. This statement confirms our calculations in which one of the side maxima was selected on the spherical mirror of the semi-confocal resonator (Fig. 4e), which was again injected into the resonator and performed up to 1000 round-trip transits in the resonator virtually without losses and any considerable intensity redistribution (Fig. 4f). The possibility of generating such beams in experiments with a semi-confocal resonator was considered in paper [21], where they were represented as a sum of degenerate modes.

Despite the above discussion, nothing prevents us to consider the field on each of the mirrors of the semi-confocal resonator (Figs 4c or e) as a unified structure. A similar field configuration, as we have seen, can appear due to the successive round trip of one Gaussian beam in the resonator, i.e. the modes of the semi-confocal or any other degenerate resonator have a dualism in some sense.

The question of whether there is the field in the semi-confocal resonator that is repeated after each four round trips or after one round trip in the resonator can be probably solved experimentally by studying the fine temporal structure of the intensity variation. In the first case, the time period for any of the off-axis maxima on the resonator mirrors should be $8L/c$, while in the second case, this period should be $-2L/c$.

It was shown above that the beams forming closed paths upon symmetric off-axis pumping can also exist in the nondegenerate resonator if the gain is high enough ($G_0 \geqslant 4$). To explain this effect, we consider the propagation of a Gaussian beam of radius $w_0$ outside the gain maximum given by (1):

$$I_{\rm t} = I_0 \exp\left[-2\frac{(x-x_0)^2}{w_0^2}\right] \times\left\{1 + (G_0 - 1)\exp\left[-2\frac{(x-x_{\rm g}^{\max})^2}{w_{\rm g}^2}\right]\right\}; \quad (2)$$

we assume for simplicity that $G_0 \gg 1$, which gives after transformations

$$I_{\rm t} \approx I_0 G_0 \exp\left[-2\frac{(x_{\rm g}^{\max} - x_0)^2}{w_{\rm g}^2 + w_0^2}\right] \exp\left[-2\frac{(x - x_{\rm t})^2}{w_{\rm t}^2}\right], \quad (3)$$

where

$$x_{\rm t} = \frac{x_0 w_{\rm g}^2 + x_{\rm g}^{\max} w_0^2}{w_{\rm g}^2 + w_0^2}; \quad (4)$$

$$w_t = \frac{w_0 w_g}{(w_g^2 + w_0^2)^{1/2}}. \quad (5)$$

The first exponential in expression (3) for $x_g^{max} \neq x_0$ determines a decrease in the Gaussian beam intensity due to the displacement of this centre from the gain maximum. The second exponential determines the position (4) of the beam after its propagation through the active medium and its radius (5). Note that the beam radius is independent of $x_g^{max}$ and $x_0$. Thus, it becomes clear why the positions of the beam incident on the plane mirror and reflected from it (Fig. 8f) do not coincide with each other. During the establishment of the field in the nondegenerate resonator upon symmetric off-axis pumping (Figs 8a, d), the maxima of the Gaussian beam are automatically detuned from the gain maxima so that the beam path in the resonator proves to be closed. It is in this way that the principle of the maximum gain is manifested for a high enough $G_0$ in the nondegenerate resonator. Note also that the points of intersection of incident and reflected beams in Fig. 8f lie behind the plane mirror, i.e. the beam path approaches the path that would be in a semi-confocal resonator with the spherical mirror radius $R = 22$ cm. Calculations show that for $R \approx 18$ cm these points of intersection are located in front of the plane mirror. As a whole, this effect can be compared with the action of a GA in an unstable resonator. The use of the GA (or a mirror with the Gaussian reflection profile) in such a resonator leads to the self-reproduction of the caustic of the beam during its successive passages in the resonator due to the change in the beam radius in the GA plane. Due to the change in the position of the Gaussian beam centre (an in it size) in the plane of Gaussian gain distributions in the nondegenerate resonator, the beam path is self-reproduced. In both cases, the effect is accompanied by additional radiation losses.

## 4. Conclusions

The numerical simulations of modes generated in a laser with plano–spherical degenerate and nondegenerate resonators upon diode pumping producing the Gaussian gain distribution in the active medium have been performed. It has been shown that in the case of axial pumping of the active medium, the lowest Hermite–Gaussian mode is excited both in the degenerate and nondegenerate resonators with the maximum weight if the pump level is sufficiently high or the characteristic size $w_g$ of the gain region of the active medium considerably exceeds the mode radius $w_0$.

In the nondegenerate resonator, upon a high enough level of the off-axis pumping ($G \geqslant 4$) producing two gain regions located symmetrically with respect to the laser axis, the types of modes can exist which have the parameters of the $TEM_{00}$ mode and periodically make round-trip transits in the resonator. This effect, as has been shown, is related to the different positions of the Gaussian beam centre at the input and output of the active medium with the Gaussian gain distribution.

The highest-order modes of the Ince–Gaussian type are excited in the nondegenerate resonator at low off-axis pump levels ($G_0 \approx 1.2$) both in the absence and presence of the symmetry of the gain distribution with respect to the resonator axis. No excitation of the modes that could be identified as Hermite–Gaussian modes have been observed in numerical experiments on the off-axis pumping of the active medium. The experimental identification of these modes upon such pumping performed in [2, 3] is most likely related to the insufficient dynamic range of the equipment.

As a whole, we can conclude that a simple model of the Gaussian gain distribution in the active medium upon diode pumping gives the result that is consistent with experimental data [6, 19, 21] and calculations performed by using more complicated models [21].

***Acknowledgements.*** This work was supported by the Russian Foundation for Basic Research (Grant No. 05-02-16818) and Grant No. NSh-8223.2006.2 of the President of the Russian Federation for the Support of Leading Scientific Schools.

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