

Properties of a thermal lens in laser ceramics

I.L. Snetkov, I.B. Mukhin, O.V. Palashov, E.A. Khazanov

Abstract. A model of thermal optical effects in laser ceramics was constructed, which takes into account random orientations of crystallographic axes in ceramics grains. Analytic expressions for the thermally induced phase, its average value and dispersion were derived. The effect of the beam-phase modulation with the characteristic transverse size of the order of the grain size was predicted. It was shown that deterioration of the parameters of the beam quality caused by this effect is inversely proportional to the ratio of the length of the ceramic element to the grain size.

Keywords: polycrystalline ceramics, thermal effects, thermal lens, beam-quality parameters.

1. Introduction

The report about the first ceramic laser (Nd : Y₂O₃ThO₂) appeared already in 1973 [1], however, the quality of ceramics was poor. In 1995, Nd : YAG ceramics was produced at the Japan Fine Ceramic Center by the method of isostatic pressing, which provided lasing with the output power of about 100 mW [2]. In the subsequent years, this technology was improved due to which the ceramics density was increased up to 99.9999 % of the density of a single crystal with the average size of the ceramics grain being 50 μm and the boundary thickness between the grains being 1 nm [3]. Another breakthrough was the technology of ceramics production by the modified deposition method with the use of vacuum sintering in the absence of external pressure, which was patented in 1998 by Koloshima Chemical [4, 5] (the characteristic grain size was 3–20 μm). The output power of Nd : YAG-ceramics lasers has increased from several hundreds of milliwatts [6] to 1.5 kW [7] for the past few years.

Many properties of ceramics are close to those of single crystals: absorption and emission spectra, transition cross section, lifetime of the upper laser level, thermal conductivity, nonlinearity of the refractive index, temperature dependence of the refractive index, breakdown threshold and chemical durability. At the same time a number of

properties of ceramics coincides with properties of glasses: possibility of obtaining wide apertures, homogeneous distribution of the activator, high ultimate concentration of the activator, possibility of controlling physicochemical and spectroscopic parameters, excellent optical quality and low cost. As seen from above, ceramics possesses unique properties available neither in single crystals nor in glasses.

The use of ceramics in high-power (average and peak) lasers is very promising due to three main advantages compared to single crystals and glasses. First, a large aperture as in glasses (450 mm [8]) and a high thermal conductivity as in single crystals. Second, possibility of producing ceramics from crystals, which cannot be grown in principle in the form of single crystals, for example, Y₂O₃, TAG, TSAG crystals, etc. Third, the viscosity of the ceramics damage is three–five times higher [9] and the parameter of the thermal damage is three times higher [10] than that in single crystals. All this makes the study of thermal effects in ceramic optical elements very urgent.

The thermally-induced depolarisation in Nd : YAG ceramics was first experimentally studied in [11, 12], where it was shown that depolarisation in ceramics is qualitatively similar to depolarisation in a single crystal with the [111] orientation. However, the experimental data were interpreted by the authors [11, 12] based on an erroneous assumption that the thermally-induced birefringence is independent of the orientation of crystallographic axes. A theoretical model of thermally-induced birefringence in ceramics was developed in [13, 14], which takes into account that the orientation of crystallographic axes is random in each grain. Fundamental effects were predicted in [13, 14] which have no analogues either in single crystals or glasses. In particular, it was shown that the polarisation of a beam propagated through a ceramic element has a spatial modulation with the characteristic transverse size, which does not exceed the average grain size. The predicted effects were observed in [15]. The results on depolarisation in ceramics and its compensation were generalised to the gyrotropic medium in [16]. At the same time, the properties of thermal distortions in ceramics have not been discussed so far.

In this paper, we studied the properties of phase distortions (thermal lens) in ceramics.

2. A model of thermal optics of a ceramic medium

Polycrystalline ceramics represents a set of single-crystal grains with the characteristic size from units to a hundred micrometers with very thin (less than 1 nm) boundaries

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between them. The main difference of ceramics from a single crystal consists in the fact that the orientation of crystallographic axes in each grain is random. Because a cubic single crystal is optically isotropic, the ceramics from any cubic crystal is also isotropic. However, the photoelastic effect caused by the temperature gradient leads to anisotropy, this effect in ceramics being totally different from the analogous effect in a single crystal. The matter is that the change in the permittivity tensor depends not only on the deformation tensor but also on orientation of crystallographic axes. Therefore, the birefringence caused by the photoelastic effect in each grain is characterised by a random orientation of eigen polarisations inherent in this grain and phase difference of waves with these polarisations (the phase difference from eigen polarisations), i.e. by its own Jones matrix.

Therefore, the thermally-loaded ceramic element represents sequentially placed phase plates with a random orientation of axes and random phase incursions of eigen polarisations (a single crystal represents a one deterministic phase plate). These values depend on transverse coordinates both deterministically (similarly to the single crystal) and randomly, because beams spaced at a distance of the order of the grain size propagate through a statistically independent set of grains. Thus, a thermally-induced phase (thermal lens) is a function a large number of random quantities.

In a coordinate system coinciding with the crystallographic one, the Jones matrix for the n th grain will have a diagonal shape and depend on the phase incursions δ_{1n} and δ_{2n} of eigen polarisations. To pass to the laboratory coordinate system, we should perform the rotation transformation, i.e. multiply the Jones matrix from the left and right by the corresponding rotation matrices \hat{R}_n and \hat{R}_n^{-1} . By multiplying the Jones matrices for all N of grains, we obtain the Jones matrix

$$\hat{M} = \exp\left(-i \sum_{n=1}^N \psi_n\right) \hat{T}, \quad (1)$$

for a ceramic sample, where

$$\hat{T} = \prod_{n=1}^N \hat{R}_n \begin{pmatrix} \exp(i\delta_n/2) & 0 \\ 0 & \exp(-i\delta_n/2) \end{pmatrix} \hat{R}_n^{-1}, \quad (2)$$

$$\psi_n = (\delta_{1n} + \delta_{2n})/2; \quad (3)$$

$\delta_n = \delta_{1n} - \delta_{2n}$. A ceramic sample is equivalent to the phase plate characterised by the Jones matrix \hat{M} . Let us find the phase incursion $\Psi = (\phi_1 + \phi_2)/2$, which is an arithmetic mean of phase incursions ϕ_1 and ϕ_2 of two eigen polarisations of this plate. Because absorption is absent, eigenvalues of the \hat{M} matrix has the form $\exp(i\phi_{1,2})$. The product of eigenvalues of the matrix is equal to its determinant. Taking into account that $\det \hat{T} = 1$, we obtain

$$\Psi = \sum_{n=1}^N \psi_n. \quad (4)$$

The mean arithmetic phase of any two orthogonal polarisations is also determined by expression (4). It can be easily verified by using the fact that the elements of the \hat{T} matrix satisfy the relations $T_{22} = T_{11}^*$ and $T_{21} = -T_{12}^*$. Therefore, the value Ψ characterises the phase distortions, i.e. a thermal lens. Note that Ψ does not depend on

orientation of eigen polarisations in grains. One can see from (1) and (4) that the problem of thermally-induced distortions can be divided into two separate problems, i.e. polarisation and phase ones. The \hat{T} matrix is responsible for polarisation distortions. Polarisation distortions are considered in detail in [13–16]. The thermal lens is determined by the phase Ψ according to (4).

3. Calculation of thermally-induced phase distortions in ceramics

To calculate the thermal phase Ψ according to (3) and (4), it is necessary to find values of phases of eigen polarisations δ_1 and δ_2 for each grain, which are determined by the change in the impermittivity tensor \hat{B} : $\delta_i = -B_{ii}n_0^3/2$, where n_0 is the ‘cold’ refractive index and $i = 1, 2$. (Hereafter till expression (12), index n denoting the grain number is omitted.) Taking into account (3), we obtain

$$\psi = -\frac{n_0^3}{4}(B_{11} + B_{22})kl, \quad (5)$$

where l is the grain length; $k = 2\pi/\lambda$; and λ is the wavelength. We consider below that the ceramic sample is a cylinder of length L , which is much larger than its radius R_0 (Fig. 1).

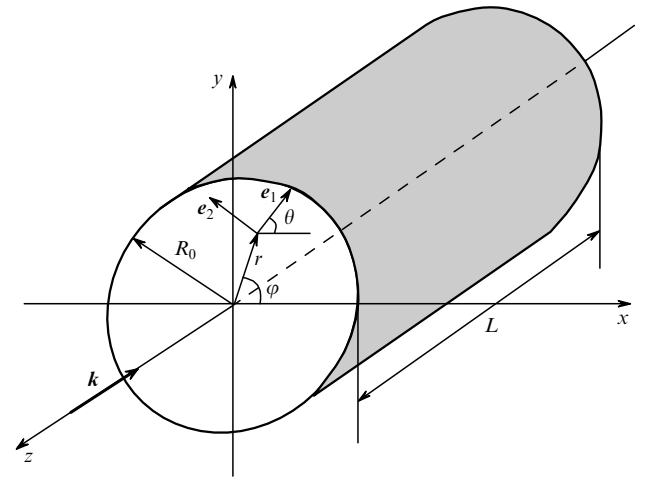


Figure 1. A cylindrical sample in the Cartesian (xyz) and cylindrical ($r\phi z$) coordinate systems: e_1, e_2 are unit vectors of the eigen polarisations at the point (r, ϕ) ; θ is the inclination angle to the x axis of eigen polarisation at the point (r, ϕ) .

To calculate the tensor \hat{B} for a grain with a random orientation of crystallographic axes a, b and c we will use the following procedure proposed in [17] for a single crystal and generalised in [13–16] for ceramics. The deformation tensor $\hat{\epsilon}$ is transformed from the cylindrical coordinate system $r\phi z$ (where it has a diagonal shape) into a Cartesian system xyz and then into a Cartesian system rotated with respect to the xyz system by the Euler angles γ, β and α so that its axes coincided with the crystallographic axes a, b, c . In this coordinate system, the tensor \hat{B} is determined by multiplying the photoelastic tensor $\hat{\rho}_a$ by the deformation tensor $\hat{\epsilon}$ and by subtracting the scalar term, which depends only on temperature T [18]:

$$\hat{B} = \hat{p}_a \hat{\varepsilon} - 2\beta_a T \hat{I} / n_0^3, \quad (6)$$

where $\beta_a = dn/dT$; n is the refractive index; and \hat{I} is the unit tensor. After this, the \hat{B} tensor upon rotating the coordinate system by the angles α , β and γ is transformed into a Cartesian coordinate system xyz . According to the procedure described in [19], analytic expression were obtained in the general form for all elements of the tensor B_{ij} ($i, j = 1, 2, 3$) including for the sum $B_{11} + B_{22}$:

$$B_{11} + B_{22} = (p_{11} + p_{12})(\varepsilon_{rr} + \varepsilon_{\varphi\varphi}) + 2p_{12}\varepsilon_{zz} + (p_{11} - p_{12}) \times (1 - \xi)(V\Delta + W\Sigma) - 4\beta_a T / n_0^3, \quad (7)$$

where

$$V = \frac{1}{4} \{ [(1 + \cos^2 \beta) \sin^2 2\alpha \sin^2 \beta - \sin^2 2\beta] \times \cos[2(\gamma - \varphi)] + \sin 4\alpha \cos \beta \sin^2 \beta \sin[2(\gamma - \varphi)] \}; \quad (8)$$

$$W = -\frac{1}{4} (\sin^2 2\alpha \sin^4 \beta + \sin^2 2\beta);$$

$$\Delta = \varepsilon_{rr} - \varepsilon_{\varphi\varphi}; \quad \Sigma = \varepsilon_{rr} + \varepsilon_{\varphi\varphi} - 2\varepsilon_{zz}; \quad (9)$$

$$\xi = \frac{2p_{44}}{p_{11} - p_{12}}; \quad (10)$$

p_{ij} ($i, j = 1, 2, \dots, 6$) are the elements of the photoelasticity tensor in the two-index notation of Nye (photoelastic coefficients). The value of ξ is called the parameter of the optical anisotropy of a crystal [18]. For all glasses $\xi = 1$ and for YAG we have $\xi = 3.2$.

Let the density of the heat release power $q(r)$ in the cylindrical sample (Fig. 1) depend only on the polar radius r and be independent either of z or polar angle φ :

$$q(r) = \frac{P_h F(u)}{L\pi r_h^2 \int_0^\rho F(u) du}, \quad (11)$$

where P_h is the power of heat release in the total volume of the sample; r_h and $F(u)$ are the radius and profile of the heat source; $u = (r/r_h)^2$; $\rho = (R_0/r_h)^2$. Without loss of generality, the integral in the denominator is considered equal to unity. The isotropy of thermal conductivity of ceramics follows from the isotropy of the thermal conductivity of a single crystal. The thermal conductivity coefficient κ for ceramics is the same as for a single crystal, which is a physical consequence of small thickness of boundaries between grains. Therefore, the equation for thermal conductivity and its solution – distribution of the temperature $T(r)$ – will be the same as for a single crystal. The coefficient α_T of the linear expansion of ceramics from a cubic crystal will be considered also isotropic. Thus, the equation for elasticity and its solution – deformation tensor – will be the same as for a single crystal. The temperature distribution $T(r)$ can be easily found from the thermal conduction equation and the expressions for the deformation tensor ε for the specified $T(r)$ can be found in [20]. By substituting them in (7) and the result in (5), we obtain the phase ψ for one grain, after which we determine from (4)

$$\Psi = \text{const} - \frac{P_h P}{2\lambda\kappa} f(u) + \frac{P_h Q}{2\lambda\kappa L} (1 - \xi) \times \sum_{n=1}^N l_n [V_n h(u) + W_n g(u, \rho)], \quad (12)$$

where

$$Q = \alpha_T \frac{n_0^3}{4} \frac{1+v}{1-v} (p_{11} - p_{12}); \quad (13)$$

$$P = \beta_a - \alpha_T \frac{n_0^3}{4} \frac{1+v}{1-v} (p_{11} + p_{12}); \quad (14)$$

$$f(u) = \int_0^u \frac{dz}{z} \int_0^z F(\eta) d\eta; \quad g(u, \rho) = \frac{1}{\rho} \int_0^\rho dz \int_0^z F(\eta) d\eta - \int_u^\rho \frac{dz}{z} \int_0^z F(\eta) d\eta; \quad h(u) = \frac{1}{u} \int_0^u dz \int_0^z F(\eta) d\eta; \quad (15)$$

v is the Poisson coefficient; l_n is the length of the n th grain; V_n , W_n are coefficients (8) for the n th grain. Constants P and Q are called thermo-optical constants of the medium [18], Q characterising the value of the thermally-induced anisotropy and P – the value of isotropic distortions. For a uniform distribution of the heat-release power $F = 1$, $\rho = 1$, expressions (15) are considerably simplified:

$$f = \frac{r^2}{R_0^2}, \quad g = \frac{r^2}{R_0^2} - \frac{1}{2}, \quad h = \frac{r^2}{2R_0^2}. \quad (16)$$

The first term in (12) depends neither on the transverse coordinates nor on the direction of crystallographic axes. Therefore, the phase incursion constant over the cross section of the sample, which does not introduce any distortions in the laser beam, corresponds to this term. We will omit this term below. The second term in (12) depends on the transverse coordinates but is independent of the direction of crystallographic axes. Therefore, a deterministic thermal lens analogous to the lens in a single crystal corresponds to it. In this case as for a single crystal this term is proportional to the thermo-optical constant P . The third term in (12) depends on the transverse coordinates and the direction of crystallographic axes. Therefore, it has a random value in each point of the transverse cross section. Note that this term is proportional to the thermo-optical constant Q , which is responsible for the anisotropic effects. As expected, upon passage from a cubic crystal to glass ($\xi \rightarrow 1$) this term vanishes.

Expression (12) fully determines the thermal lens not only for ceramics but for a single crystal (the sum will consist of one term) with an arbitrary orientation for any distribution of the power of the heat release (11). In particular, for the [001] (α - any, $\beta = 0$) and [111] orientations ($\alpha = \pi/4$, $\tan^2 \beta = 2$) we obtain from (12)

$$\Psi_{[001]} = -\frac{pP}{2Q} f(u), \quad (17)$$

$$\Psi_{[111]} = -\frac{p}{2} \left[\frac{P}{Q} f(u) + \frac{1}{3} (1 - \xi) g(u, \rho) \right], \quad (18)$$

where

$$p = \frac{QP_h}{\lambda\kappa}. \quad (19)$$

The parameter p means the heat-release power in the entire sample normalised to the medium constants. Estimates show that for $p > 20$ the mechanic damage of the sample is highly probable. For a homogeneous heat release, the parameters g and f are determined by relations (16) and expressions (17) and (18) are transformed into expressions presented in [18, 21].

4. Calculation of the average value (mathematical expectation) and dispersion (root-mean-square deviation) of the phase

Let us obtain the averaged parameters of phase distortions of a beam (12): mathematical expectation and dispersion. We assume that Euler angles α , β and γ are distributed uniformly within its limits ($\alpha \in [-\pi, \pi]$, $\beta \in [-\pi/2, \pi/2]$, $\gamma \in [-\pi, \pi]$) and the grain length l is a random quantity, which has an average value l_g , i.e. the average number $N_g = L/l_g$ of grains on the beam path. In addition, we assume that random quantities α , β , γ and l are independent for different grains. In other words, the distribution function for a set of grains is the product of distribution functions for each grain. Note that because the length L of the sample is specified, the number N of grains, i.e. the number of terms in sum (12) is also a random quantity depending on l . By averaging expression (12) over the values of α , β , γ and l , we obtain

$$\langle \Psi \rangle = -\frac{p}{2} \left[\frac{P}{Q} f(u) + (1 - \xi) \frac{11}{64} g(u, \rho) \right]. \quad (20)$$

One can see from (20) that the average value of the phase incursion over the length of a ceramic element does not depend on the number N_g of grains on the beam path and is directly proportional to p . It follows from comparison of (18) and (20) that the only difference of $\langle \Psi \rangle$ from $\Psi_{[111]}$ consist in the substitution of $(1 - \xi)/3$ by $11(1 - \xi)/64$. In other words, the average value of phase distortions in ceramics is equal to distortions in a single crystal with the [111] orientation and effective constant ξ_{eff} determined by the expression

$$1 - \xi_{\text{eff}} = \frac{64}{33} (1 - \xi). \quad (21)$$

For a YAG crystal, we have $\xi = 3.2$ and $\xi_{\text{eff}} = 5.3$. The calculation of the phase dispersion (12) by the formular $D = \langle \Psi^2 \rangle - \langle \Psi \rangle^2$ yields

$$D = \frac{p^2 (1 - \xi)^2}{2^{17} N_g} \left(1 + \frac{D_l}{l_g^2} \right) [265h^2(u) + 330g^2(u, \rho)], \quad (22)$$

where D_l is the dispersion of the grain length. One can see from (22) that the phase dispersion is proportional to the square p and inversely proportional to the average number N_g of grains on the beam path.

Along with the analytic analysis we simulated numerically a ceramic sample. For a numerical averaging for each beam with coordinates r and φ , the computer generated random quantities α , β , γ and l for each grain and calculated the phase incursion according to (12). As soon as the total length of grains exceeded the length L of the sample, the length of the last grain decreased to the value for which the total length of all grains was equal to L . The power distribution of the heat release was assumed homogeneous

over the volume, i.e. f , g and h were determined by (16). The phase incursions were calculated for 40000 points of the transverse cross section. 100 realisations were calculated in each point after which the average values of the phase $\langle \Psi \rangle$ and its dispersion D were calculated.

The difference of $\langle \Psi \rangle$ from the values yielded by expressions (16) and (20) was less than 1%. Figure 2 presents radial dependences of Ψ for a random realisation and Fig. 3 presents radial dependences of D for a numerical averaging over 100 realisations and analytical averaging, i.e. by using expressions (16) and (22). One can see from Fig. 3 that the analytic results well agree with the numerical results. Note that in the beam centre where the thermal phase has a minimal gradient (i.e. distortions are minimal) the phase dispersion has a local maximum. At the same time, for $r = 0.65R_0$ the phase dispersion is minimal.

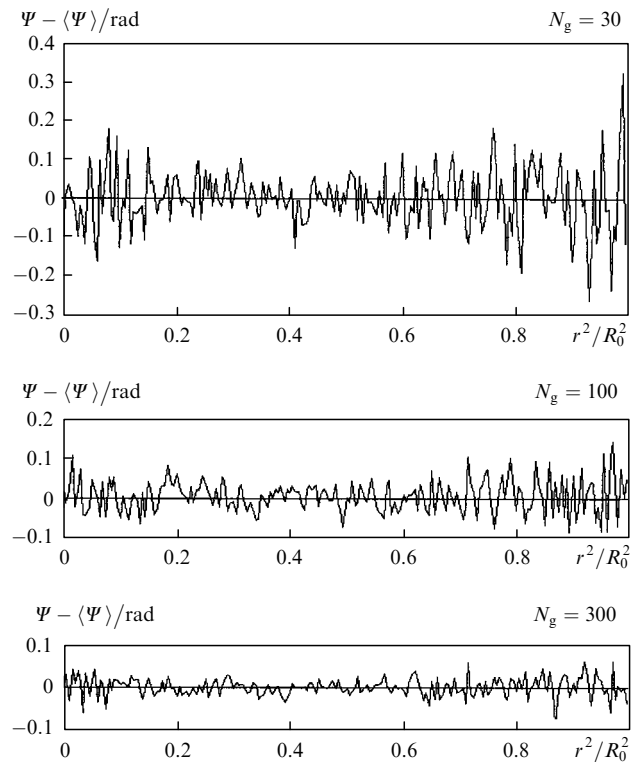


Figure 2. Random realisations of the phase $\Psi - \langle \Psi \rangle$ for different N_g and $p = 10$.

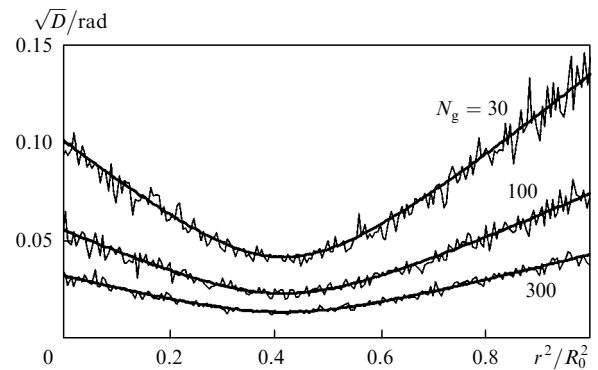


Figure 3. Phase dispersion D obtained analytically (thick curves) and for numerical simulation of 100 realisations (thin curves) for different N_g and $p = 10$.

5. Deterioration of the beam quality due to the thermal lens

The thermally-induced phase (12) can be divided into the value $\langle\Psi\rangle$ averaged over realisations and random deviations $\Psi - \langle\Psi\rangle$. The average thermally-induced phase $\langle\Psi\rangle$ can be compensated for to some extent as in single crystals. When the source of heat release is uniform, the parameter $\langle\Psi\rangle$ parabolically depends on the radius and for its compensation it is enough to change the resonator configuration or to use an ordinary lens or a telescope with the focal distance, which is the same in the module but opposite in sign. In the general case, an absorbing medium with the value of P opposite in sign [21, 22] or deformable adaptive mirrors are efficiently used to compensate for the thermal lens. We will be interested in specific ceramic effects, which cannot be compensated for by the above methods that is why we will calculate the deterioration of the beam quality caused by the phase $\Psi - \langle\Psi\rangle$. In addition, as we are interested in phase effects, we will assume that neither the amplitude nor beam polarisation was distorted after propagation through the ceramic element.

As the quality criterion, we will use the overlap integral χ , the Strehl number S and the parameter M^2 , whose definitions are well known and can be found in [21, 23–27]. While calculating $\langle\chi\rangle$ and $\langle S\rangle$, we assumed that $|\Psi - \langle\Psi\rangle| \ll 1$, which is, as shown above, nearly always valid for reasonable thermal loads. To calculate the parameter $\langle M^2\rangle$, it is convenient to use not the definition of M^2 given in [25] but the expressions obtained by the method of moments [27]. For a beam with a complex field amplitude $E(r)$, we have

$$\begin{aligned} \langle\chi\rangle &= 1 - \int_0^\infty DE^2 r dr / \int_0^\infty E^2 r dr, \\ \langle S\rangle &= 1 - \int_0^\infty DEr dr / \int_0^\infty Er dr, \\ \langle M^4\rangle &= M_{\text{dif}}^4 + \left(\frac{2a_{\text{eff}}}{l_g}\right)^2 \int_0^\infty DE^2 r dr / \int_0^\infty E^2 r dr, \end{aligned} \quad (23)$$

where

$$a_{\text{eff}} = \left(\int_0^\infty E^2 r^3 dr / \int_0^\infty E^2 r dr \right)^{1/2}$$

is the effective beam radius; M_{dif}^2 is the parameter M^2 for a undistorted beam (a beam with the plane phase). One can see from (23) that all the three quality parameters are deteriorated, i.e. $1 - \langle\chi\rangle$, $1 - \langle S\rangle$ and $\langle M^4\rangle - M_{\text{dif}}^4$, which are proportional to the phase dispersion D . It follows from (22) that this deterioration is proportional to the square of the power p and is inversely proportional to the number N_g of grains on the beam path. Unlike $\langle\chi\rangle$ and $\langle S\rangle$, the value of $\langle M^4\rangle$ depends also on the number of grains on the beam diameter $2a_{\text{eff}}/l_g$. This has a simple physical explanation. The overlap integral characterises the decrease in the power in the initial transverse mode and the Strehl number – a decrease in the power on the axis in the focal plane. In what modes and at what distances from the beam axis the power ‘spread’ due to thermal distortions does not play any role. That is why both these parameters do not depend on the characteristic transverse size of the phase modulation and are determined only by its depth, i.e. the phase dispersion

D . At the same time, the increase in the parameter M^2 due to phase distortions is determined by an increase in the second moment (effective radius) of the beam in the waist. For small phase distortions, the main part of the power will be localised in the diffraction region and only its negligible part (proportional to D) will be distributed over a larger area. Inherently, the second moment substantially depends on how large is this area. Its characteristic size is determined by the square of the ratio of the initial beam diameter $2a_{\text{eff}}$ to the spatial scale of distortions equal to l_g , which one can see from (23). Because this ratio is large, the thermally-induced distortions lead to an insignificant change in the overlap integral and the Strehl number and at the same time to a dramatic increase in the parameter M^2 .

Let us illustrate this for the case of a uniform heating and a Gaussian beam with the complex field amplitude $E(r) = E_0 \exp\{-[r/(2a_{\text{eff}})]^2\}$. By substituting (16) into (22), and the result into (23), it is easy to take analytically all the integrals and obtain expressions for the quality parameters. Figures 4 and 5 show the dependences of the quality parameters on the ratio of the beam radius a_{eff} to the radius R_0 of the ceramic sample.

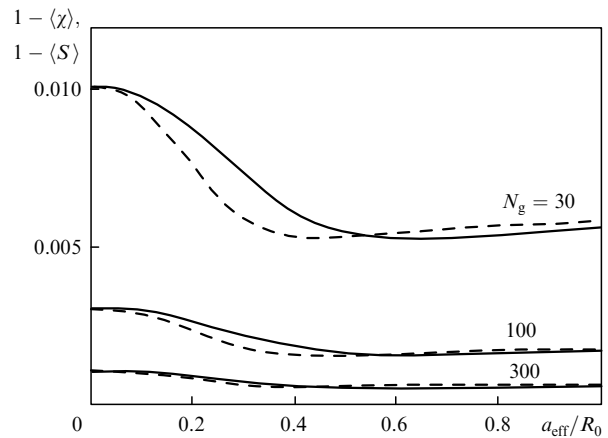


Figure 4. Dependences of $1 - \langle\chi\rangle$ (solid curves) and $1 - \langle S\rangle$ (dashed curves) on the beam radius a_{eff} for different N_g and $p = 10$.

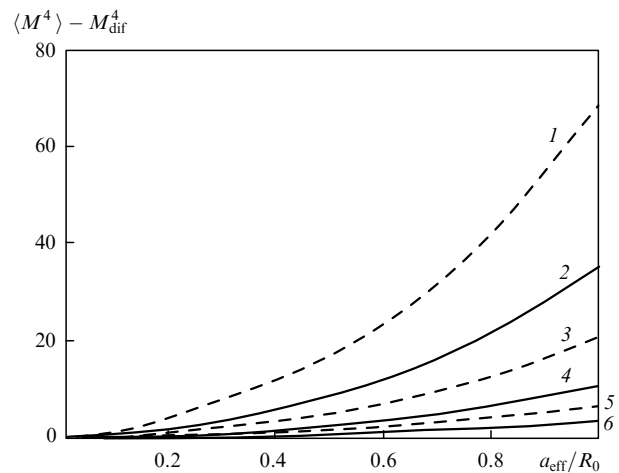


Figure 5. Dependences of $\langle M^4\rangle - M_{\text{dif}}^4$ on the beam radius a_{eff} for $N_g = 30$ (1, 2), 100 (3, 4) and 300 (5, 6), $l_g = 0.018R_0$ (1, 3, 5) and $0.025R_0$ (2, 4, 6), $p = 10$.

In lasers with a high peak power, apart from the distortion of the beam quality described above thermally-induced phase distortions in ceramics can lead to the development of electronic small-scale self-focusing of radiation for a relatively low values of the B integral. For a quantitative description of this effect, it is necessary to take into account polarisation distortions along with the phase distortions [13–16]. The problem of self-focusing in laser ceramics is the subject to future studies.

6. Conclusions

Let us sum up the main results of this study:

(i) A model of thermo-optical distortions (both polarisation and phase distortions) in laser ceramics has been constructed, which takes into account random orientation of crystallographic axes in grains.

(ii) Expressions for the thermally-induced phase incursion, its average value (mathematical expectation) and root-mean-square deviation have been derived for any axially-symmetric heat release in a rod ceramic element.

(iii) The average value of the thermally-induced phase incursion in ceramics is the same as in a single crystal with the [111] orientation with the accuracy to the change of the parameter ξ by ξ_{eff} according to expression (21). In particular, the average phase incursion does not depend on the number of grains on the beam path and is directly proportional to the power of the heat release.

(iv) The thermally-loaded ceramics introduced the small-scale phase modulation with the characteristic size of the order of the grain in the beam. The root-mean-square deviation of the phase is directly proportional to the power of the heat release and is inversely proportional to the root from the number of grains on the beam path.

(v) The difference of the Strehl number and the overlap integral from unity, which is caused by the small-scale phase modulation, is insignificant. At the same time, the parameter M^2 increases dramatically. In addition, this modulation can serve as a source of self-focusing in lasers with a high peak power.

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