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Excitation of leaky modes in a system of coupled waveguides

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***Abstract*. A system of coupled single-mode waveguides with the number M of guided modes lower than the number N of single-mode waveguides is studied. Leaky modes in this system are investigated in detail. It is shown, in particular, that these modes can be excited by light incident on the side surface of the system when the reflection coefficient vanishes. It is found that the angular dependence of the coefficient of reflection from the side surface of the system can be used to refine the dispersion curve for leaky modes. It is shown that light incident at a grazing angle can propagate in the system in the direction considerably different from the propagation direction of a beam incident from a substrate, even in the case of a small difference in the refractive indices.**

Keywords: *leaky modes, coupled waveguides, propagation of light in channel waveguides, diffraction of light.*

1. Introduction

A system of tunnel-coupled waveguides is called homogeneous if it consists of identical equidistant waveguides in which light propagation constants are independent of longitudinal and transverse coordinates. The simplest example of such a system is a usual dielectric multilayer interference mirror. Another example is an array of channel waveguides formed in a planar waveguide film lying on a substrate with a low refractive index. Channel-waveguide systems attract special recent interest in waveguide laser optics due to the possibility of increasing their output power [1]. The propagation of light in homogeneous systems of channel waveguides was studied earlier in papers [2, 3]. It was pointed out, in particular, that the Bragg diffraction of light in them is of special interest [4]. In this connection we studied in this paper the Bragg diffraction of light incident both on the side surface of a waveguide system and on waveguide ends. The aim of the paper is to find the conditions providing a high selection of modes due to Bragg diffraction in a system of coupled waveguides.

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2. Theory

We showed earlier [2] that the number of modes in a system of homogeneous tunnel-coupled single-mode waveguides is determined by the number N of waveguides and the refractive index n_f of the waveguide. In this paper, we consider a waveguide system with the following parameters: $N = 50$, the width of waveguide layers is $h = 1.1$ μm, $n_f = 1.465$, the width of spacings between two layers is $s = 1.3$ μm, and the refractive index of spacings is $n_s = 1.46$. We assume that the refractive index of the environment is also 1.46. If a single-mode waveguide is sufficiently 'weak', the number M of guided modes in the system is smaller than the number of waveguides ($M < N$). In our system, M proved to be equal to 34, while the rest of the modes are leaky modes. Figure 1 presents the dependences of the effective refractive indices n^* of modes on the mode number m [5]. The leaky modes are radiation modes that carry energy out of the waveguide system through its side surface. The angle of radiation escape through the side surface to a substrate (the angle of radiation propagation in the substrate) is described by the expression

$$\cos\theta = \frac{n^*}{n_s}. \qquad (1)$$

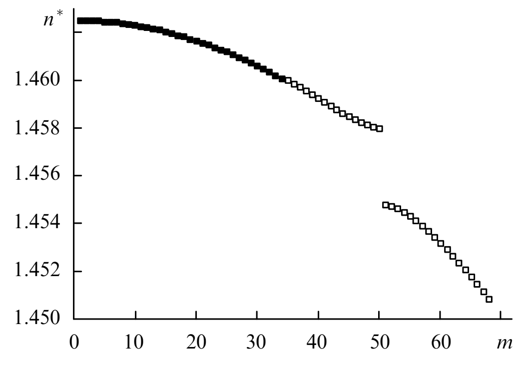


Figure 1. Dependences of the effective refractive index n^* of guided (■) and leaky (□) modes in a system of tunnel-coupled waveguides on the mode number m for $N = 50$, $s = 1.3$ μm, $h = 1.1$ μm, $\Delta n = n_f - n_s = 0.005$, and $n_s = 1.46$.

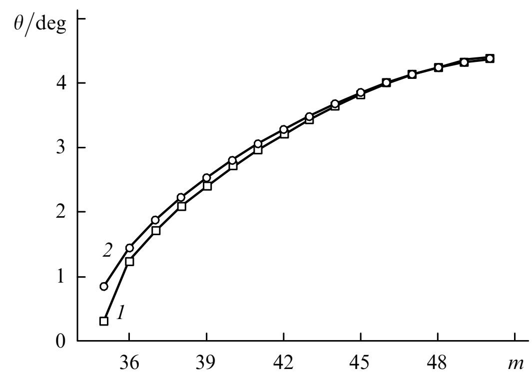


Figure 2. Dependence of the angle of propagation θ of leaky modes on the mode number m calculated from data in Fig. 1 (*1*). Curve (*2*) is angular positions of the zero values of light reflection from the side surface of a system of coupled weaveguides.

In our case, the substrate is a reference tube [6] with the refractive index $n = 1.46$. Curve (*1*) in Fig. 2 shows the dependence of the angle of propagation of leaky modes on the mode number m.

If a waveguide system has leaky modes, these modes can be excited by light incident on the side surface of the system from the environment, i.e. from a substrate. Because we consider a system of periodically arranged waveguides, we calculated first the Bragg diffraction from the side surface of the system for a light beam incident from a substrate at different glancing angles θ [7]. Figure 3 shows this dependence. First, note that for small glancing angles ($0 < \theta < 3°$), the dependence of the reflection coefficient $R(\theta)$ has the same number of maxima and minima equal to $N - M = 16$, which corresponds to the number of leaky modes of the system of waveguides located in the first allowed band of the photon system. Second, we compared the dependences of the angular positions of the zeroes of the function $R(\theta)$ and the angle of propagation of leaky modes on the mode number and found that these dependences coincide with good accuracy (Fig. 2). These two circumstances suggest that the Bragg reflection of light from the side surface of the waveguide system leads to excitation of leaky modes if the angle of incidence of light corresponds to the minimum of the function $R(\theta)$.

By considering the waveguide system as a one-dimensional photonic crystal, it is convenient to represent the propagation of light in it by the Floquet–Bloch waves with the Bloch wave vector $\boldsymbol{K}_{\mathrm{B}}$ whose modulus is described by the expression [7]

$$\cos(K_{\mathrm{B}}\Lambda) = \cos(\kappa_{\mathrm{f}}h)\cos(\kappa_{\mathrm{s}}s) - \frac{1}{2}\left(\frac{\kappa_{\mathrm{s}}}{\kappa_{\mathrm{f}}} + \frac{\kappa_{\mathrm{f}}}{\kappa_{\mathrm{s}}}\right)\sin(\kappa_{\mathrm{f}}h)\sin(\kappa_{\mathrm{s}}s), \quad (2)$$

where $\Lambda = h + s$ is the structure period; $\kappa_{\mathrm{f}} = k(n_{\mathrm{f}}^2 - n^{*2})^{1/2}$ and $\kappa_{\mathrm{s}} = k(n_{\mathrm{s}}^2 - n^{*2})^{1/2}$ are the moduli of the transverse wave vectors calculated for plane waves propagating in the waveguide and intermediate layers of the system; $k = 2\pi/\lambda$; and λ is the wavelength of light in vacuum. According to [7], the reflection coefficient upon the side incidence of light is

$$R(\theta) = \frac{|C|^2}{|C|^2 + \left[\sin(K_{\mathrm{B}}\Lambda)/\sin(NK_{\mathrm{B}}\Lambda)\right]^2} \quad (3)$$

and is determined by the rapidly oscillating function $\sin(K_{\mathrm{B}}\Lambda)/\sin(NK_{\mathrm{B}}\Lambda)$. The coefficient

$$|C|^2 = \frac{1}{4}\left(\frac{\kappa_{\mathrm{f}}}{\kappa_{\mathrm{s}}} - \frac{\kappa_{\mathrm{s}}}{\kappa_{\mathrm{f}}}\right)^2 \sin^2(\kappa_{\mathrm{f}}h) \quad (4)$$

plays the role of the structural factor of a crystal cell and varies slowly with θ. For $|C|^2 \neq 0$, the condition for the reflection minima

$$\frac{\sin(NK_{\mathrm{B}}\Lambda)}{\sin(K_{\mathrm{B}}\Lambda)} = 0 \quad (5)$$

following from (3) is in fact the dispersion equation for the leaky modes of the waveguide system.

We calculated the field inside the waveguide system for light incident on the side surface and found that the reflection minima correspond to the maximum accumulation of optical energy inside the waveguide system, i.e., in other words, to excitation of the leaky modes. For the angles corresponding to the reflection maxima, the energy was not accumulated in the system. This behaviour of the system is completely similar to that of a Fabry–Perot resonator, where energy is also accumulated in the reflection minima. The recalculation of the angles corresponding to the reflection minima to the effective refractive index allows us to refine the dispersion

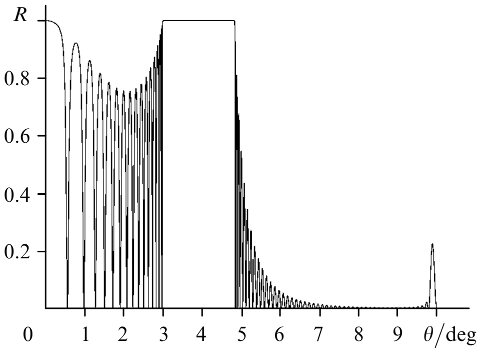


Figure 3. Angular dependence of the reflection coefficient for light incident on the side surface of a waveguide system from a supporting tube.

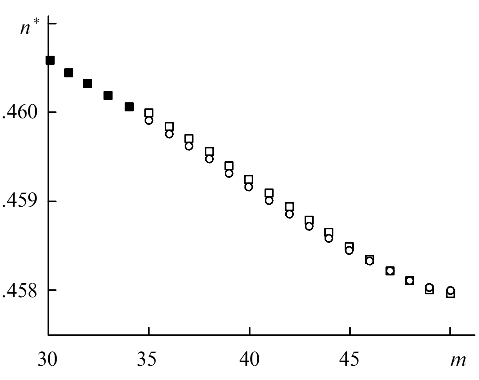


Figure 4. Refined dispersion curve (○) for the waveguide system studied; (■) guided modes; (□, ○) leaky modes.

curves shown in Fig. 1. The effective refractive indices for leaky modes calculated in this way are in good agreement with those obtained earlier (Fig. 4).

3. Experiment

We observed earlier in [8] the excitation of leaky modes by light incident on the side surface of a waveguide system. In this paper, we studied this phenomenon in more detail. Figure 5 shows schematically the setup used in experiments. To observe the picture of light beams reflected from a sample and transmitted through it, a screen was placed at a distance of 2.5 m from the sample. The intensity distribution of the excited modes was observed by projecting the image of the output end of the waveguide system by a microobjective (with a magnification of 300) on the screen.

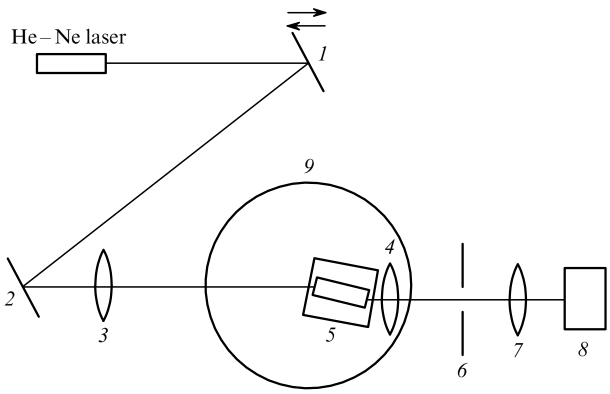
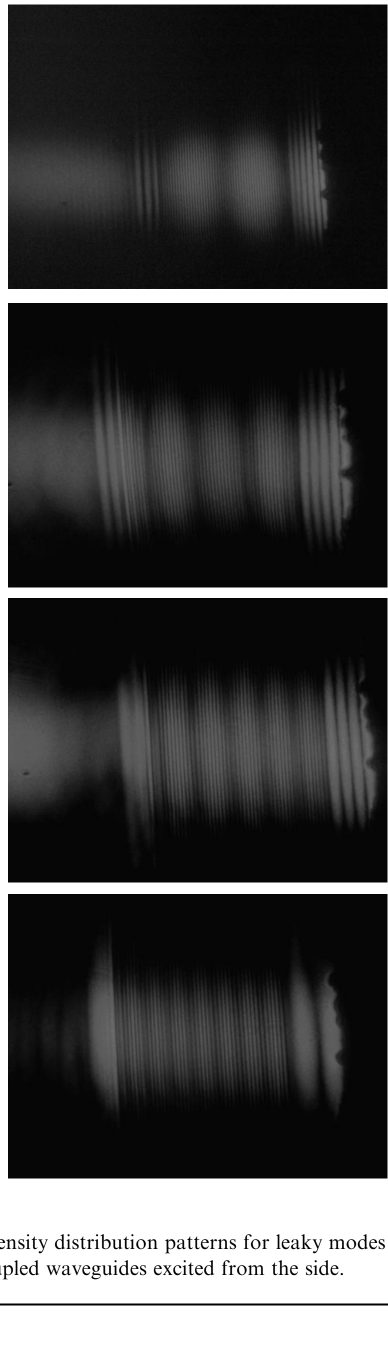


Figure 5. Optical scheme of the experimental setup for exciting modes in a system: (*1*) movable mirror; (*2*) fixed mirror; (*3*) long-focus lens; (*4*) microobjective; (*5*) sample cell; (*6*) aperture; (*7*) focusing lens; (*8*) photodetector; (*9*) G5M goniometer.

For angles $3^\circ < \theta < 4.8^\circ$ corresponding to the forbidden photonic band of the waveguide structure (see Fig. 3), a bright spot formed by the reflected wave was observed on the screen, while the intensity of the transmitted wave, whose distribution was also observed on the screen, was almost zero. For angles $\theta < 3^\circ$, the brightness of the reflected-beam spot decreased due to the appearance of narrow dips (down to zero) in the spot intensity distribution. Simultaneously, the intensity of a spot in the form of high-intensity bands produced by the transmitted beam increased. The leaky modes propagating in the waveguide system lose their energy, emitting it in the form of directed light beams; therefore, we explain the appearance of bright fringes in the transmitted beam by the excitation of leaky modes.

The number and angular positions of high-intensity emission bands in the transmitted-beam spot strictly corresponded to the number and angular positions of dips in the intensity distribution of the reflected-beam spot, i.e. we observed one or more dips and, correspondingly, one or more bright bands, depending on in which region of the first allowed band the leaky modes of the system were excited. For leaky modes located near the forbidden-band edge, the intermode distance measured in angles θ was $\sim 0.1^\circ$ and it was a few times smaller than the angular aperture of the beam incident on the side surface of the waveguide system, which was $\sim 0.6^\circ$ for a focal spot of the laser beam of diameter 50 μm. For this reason, several dips were observed in the intensity distribution in the reflected-beam spot, indicating the excitation of several leaky modes. For modes located at the centre of the first allowed band, the intermode distance is approximately equal to the angular aperture of the beam incident on the side surface of the waveguide system, and for angles $\theta \sim 1^\circ – 2^\circ$, no more than one – two dips were observed in the reflected-beam-spot intensity distribution. For glancing angles $4.8^\circ < \theta < 9.9^\circ$ corresponding to excitation of leaky modes in the second allowed band of the waveguide structure, similar effects were observed.

Figure 6 presents the intensity distributions for leaky modes observed from the end of the waveguide system illuminated from the side by a focused light beam of diameter 230 μm in the focal plane. These photographs correspond to the 2nd, 3rd, 5th, and 7th reflection minima, corresponding to excitation of the leaky modes with $m = 49, 48, 46$, and 44. The 50th mode, nearest to the forbidden-band boundary, is weakly excited, and we do not present its intensity distribu-



$m = 49$

$m = 48$

$m = 46$

$m = 44$

Figure 6. Intensity distribution patterns for leaky modes at the end of a system of coupled waveguides excited from the side.

tion. One can see that the number of periods of the field intensity envelope is determined by the mode detuning from the forbidden-band edge and corresponds to the results of calculations.

When the angle of incidence of light on the side surface of the waveguide system lies in the forbidden band ($\theta = 3^\circ - 4.8^\circ$), modes in the systems are not excited. However, if a guided mode propagating at the Bragg angle

$$\varphi_{\mathrm{Br}} = \arcsin\frac{\lambda}{2\Lambda} \qquad (6)$$

is excited from the end, the output emission is recorded as a peak on the transmission curve (Fig. 7a). The spatial Fourier analysis of this emission shows that two modes propagate at the Bragg angle in the waveguide system excited from the end [6], and both these modes are leaky modes.

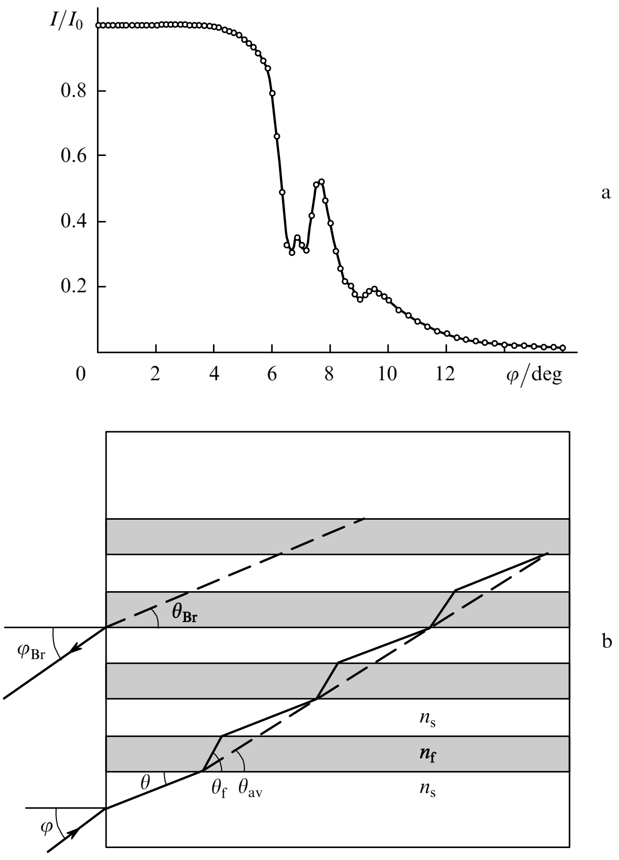


Figure 7. Dependence of the light intensity I/I_0 at the output of the end-excited waveguide system on the angle of incidence φ of an excitation beam (a) and a scheme illustrating the propagation of radiation in this system (angles are enlarged for clearness) (b).

Let us, however, call attention to an interesting fact. If, by using the law of refraction, we recalculate the angles corresponding to the forbidden-band edges to analogous angles in air ($4.38^\circ - 7.02^\circ$), then the angle $\varphi_{\mathrm{Br}} = 7.58$ will fall outside the forbidden band with boundaries calculated from the theoretical dependence $R(\theta)$ (Fig. 7b). At first glance this contradicts the common sense; however, this fact can be simply explained. The matter is that upon the grazing incidence of light on a structure, the angle θ_f of light propagation in waveguide layers considerably differs from the angle θ despite a small difference in the refractive indices ($n_\mathrm{s}\cos\theta = n_\mathrm{f}\cos\theta_\mathrm{f}$). The average angle θ_{av} of light propagation in the waveguide system can be determined from simple geometrical considerations [$(h + s)\cot\theta_{\mathrm{av}} = h\cot\theta_\mathrm{f} + s\cot\theta$] and is $3.81^\circ - 5.53^\circ$. By recalculating the Bragg angle for a mode propagating in the waveguide system by using the averaged refractive index, we obtain 5.17°. Therefore, there is no contradiction, and the Bragg angle falls into the forbidden band, as should be. Note that the averaged refractive index in this system calculated by the expression $n_{\mathrm{av}}\cos\theta_{\mathrm{av}} = n_\mathrm{s}\cos\theta$ depends not only on the widths and refractive indices of layers but also on the angle of incidence of light.

4. Conclusions

The study of the Bragg diffraction in a system of coupled waveguides and Bragg reflection from the surface of this system supplement each other and give a complete picture of the phenomenon. In particular, when the number M of guided modes is smaller than the number N of single-mode waveguides, $N - M$ leaky modes are excited in the system illuminated from the side, so that the total number of modes before the first Bragg reflection is N. The study of the Bragg reflection of light from the surface of a waveguide system near the transition from the guided modes to leaky modes allows one to refine the dispersion curve of the system.

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