

Efficiency of nonstationary transformation of the spatial coherence of pulsed laser radiation in a multimode optical fibre upon self-phase modulation

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Abstract. The model scheme of the nonlinear mechanism of transformation (decreasing) of the spatial coherence of a pulsed laser field in an extended multimode optical fibre upon nonstationary interaction with the fibre core is theoretically analysed. The case is considered when the spatial statistics of input radiation is caused by phase fluctuations. The analytic expression is obtained which relates the number of spatially coherent radiation modes with the spatially energy parameters on the initial radiation and fibre parameters. The efficiency of decorrelation of radiation upon excitation of the thermal and electrostriction nonlinearities in the fibre is estimated. Experimental studies are performed which revealed the basic properties of the transformation of the spatial coherence of a laser beam in a multimode fibre. The experimental results are compared with the predictions of the model of radiation transfer proposed in the paper. It is found that the spatial decorrelation of a light beam in a silica multimode fibre is mainly restricted by stimulated Raman scattering.

Keywords: spatial coherence, multimode optical fibre, nonlinear interaction, self-phase modulation.

1. Introduction

Many problems of optical data processing in coherent radiation require the averaging of field realisations to increase the signal-to-noise ratio during the recording time [1–3]. If this time is long enough, the efficient averaging can be performed by classical mechanical methods with the help of special mirrors, phase plates, and other elements moving in the radiation field, which produce the random phase modulation of the wave fronts being recorded [2, 3]. If the recording time is short (no more than 10 ns), these methods cannot be applied because of a large inertia of modulators. In this case, the use of the piezoelectric and acousto-optic modulation of a light beam is also inefficient

[4, 5]. Under such conditions, methods based on a rapid decomposition of the light field into orthogonal modes with the help of delay lines of different types can be used [6, 7], as well as quick-response nonlinear phenomena such as the radiation self-phase modulation [8, 9] and stimulated scattering [10]. The averaging process can be considered in this case as the transformation (decreasing) of the initial spatial coherence of radiation.

Recently multimode optical fibres have been successfully applied to suppress the coherent noise (speckle noise) in photometric schemes for selecting the informative signal [11, 12]. The fibres provide the efficient transformation of the spatial coherence of radiation and can be also used for radiation transport to any place. Despite wide applications of multimode fibres for suppressing noises, the mechanisms of transformation of coherence in them are studied insufficiently. Along with the known reasons for the spatial decorrelation of radiation in a fibre such as the dispersion of the beam propagation [13] and polarisation averaging [14], recently a new mechanism of decreasing the coherence of radiation related to the self-action of radiation was found [9, 15].

In this paper, we continue the study [15] of the degree of transformation of the spatial coherence of radiation in a multimode fibre upon the nonlinear interaction of radiation with the fibre core. The aim of the paper is to develop the model scheme of the nonstationary interaction of radiation with the fibre core, to estimate the transformation efficiency of the spatially angular characteristics of radiation for different nonlinear mechanisms of variations in the refractive index, and to compare the calculations with the results of the experimental study of the angular structure of radiation at the fibre output.

2. Theoretical model

It was found in [15] that upon stationary nonlinear interaction of radiation with the core of an ideal optical fibre, the degree of spatial coherence of radiation does not change. By an ideal multimode fibre we mean an absolutely direct fibre without any fluctuations in the refractive-index distribution, both in the transverse and longitudinal directions. Nonlinear stationary interaction in such a fibre gives rise only to the induced addition to the refractive index of the fibre core. It is interesting to find the spatial coherence of radiation at the output of the ideal multimode fibre in the case of the stationary nonlinear interaction of radiation with its core.

To simplify calculations, we make several assumptions:

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(i) The distribution of the permittivity ε of the optical fibre is described by the parabolic law:

$$\varepsilon(\mathbf{r}) = \varepsilon_0(1 - q^2\mathbf{r}^2), \quad (1)$$

where ε_0 is the permittivity on the fibre axis; $q^2 = \Delta/a^2$; $\Delta \approx (\varepsilon_0 - \varepsilon_1)/\varepsilon_0$ is the parameter characterising the profile height [16]; ε_1 is the permittivity of the fibre core; a is the radius of the fibre core; and \mathbf{r} is the radius vector in the plane perpendicular to the fibre axis;

(ii) the complex amplitude of radiation $A(\mathbf{r}, z, t)$ slowly varies with time t and along the fibre axis z . The dispersion of the refractive index of the fibre core and absorption of radiation are absent;

(iii) the input radiation is spectrally pure, i.e. its amplitude can be written in the form

$$A_0(\mathbf{r}, z = 0, t) = U(\mathbf{r})U(t), \quad (2)$$

where $U(\mathbf{r})$ and $U(t)$ are complex quantities.

Then, the evolution of the amplitude $A(\mathbf{r}, z, t)$ in the infinite medium approximation (the half-width of the light beam in the fibre is smaller than a) can be described by the scalar equation

$$\frac{\partial A}{\partial z} + \frac{i}{2k_0} \Delta_{\perp} A - i \left(\frac{1}{2} k_0 q^2 \mathbf{r}^2 - \mu \right) A = 0. \quad (3)$$

Here, Δ_{\perp} is the transverse Laplacian; $\mu = k_0 n_{nl}/n_0$; $n_{nl} = \delta \times \int_{-\infty}^t |A(\mathbf{r}, t')|^2 dt'$; $k_0 = 2\pi n_0/\lambda$ is the wave number; λ is the average radiation wavelength; n_0 is the refractive index of the fibre core; and δ is the nonlinear interaction parameter.

Under assumptions made above, the nonstationary nature of the radiation self-modulation process in (3) is taken into account only by the time dependence of the nonlinear refractive index n_{nl} . The interaction regime under study is manifested upon excitation of the thermal nonlinearity and nonlinearities appearing in nonabsorbing media if their relaxation time is longer or of the order of the exciting pulse duration [17].

We seek the solution of Eqn (3) in the form

$$A(x, y, z, t) = f(x)g(y) \exp[ip(t)z], \quad (4)$$

where $p(t)$ is the propagation constant; and $f(x)$ and $g(y)$ are unknown functions. By assuming that $|A(\mathbf{r}, z, t)|^2$ remains virtually constant and equal to the radiation intensity at the fibre input, solution (4) in the mode representation can be written in the form [18]

$$A(x, y, z, t) = \sum_{m,n} J_{mn}(x, y, t) \exp(ip_{mn}z). \quad (5)$$

Here, the function $J_{mn}(x, y, t)$ characterises the distribution of the field amplitude of a mode of the fibre in the plane xy of its output end; $p_{mn} = \mu - q(m+n+1)$; m and n are the numbers of the mode functions in the light field expansion. By using the Meler formula [19], we write solution (5) in the form

$$A(\mathbf{r}, z, t) = \int F(\mathbf{r}, \mathbf{r}') A_0(\mathbf{r}', t) \exp(-i\mu z) d^2\mathbf{r}', \quad (6)$$

where

$$A_0(\mathbf{r}', t) = A(\mathbf{r}', t, z = 0);$$

$$F(\mathbf{r}, \mathbf{r}') = \frac{ik_0 s(z)}{2\pi z} \exp \left\{ -i \left[\psi(\mathbf{r}, z) + \psi(\mathbf{r}', z) - \frac{k_0}{2z} s(z) \mathbf{r} \mathbf{r}' - 2qz \right] \right\};$$

$$\psi(\mathbf{r}, z) = \frac{k_0}{2z} [s(z) \cos(qz)] \mathbf{r}^2; \quad s(z) = \frac{qz}{\sin(qz)}.$$

Let us find the spatial correlation function

$$B(\mathbf{r}_1, \mathbf{r}_2, z) = \int_{-\infty}^{\infty} \langle A(\mathbf{r}_1, z, t) A^*(\mathbf{r}_2, z, t) \rangle dt. \quad (7)$$

By substituting expression (6) into (7), we perform spatial averaging by assuming that only the phase of input radiation fluctuates and its statistics is described by the Gaussian law. Then, we perform integration over the variables \mathbf{r}' and t by assuming that the time dependence of the radiation intensity is described by a rectangular pulse of duration T . Let us pass to the variables $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and calculate the homogeneous correlation function $b(\mathbf{r}, z)$ [19]:

$$b(\mathbf{r}, z) = \int_{-\infty}^{\infty} B(\mathbf{R}, \mathbf{r}) d^2\mathbf{R}. \quad (8)$$

The Fourier transform of expression (8) allows us to obtain the function $G(\mathbf{v}, z)$ determining the angular spectrum of radiation emerging from the fibre:

$$G(\mathbf{v}, z) = \int_{-\infty}^{\infty} b(\mathbf{r}, z) \exp(i\mathbf{v} \mathbf{r}) d^2\mathbf{r}, \quad (9)$$

where \mathbf{v} is the angular frequency vector. By using $G(\mathbf{v}, z)$, we can find the width Δv of the angular spectrum of the give radiation:

$$\Delta v = \left[\int_{-\infty}^{\infty} \mathbf{v}^2 |G_n(\mathbf{v}, z)| d^2\mathbf{v} / \int_{-\infty}^{\infty} |G_n(\mathbf{v}, z)| d^2\mathbf{v} \right]^{1/2} = \left\{ - \left[\frac{d^2 |b(\mathbf{r})|}{d\mathbf{r}^2} \right]_{\mathbf{r}=0} \left[|b(\mathbf{r})| \right]_{\mathbf{r}=0}^{-1} \right\}^{1/2}, \quad (10)$$

where $G_n(\mathbf{v}, z) = G(\mathbf{v}, z)/G(\mathbf{v}, 0)$.

For a small enough width Δv of the spectrum ($\Delta v \ll k_0$), which is obviously valid for the radiation transfer scheme under study, we can pass to the number N of spatially coherent modes by using a simple relations [20]

$$N \simeq \Delta v^2 S. \quad (11)$$

Here, S is the light-beam cross section at the fibre output.

The parameter N can be estimated by measuring the contrast C of the speckle pattern of radiation emerging from the fibre [21]:

$$C = \frac{\sigma}{\langle I \rangle} = \frac{1}{\sqrt{N}}, \quad (12)$$

where $\langle I \rangle$ is the average intensity of the speckle pattern and σ is its mean-root-square deviation. Thus, analysis of the coherent properties of radiation transmitted through the fibre is reduced to the determining the number of spatially coherent modes contained in it. In this case, the greater is N , the smaller is C and the lower is the spatial coherence of radiation.

By calculating Δv (10) with the use of (8), (9), and the Gaussian correlation function of the input field, we obtain the expression

$$\Delta v^2 = 4 \left(\frac{1}{r_{\text{eff}}^2} + \frac{1}{6} M^2 \right), \quad (13)$$

where

$$r_{\text{eff}}^2 = \frac{2a_0^2 r_0^2}{2a_0^2 + r_0^2}; \quad M = \frac{d}{a_0 \sqrt{h}}; \quad d = \frac{k_0}{n_0} \delta |A_0|^2 z;$$

$$h = \cos^2(qz) + \frac{\sin^2(qz)}{l_d^2 q^2};$$

$l_d = ka_0 r_{\text{eff}} / (2\sqrt{2})$ is the diffraction length of the beam spread in a linear medium; $k = 2\pi/\lambda$; and a_0 is the radius of the light beam at the fibre input, which is approximately equal to the fibre core radius; r_0 is the radius of coherence of the input radiation. In this case, the number of radiation modes is

$$N = N_0 + \frac{2}{3} M^2 S. \quad (14)$$

Here, $N_0 = 4S/r_{\text{eff}}^2$ is the number of radiation modes at the fibre input.

One can see from (14) that upon the nonstationary interaction of the light beam with the fibre core, even in the case when the fibre is ideal, the spatial coherence of the beam is transformed (reduced). This is obviously explained by the appearance of coupling between fibre modes, which is demonstrated by the loss of the discreteness of the mode propagation constant p_{mn} due to its dependence on the parameter μ , which can take any real values. The coupling between modes with different phases can lead due to their interference to the inhomogeneous distribution of the radiation intensity over the fibre cross section and induce random variations in the refractive index in the fibre core. Scattering by these inhomogeneities causes the disappearance of coherence during the pulse.

The characteristic result obtained upon the nonstationary transformation of the spatial coherence of radiation, whose statistics caused only by phase fluctuations, is that the transformation efficiency determined by the ratio N/N_0 depends, as in the case of thin nonlinear medium [8], on the degree of coherence of radiation coupled into the fibre.

The main nonlinear mechanisms of variations in the refractive index of the fibre core in the nanosecond range of durations of transmitted radiation pulses are the heating of the medium and electrostriction [22]. The relaxation time τ_p of the electrostriction pressure is determined by the propagation time of a sound wave through the mode spot with the transverse size $w = 2[2a/(n_0 k \sqrt{2A})]^{1/2}$. For $a = 50 \mu\text{m}$, $\lambda = 532 \text{ nm}$, $A = 0.01$, and the sound speed in quartz $v_s = 10^5 \text{ cm s}^{-1}$, we have the time $\tau_p = 1.2 \times 10^{-8} \text{ s}$. For pulse

durations $t_p \sim 10 - 15 \text{ ns}$ used in experiments, the nonlinear modulation of the refractive index of the fibre core caused by electrostriction can be considered locally nonstationary. In this case, the nonlinear interaction parameter is $\delta = \delta_{\text{st}} = n_{2\text{st}} t_p / \tau_p$ [17], where $n_{2\text{st}}$ is the nonlinearity coefficient for the electrostriction polarisation of the medium.

The estimates of the parameter d performed for the case of excitation of the striction pressure and thermal nonlinearity showed that for nanosecond pulses the obtained values of d differ from each other by more than two orders of magnitude. In this case, the nonlinear phase incursion caused by electrostriction dominates. Therefore, we can assume that electrostriction is the key nonlinear mechanism of decorrelation of radiation in silica fibres. Nonlinear phase incursions were calculated by using the following parameters $n_{2\text{st}} = 0.57 \times 10^{-13} \text{ cgs units}$ [23], $n_0 = 1.6$ is the refractive index of silica, $\delta_T = (\partial n / \partial T) \alpha t / (\rho c_p n_0)$, $\partial n / \partial T \sim 10^{-6} \text{ K}^{-1}$ is the temperature coefficient of the refractive index of silica, $\alpha \sim 10^{-4} \text{ cm}^{-1}$ is the absorption coefficient of a silica fibre measured at 532 nm, $\rho = 2.2 \text{ g cm}^{-3}$ is the density of silica, $c_p = 0.728 \text{ J g}^{-1} \text{ K}^{-1}$ is the thermal capacity of silica, $|A_0| = 8\pi P_0 \exp(-\alpha z) / (cn_0 S_{\text{eff}})$, P_0 is the radiation power at the fibre input, $S_{\text{eff}} = \pi w^2 / 4$ is the effective area of the fibre cross section, $a = 50 \mu\text{m}$, and $t_p = 10 \text{ ns}$.

3. Experiment

The basic properties of nonlinear transformation of the spatial coherence of radiation in a fibre were studied on the experimental setup presented schematically in Fig. 1. The 532-nm second harmonic radiation from pulsed Nd:YAG laser (1) propagated through a set of calibrated neutral filters (2) and was coupled with the help of microobjective (3) into short ($\sim 1 \text{ m}$) multimode fibre (4) and then to main step-index fibre (5) of length $\sim 100 \text{ m}$ with the core of diameter $\sim 100 \mu\text{m}$. A part of radiation emerging from the fibre was directed by beamsplitter (6) through interference filter (7) with the transmission maximum at 532 nm to power meter (8). Radiation transmitted through the beamsplitter was incident on mat plate (9), scattered in it, propagated through filter (10), similar to filter (7), and recorded with CCD array (11) with 2048 pixels (the spatial resolution was $\sim 14 \mu\text{m}$).

We measured the contrast of radiation speckles at a wavelength of 532 nm behind mat plate (9). Speckle patterns were recorded for different input radiation powers and the specified spatial coherence of radiation. The spatial coherence of radiation directed to the fibre was changed by introducing apertures of different diameters into the laser

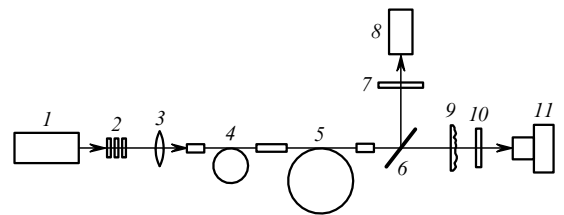


Figure 1. Optical scheme of the experimental setup: (1) pulsed Nd:YAG laser (532 nm); (2) set of neutral optical filters; (3) microobjective; (4) short fibre; (5) main fibre; (6) beamsplitter; (7, 10) interference filters; (8) power meter; (9) mat plate; (11) CCD array.

resonator. If the divergence of radiation exceeds the numerical aperture of a fibre, the angular selection of radiation can occur and its spatial coherence can improve. For this reason, we estimated the initial number of spatially coherent modes from the contrast of speckle patterns obtained at the output of short fibre (4) (Fig. 1). This eliminated the inaccuracy in determining the number N_0 admitted in paper [15] in calculations N_0 from the contrast of the speckle pattern formed by radiation emitted directly from a laser.

A specific feature of the nonlinear process of radiation self-action in a multimode silica fibre is that, beginning from a characteristic (threshold) input power for the specified fibre length, stimulated Raman scattering (SRS) is excited. This scattering leads to the transfer of the input energy into the Stokes components. As a result, the efficiency of transformation of the spatial coherence of radiation at the initial frequency decreases when the input power exceeds the threshold. Figure 2 shows the experimental dependences of the integrated coefficient $K = W_S/W_p$ (where W_S is the total energy of the Stokes SRS waves and W_p is the pump wave energy) of transformation of the 532-nm radiation energy to the Stokes components on the input power for two values of N_0 . One can see that the energy transfer efficiency can be close to unity and depends on the initial spatial coherence of radiation.

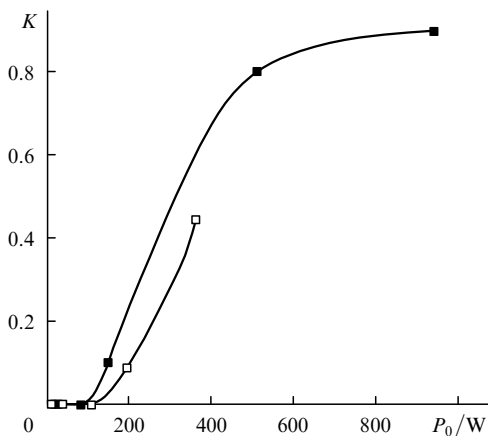


Figure 2. Experimental dependences of the integrated coefficient K of transformation of the radiation energy to the Stokes SRS components on the input radiation power P_0 for $N_0 = 4$ (■) and 6 (□).

The influence of SRS on the transformation efficiency of the spatial coherence of radiation in a fibre is illustrated by the dependences presented in Fig. 3. One can see that, as the radiation power coupled into main fibre (5) (Fig. 1) increases, the rate of the increase in the number of spatially coherent modes slows down beginning from some value of the input power.

Figure 4 presents the dependences of the transformation efficiency of the spatial coherence of radiation not subjected to SRS in a fibre on its average power measured at the output of fibre (5) with power meter (8) (Fig. 1). Dependences (1) and (2) correspond to the experimental data, and dependences (3) and (4) are constructed by the values N/N_0 found from (14) for the same average radiation powers as in the experiment and $\delta_{st} = n_{2st}t_p/\tau_p$ and $l_dq = 1$. It follows from these dependences that the spatial

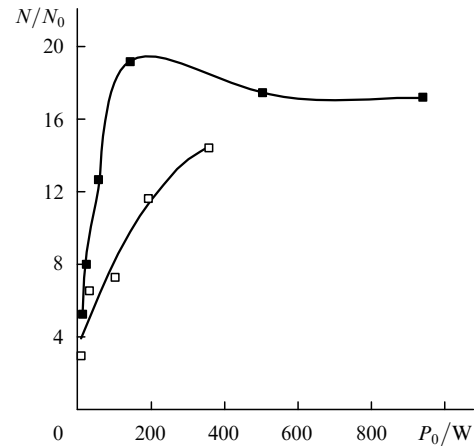


Figure 3. Dependences of the transformation efficiency N/N_0 of the spatial coherence of radiation on the input power P_0 for $N_0 = 4$ (■) and 6 (□).

decorrelation of radiation increases with increasing the input radiation power. However, the theoretical dependence of the spatial coherence of radiation does not coincide with experimental dependences. While the theoretical transformation efficiency of the spatial coherence of radiation quadratically depends on the radiation power, this dependence is not observed in experiments. This is obviously explained by excitation of SRS in the fibre, which not only reduces the effective power of the input radiation but also changes the type of the decrease in coherence, which is not taken into account in the theoretical model [24]. At the same time, one can see from Fig. 4 that our model correctly predicts the type of the dependence of the number of modes produced in a fibre on the initial radiation coherence.

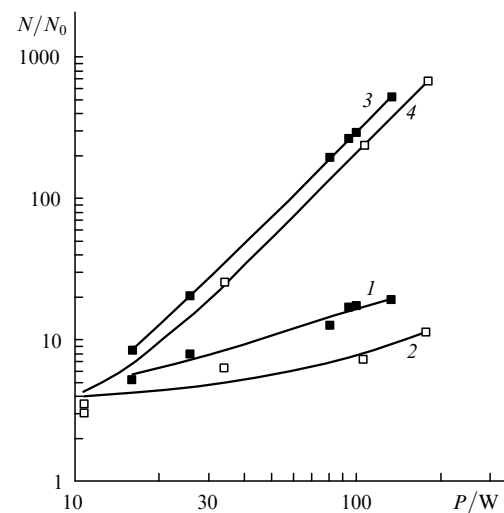


Figure 4. Experimental (1, 2) dependences of the transformation efficiency N/N_0 of the spatial coherence of radiation on its average power and theoretical dependences calculated by (14) (3, 4) for $N_0 = 4$ (■) and 6 (□).

4. Conclusions

We have considered the model scheme of the nonlinear nonstationary process of transformation of the spatial

coherence of a light field in a multimode fibre for the case of input-radiation statistics caused by phase fluctuations. The theoretical analysis of the mode structure of the transformed radiation has shown that the degree of its decorrelation is determined by the average linear phase incursion along the fibre length and depends quadratically on it. A specific feature of the model scheme is the dependence of the transformation efficiency of spatial modes on the number of modes of radiation coupled into the fibre, which was observed experimentally. It has been found experimentally that the decorrelation of light beams in a multimode silica fibre is decreased mainly due to SRS, which reduces the effective input radiation power and changes the dynamics of amplitude–phase transformations of light.

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