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Nonreciprocal dynamics of pulses in a nonlinear inhomogeneous fibre

M.S. Adamova, I.O. Zolotovskii, D.I. Sementsov

***Abstract*. The conditions, under which the nonreciprocity of the frequency modulation rate and pulse duration as well as the spectral nonreciprocity in fibres with different types of inhomogeneity of nonlinearity and group-velocity dispersion appear, are studied for the Gaussian and hyperbolic secant frequency-modulated pulses. Strong compression nonreciprocity is found in fibres with an alternating group-velocity dispersion periodically changing over its length.**

Keywords: *nonreciprocal effects, inhomogeneous fibres, compression and spectral nonreciprocities.*

1. Introduction

Some specific nonreciprocal effects are caused by the nonlinear character of interaction of counterpropagating waves with the active medium and determine the radiation dynamics in the resonator of ring lasers. Many precision measurements of fundamental quantities are based on the nonreciprocal effects [1 – 6]. Nonreciprocal effects can be realised both in integral schemes and in fibre lasers and waveguides. One of the reasons leading to the propagation asymmetry of forward and backward waves can be longitudinal inhomogeneity of the active nonlinear medium, which is frequently used to control laser radiation in optical fibres [7 – 10]. Moreover, it is optical fibres with the group-velocity dispersion (GVD) alternating along the fibre length that have been considered recently as the most promising systems for creating efficient fibreoptic communication lines, in particular, soliton ones [11 – 14].

The possibility of appearance of spectral nonreciprocity upon propagation of a Gaussian pulse without the initial frequency modulation (FM) was shown in [15] for a spatially inhomogeneous medium with a cubic nonlinearity and dissipation neglecting the GVD. In paper [16], the nonreciprocal dynamics of formation of a shock wave was studied by the example of a Gaussian pulse in an inhomogeneously amplifying medium with the nonlinearity dispersion and a negligible average GVD over the fibre length. The presence of the GVD and initial FM not only significantly changes the character of the nonreciprocal dynamics but also substantially complicates the solution of corresponding dynamic equations. In this paper, we analyse the FM dynamics of forward and backward pulses in fibres with both zero and non-zero GVD. We also study the conditions under which compression and spectral nonreciprocities of Gaussian and hyperbolic secant pulses appear for different types of the inhomogeneity of dispersion and nonlinearity over the fibre length. In the case of a negligible GVD, by using exact solutions of the nonlinear Schrödinger equation (NSE) for a fibre with the inhomogeneous amplification and nonlinearity, the nonreciprocity of the FM rate is studied, which determines the spectral width of the pulse. We solved the NSE taking into account the GVD by using the variation approach and obtained equations determining the dynamics of individual pulse parameters. The obtained solutions are used to study the nonreciprocal dynamics of pulses (the FM rate and duration).

2. General equations

Consider the dynamics of an optical pulse in an inhomogeneous nonlinear amplifying fibre. In this case, the field of a wave packet propagating in the fibre can be written in the form

$$\boldsymbol{E}(t,r,z)=\frac{1}{2}\boldsymbol{e}U(r,z)\left(B(t,z)\exp\left\{\mathrm{i}\left[\omega_0 t-\int_0^z\beta'(\xi)\mathrm{d}\xi\right]\right\}+\text{c.c.}\right), \quad (1)$$

where $\boldsymbol{e}$ is the unit vector of the light field polarisation; the function $U(r,z)$ describes the radial distribution of the field in the fibre; ω_0 is the carrier frequency of the wave packet; and $\beta'(z)$ is the real component of the complex propagation constant. For the complex pulse envelope $B(t,z)$, the NSE with coefficients variable over the fibre length is valid, which are assumed to change slowly:

$$\frac{\partial B}{\partial z}-\mathrm{i}\frac{D(z)}{2}\frac{\partial^2 B}{\partial\tau^2}+\mathrm{i}R(z)|B|^2B=g(z)B. \quad (2)$$

Here, the following parameters are introduced:

$$\tau=t-\int_0^z\frac{\mathrm{d}\xi}{u(\xi)}$$

is the time in the running coordinate system;

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$$u(z) = \left[\frac{\partial\beta(z)}{\partial\omega}\right]_0^{-1} \text{ и } D(z) = \left[\frac{\partial^2\beta(z)}{\partial\omega^2}\right]_0$$

is the group velocity of the wave packet and the fibre GVD; $R(z)$ is the Kerr nonlinearity coefficient; and $g(z)$ is the gain increment of the fibre. The slow change in the parameters (for example, for R) over the fibre length means that the condition $\lambda_0|\partial R/\partial z| \ll R$ is fulfilled for each of them, i.e. at the distances of the order of the carrier wavelength λ_0, the change in the value of R is negligible. By performing substitution in (2)

$$B(\tau, z) = A(\tau, z)\exp\left[\int_0^z g(\xi)\mathrm{d}\xi\right], \tag{3}$$

we obtain the expression for the envelope amplitude $A(\tau, z)$:

$$\frac{\partial A}{\partial z} - \mathrm{i}\frac{D(z)}{2}\frac{\partial^2 A}{\partial\tau^2} + \mathrm{i}R_{\text{eff}}(z)|A|^2 A = 0, \tag{4}$$

where the effective nonlinearity parameter

$$R_{\text{eff}}(z) = R(z)\exp\left[2\int_0^z g(\xi)\mathrm{d}\xi\right]$$

is introduced.

For $D(z) < 0$ and $R_{\text{eff}}(z) > 0$, one of the possible solutions of (4) describes hyperbolic secant solitons [17].

3. 'Dispersionless' approximation

In the general case, NSE (4) can be solved only numerically. However, its exact analytic solution can be obtained for the zero or negligible GVD, when the 'dispersionless' approximation is valid [the dispersion term of the first order is taken into account in (2) by introducing the running time τ]. This situation is possible for the fibre lengths that are much shorter that the dispersion length, i.e. for $L \ll L_D = \tau_0^2/|D|$. In this case, the pulse duration $\tau_\mathrm{p}(z)$ can be assumed constant over the entire length of the fibre and equal to the duration τ_0 of the coupled pulse. In this case, the solution of Eqn (4) can be presented in the form

$$A(\tau, z) = A(\tau, 0)\exp[\mathrm{i}\varphi(\tau, z)]. \tag{5}$$

Here, $A(\tau, 0)$ is the pulse envelope at the input to the fibre (for $z = 0$) and the phase of the propagating pulse depends on the longitudinal coordinate:

$$\varphi(\tau, z) = \frac{\alpha_0\tau^2}{2} - |A(\tau, 0)|^2\int_0^z R(z')\exp\left[2\int_0^{z'} g(\xi)\mathrm{d}\xi\right]\mathrm{d}z'. \tag{6}$$

The following analysis will be performed for frequency-modulated pulses, whose input profiles can be described by functions

$$A(\tau, 0) = A_0 G(\tau)\exp\left(-\frac{\mathrm{i}\alpha_0\tau^2}{2}\right), \tag{7}$$

where A_0 is the peak value of the pulse amplitude; α_0 is the FM rate at the input of the fibre; $G(\tau) = \exp[-\tau^2/(2\tau_0^2)]$ for the Gaussian pulse and $G(\tau) = \mathrm{sech}(\tau/\tau_0)$ for the hyperbolic secant pulse. The FM rate characterises the spectral width of the wave packet and, in the general case, is determined by the expression

$$\alpha(\tau, z) = \frac{\partial^2\varphi}{\partial\tau^2} = \alpha_0 - \int_0^z R(z')\exp\left[2\int_0^{z'} g(\xi)\mathrm{d}\xi\right]\mathrm{d}z' \times \frac{\partial^2|A(\tau, 0)|^2}{\partial\tau^2}. \tag{8}$$

By taking (7) and (8) into account, the expressions for the FM rate of the forward and backward pulses after their propagation through the fibre of length L can be written in the form

$$\alpha_\pm(\tau) = \alpha_0 + 2I_0 G_\pm^2\Theta_\pm\tau_0^{-2}\int_0^L R_\pm(z)\exp\left[2\int_0^z g_\pm(\xi)\mathrm{d}\xi\right]\mathrm{d}z, \tag{9}$$

where $G_\pm = G(\tau_\pm)$ and functions

$$\Theta_\pm = \begin{cases} 1 - 2(\tau_\pm/\tau_0)^2, \\ 1 - \tanh^2(\tau_\pm/\tau_0) \end{cases} \tag{10}$$

and parameters

$$I_0 = |A_0|^2, \tau_\pm = t - \int_0^L \frac{\mathrm{d}z}{u_\pm(z)}$$

are introduced for the Gaussian and hyperbolic secant pulses. All the functions responsible for the forward and backward pulse propagation are introduced according to the general rule: $f_+(z) = f(z)$, $f_-(z) = f(L - z)$. It follows from the above relations that the nonreciprocity of the FM rate of a propagating pulse will be absent, if the functions $R(z)$ and $g(z)$ are even with respect to the point $z = L/2$.

As an example, consider by using expression (9) the character of nonreciprocity of the FM rate for one of possible types of the distribution of the nonlinearity parameter over the fibre length in the 'dispersionless' approximation. Let the gain increment be constant $g(z) = g_0$ over the fibre length and the nonlinearity inhomogeneity be described by the exponential dependence $R(z) = R_0\exp(\gamma z)$, where γ is the nonlinearity coefficient. In this case, the FM rate for counterpropagating pulses at the fibre output is determined by the expressions

$$\alpha_\pm(\tau) = \alpha_0 + \frac{2R_0 I_0 G_\pm^2\Theta_\pm F_\pm}{\tau_0^2}\frac{\exp[(2g_0 \pm \gamma)L] - 1}{2g_0 \pm \gamma}, \tag{11}$$

where $F_+ = 1$ and $F_- = \exp(\gamma L)$. One can see from (11) that for $\gamma = 0$, the FM rate is the reciprocal quantity, i.e. $\alpha_+ = \alpha_-$. For the central part of the pulse ($\tau_\pm = 0$, $\Theta_\pm = 1$, $G_\pm = 1$), we can obtain an analytic expression for the nonreciprocity of the FM rate at the fibre output, which will be characterised by the parameter $\eta_\alpha = (\alpha_+ - \alpha_-)\tau_0^2$:

$$\eta_\alpha(L) = 2I_0 R_0\exp\left[\left(g_0 + \frac{\gamma}{2}\right)L\right]\left\{\frac{\sinh[(g_0 + \gamma/2)L]}{g_0 + \gamma/2} - \frac{\sinh[(g_0 - \gamma/2)L]}{g_0 - \gamma/2}\right\}. \tag{12}$$

Figure 1 shows the dependence of the nonreciprocity of the FM rate $\eta_\alpha(L)$ on the fibre length, which was plotted by using (12) for different γ and the following parameters of the

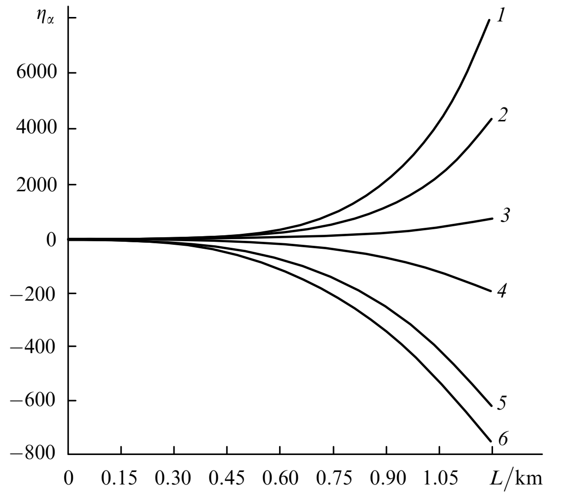

Figure 1. Dependences of the nonreciprocity of the FM rate of Gaussian pulses on the fibre length for $D(z) = 0$, $R(z) = R_0 \exp(\gamma z)$ and $\gamma = 2$ (*1*), 1.5 (*2*), 0.5 (*3*), −0.2 (*4*), −1 (*5*) and −2 km^{-1} (*6*).

fibre and the coupled pulse: $R_0 = 1$ W^{-1} km^{-1}, $g_0 = 0.5$ km^{-1}, $I_0 = 1$ kW, $\tau_0 = 1$ ps. One can see that for the nonlinearity inhomogeneity realised in the fibre, the nonreciprocity of the FM rate substantially depends on the degree and character of this inhomogeneity, i.e. on the value and sign of γ as well as on the fibre length.

Considerable nonreciprocity can also appear during the pulse propagation in a structure consisting of alternating nonlinear passive and active elements with weak nonlinearity. Thus, for the simplest two-element cascade, the linear amplifier – nonlinear passive fibre, the pulse after propagation through the amplifier at the input to the second part of the cascade has the power $I_0 \exp(2g_1L_1)$, therefore, at the output from the cascade (at the length $L = L_1 + L_2$) for $\tau_+ = 0$, its FM rate is

$$\alpha_+(L) = \alpha_0 + 2I_0R_2L_2\tau_0^{-2}\exp(2g_1L_1), \tag{13}$$

where L_i, R_i, g_i are the parameters of the corresponding elements of the cascade ($i = 1, 2$). In this case, $R_1 = 0$ and $g_2 = 0$. If the pulse is first coupled into the nonlinear fibre and only after it into the amplifier, its FM rate at the output from the cascade for $\tau_- = 0$ is

$$\alpha_-(L) = \alpha_0 + 2I_0R_2L_2\tau_0^{-2}. \tag{14}$$

It follows from the above relations that the nonreciprocity of the FM rate for the fibre cascade under study is determined by the expression

$$\eta_\alpha = 2I_0R_2L_2\left[\exp(2g_1L_1) - 1\right] \tag{15}$$

and for $g_1L_1 > 1$ can be substantial.

The nonreciprocity of the FM rate of the pulse should lead to the nonreciprocity of its spectral width considered in [15] for a Gaussian pulse without the initial FM ($\alpha_0 = 0$). In the case under study, $\alpha_0 \neq 0$ and the root-mean-square width of the Gaussian pulse propagating in the forward and backward directions is described by the expression

$$\Delta\omega_\pm = \left[1 + \frac{4}{3\sqrt{3}}\left(\varphi^\pm_{\max}\right)^2\right]^{1/2}\Delta\omega_0, \tag{16}$$

where the maximum phase shift $\varphi^\pm_{\max} = [\alpha_\pm(L) - \alpha_0]\tau_0^2/2$ is determined by the FM rate of the central part of the pulse (for $\tau_\pm = 0$) at the fibre output; $\Delta\omega_0 = (\tau_0^{-2} + \alpha_0^2\tau_0^2)^{1/2}$ is the initial width of the pulse with the nonzero initial FM rate, which is coupled into the fibre. Taking (16) into account, the spectral nonreciprocity $\eta_{\Delta\omega} = (\Delta\omega_+ - \Delta\omega_-)/\Delta\omega_0$ is determined as:

$$\eta_{\Delta\omega} \simeq \left[1 + 0.19(\alpha_+ - \alpha_0)^2\tau_0^4\right]^{1/2}$$

$$-\left[1 + 0.19(\alpha_- - \alpha_0)^2\tau_0 4\right]^{1/2}. \tag{17}$$

In the case $|\alpha_\pm - \alpha_0|\tau_0^2 \ll 1$, the expression for the spectral nonreciprocity takes the from $\eta_{\Delta\omega} \simeq 0.1\tau_0^2(\alpha_+ + \alpha_- - 2\alpha_0)\eta_\alpha$. If the forward and backward pulses have substantially different FM velocities and, hence, different spectral widths, then after their coupling into a dispersion medium (or propagation through a dispersion element, for example, diffraction grating), we can obtain a strong compression nonreciprocity. For the pulses under study, this nonreciprocity can be defined by the parameter

$$\eta_\tau = \frac{\tau_p^+ - \tau_p^-}{\tau_0} = \frac{\chi}{\tau_0}\left(\frac{1}{\Delta\omega_+} - \frac{1}{\Delta\omega_-}\right), \tag{18}$$

where we take into account that the root-mean-square duration of the forward and backward pulses is $\tau_p^\pm = \chi/\Delta\omega_\pm$ and χ depending on the shape of the wave packet, is constant ($\chi = 1/2$ and $\pi/6$ for the Gaussian and hyperbolic secant pulses, respectively).

4. General case of the nonzero GVD

The considered particular cases of the nonreciprocity of parameters have been studied in the approximation of the 'dispersionless' medium, i.e. for $D = 0$. For the non-zero GVD, the nonreciprocal effects should be even more pronounced; however, the analytic study of this problem is complicated by the fact that the equation describing the propagation dynamics of a pulse is not integrable and, hence, requires a numerical solution. Another way is the application of the variation procedure to solve the NSE describing the transformation of a frequency-modulated pulse in the nonlinear single-mode fibre with amplification. It is known [18] that in systems with inhomogeneous dispersion parameters the solution of this equation and the dynamics of the wave packet can be described by using the variation approach. According to this approach, we can pass to Eqn (4) from the Euler–Lagrange equation

$$\frac{\partial\mathscr{L}}{\partial A^*} - \frac{\partial}{\partial z}\frac{\partial\mathscr{L}}{\partial A_z^*} - \frac{\partial}{\partial\tau}\frac{\partial\mathscr{L}}{\partial A_\tau^*} = 0, \tag{19}$$

if the Lagrangian of the system is introduced as:

$$\mathscr{L} = \frac{1}{2}\left[\mathrm{i}\left(AA_z^* - A^*A_z\right) + D|A_\tau|^2 + R|A|^4\right], \tag{20}$$

where the subscript of the amplitude means a derivative with respect to the corresponding variable. The approximate solutions of (4) can be found from the condition of the extremum of the action functional

$$S = \int_0^{\infty} \langle \mathscr{L} \rangle \mathrm{d}z,$$

which is equivalent to the system of equations

$$\frac{\partial \langle \mathscr{L} \rangle}{\partial \Theta_i} - \frac{\partial}{\partial z} \frac{\partial \langle \mathscr{L} \rangle}{\partial \Theta_{iz}} = 0. \tag{21}$$

Here, Θ_i are parameters in trial solutions of Eqn (4) depending only on z and $\Theta_{iz} \equiv \partial \Theta_i / \partial z$. To construct the averaged Lagrangian

$$\langle \mathscr{L} \rangle = \frac{1}{\tau_0} \int_{-\infty}^{\infty} \langle \mathscr{L} \rangle \mathrm{d}\tau$$

we will use the trial functions, which describe the envelopes of pulses under study:

$$A(\tau, z) = CG(\tau, \tau_{\mathrm{p}}) \exp\left[\mathrm{i}\left(\frac{\alpha\tau^2}{2} + \varphi\right)\right], \tag{22}$$

where the amplitude $C(z)$, phase $\varphi(z)$, FM rate $\alpha(z)$ and duration $\tau_{\mathrm{p}}(\tau)$ of the forward or the backward pulses play the role of the variable parameters Θ_i. After the substitution of trial solutions (22) into (20) and time integration, we find the averaged Lagrangians:

$$\langle \mathscr{L} \rangle = \frac{C^2}{\tau_0} \times \begin{cases} \left(\tau_{\mathrm{p}} \dfrac{\mathrm{d}\varphi}{\mathrm{d}z} + \dfrac{1}{4}\tau_{\mathrm{p}}^3 \dfrac{\mathrm{d}\alpha}{\mathrm{d}z} + \dfrac{1}{4}\tau_{\mathrm{p}}^3 D\alpha^2 + \dfrac{D}{4\tau_{\mathrm{p}}} + \dfrac{\sqrt{2}}{4} RC^2 \tau_{\mathrm{p}}\right)\sqrt{\pi}, \\ 2\tau_{\mathrm{p}} \dfrac{\mathrm{d}\varphi}{\mathrm{d}z} + \dfrac{\pi^2}{12}\tau_{\mathrm{p}}^3 \dfrac{\mathrm{d}\alpha}{\mathrm{d}z} + \dfrac{\pi^2}{12}\tau_{\mathrm{p}}^3 D\alpha^2 + \dfrac{D}{3\tau_{\mathrm{p}}} + \dfrac{2}{3} RC^2 \tau_{\mathrm{p}}. \end{cases} \tag{23}$$

By using the obtained expressions for $\langle \mathscr{L} \rangle$ and (21), we arrive at a system of equations, which determine the pulse parameters under study:

$$C^2 \tau_{\mathrm{p}} = I_0 \tau_0,$$

$$\frac{\mathrm{d}\tau_{\mathrm{p}}}{\mathrm{d}z} = D\alpha\tau_{\mathrm{p}}, \tag{24}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}z} = k_1 D \tau_{\mathrm{p}}^{-4} - D\alpha^2 + k_2 R_{\mathrm{eff}} I_0 \tau_0 \tau_{\mathrm{p}}^{-3}$$

(the equation for the parameter φ is omitted here as inessential for the following analysis). For Gaussian pulses, $k_1 = 1$, $k_2 = 1/\sqrt{2}$ and for the hyperbolic secant pulses, $k_{1,2} = 4/\pi^2$. Note that z in (24) is a distance propagated in the fibre by the forward or backward pulse; for the backward pulse, it is necessary to replace in (24) all the parameters, which are functions of z, by functions of $L - z$, i.e. $D_{-}(z) = D(L - z)$,

$$R_{\mathrm{eff}}^{-}(z) = R(L - z) \exp\left[2\int_0^z g(L - \xi)\mathrm{d}\xi\right].$$

System of equations (24) allows one to analyse the dynamics of the pulse parameters (duration, amplitude, FM rate and phase) for the specified parameters of the fibre, while these parameters are not contained explicitly in (4). Note, however, that the initial NSE and the variation procedure have their application limits. First, the decrease in the pulse duration till the values shorter than femtosecond ones requires the consideration of the third-order dispersion terms in initial equation (2). Second, strong compression of the pulse leads to the distortion of its shape, while the variation method assumes that the pulse shape upon its propagation does not change (which is possible if it is slightly compressed). The equations used above also neglect the dispersion of the gain increment of the fibre, which considerably affects the pulse shape. Therefore, the performed analysis is valid for optical fibres for which the gain increment $g(\omega)$ is approximately the same in a rather broad frequency range and its dispersion can be neglected [19, 20].

5. Numerical analysis

System of equations (24) is significantly simpler than initial equation (4) because it allows solutions for practically important cases of the pulse dynamics in an inhomogeneous medium [21]. In the case of $D \neq 0$, the homogeneous gain increment and nonlinearity of the medium, nonreciprocal effects can be caused only by the influence of the nonreciprocity of the material dispersion of the fibre. For the numerical analysis, we will use the characteristic parameters of the fibre and coupled Gaussian pulse: the gain increment $g_0 = 0.5$ km^{-1}, the input duration and pulse power $\tau_0 = 1$ ps, $I_0 = 1$ kW. For the simplicity of the analysis, the initial FM rate is set equal to zero, which, in fact, does not limit the generality of obtained results. The material dispersion and nonlinearity can be defined as $D(z) = D_0 f(z)$, $R(z) = R_0 \rho(z)$, where $D_0 = 10$ ps^2 km^{-1}, $R_0 = 1$ W^{-1} km^{-1} and $f(z)$ and $\rho(z)$ are functions specifying the longitudinal profile of their change.

Figures 2–4 present the dependences of the nonreciprocity of the FM rate and pulse duration on the fibre length calculated by using (24) for some most characteristic inhomogeneities of the parameters D and R, i.e. functions $f(z)$ and $\rho(z)$. Figure 2 shows the dependences obtained for the profile of the material dispersion homogeneous over the length with $f(z) = -1$ and the nonlinearity inhomogeneity of the type $\rho(z) = \exp(\gamma z)$, $\gamma = -1$ km^{-1}. At the initial stage of pulse propagation in the fibre, its dynamics according to (24) substantially depends on the ratio of the parameters D and R_{eff}. For $\alpha_0 = 0$ and $D < 0$, the pulse compression takes place for $\mathrm{d}\alpha/\mathrm{d}z > 0$, namely for $R_{\mathrm{eff}}(0) > k_1|D|/(k_2\tau_0^2 I_0)$, which is fulfilled for the selected values entering this inequality of parameters. One can see that at the initial stage $\alpha(z)$ increases while $\tau(z)$ decreases. When the FM rate achieves the value for which the condition

$$\alpha^2 + \frac{k_2 R_{\mathrm{eff}} \tau_0 I_0}{|D|\tau_{\mathrm{p}}^2} = \frac{k_1}{\tau_{\mathrm{p}}^4} \tag{25}$$

is fulfilled, the sign of the derivative $\mathrm{d}\alpha/\mathrm{d}z$ changes, after which the FM rate drastically decreases and changes its sign. Then, the pulse begins to broaden till the next change in the sign of the parameter α. Thus, the pulse dynamics in the case under study has a cyclic character, which slightly differs from the forward and backward propagation. The fibre lengths, for which the input durations of counter-propagating pulses will be equal, correspond to the

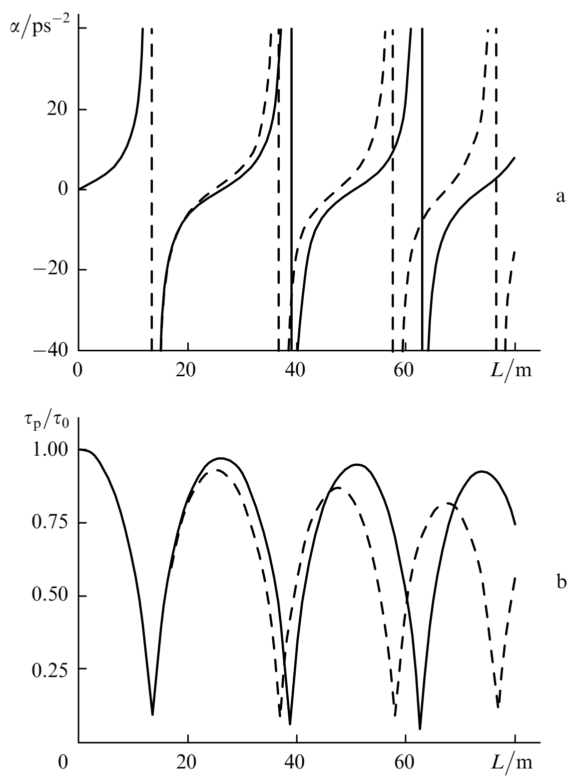

Figure 2. Dependences of the FM rate (a) and pulse duration (b) on the fibre length for $D(z) = -D_0 = -10^{-26}$ s^2 m^{-1}, $R(z) = R_0 \exp(\gamma z)$ and $\gamma = -1$ km^{-1}. Solid curves correspond to the forward pulse and dashed curves – to the backward pulse.

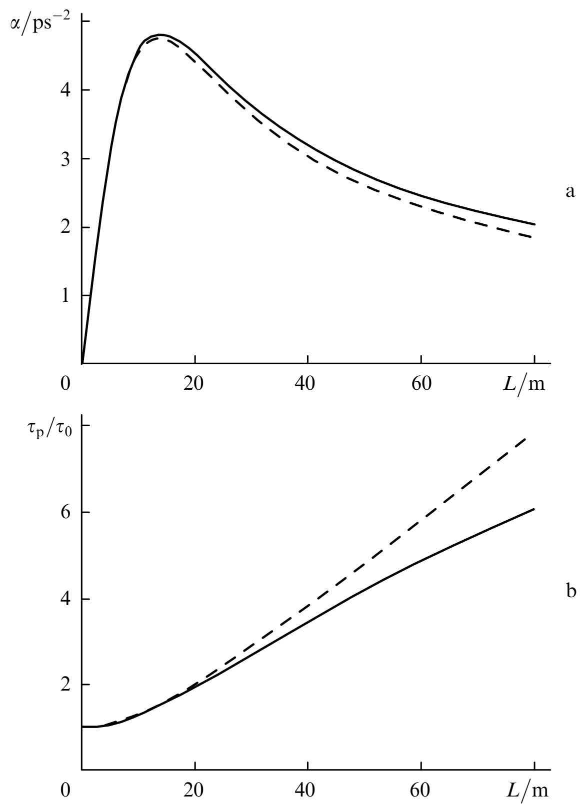

Figure 3. Dependences of the FM rate (a) and pulse duration (b) on the fibre length for $R(z) = R_0$, $D(z) = D_0 \exp(\gamma z)$ and $\gamma = -1$ km^{-1}. Solid curves correspond to the forward pulse and dashed curves – to the backward pulse.

intersection points of curves $\tau_+(L)$ and $\tau_-(L)$. Therefore, the nonreciprocity η_τ oscillates with increasing the fibre length. A similar situation takes place for the nonreciprocity of the FM rate. It follows from the above dependences that the presence of the GVD, which is homogeneous over the fibre length, significantly changes the character of the forward and backward propagation of a pulse, thus leading to a more pronounced nonreciprocity of its main parameters. Thus, in the 'dispersionless' case, for the fibre length of $L = 80$ m the nonreciprocity η_α of the FM rate is ~ 0.2, while in the case under study it is ~ 10.5.

The dependences in Fig. 3 describe the solutions of equations (24) for the exponential dispersion profile $f(z) = \exp(\gamma z)$, $\gamma = -1$ km^{-1} (decreasing for the forward and increasing for the backward pulse) and homogeneous nonlinearity $\rho(z) = 1$. Unlike the above dependences where the nonreciprocal dynamics was determined by the nonlinearity inhomogeneity, in this case, the nonreciprocity of the parameters τ and α is caused by the inhomogeneity of the dispersion parameters of the fibre. Because $D(z) > 0$, the pulse, according to (24), is compressed at $\alpha(z) < 0$. The pulse is broadened for the selected parameters because the value $\alpha(L)$ is positive everywhere. The nonreciprocity of the parameters $\alpha(L)$ and $\tau(L)$ is caused by the fact that the input parameters of the fibre for the counterpropagating pulses are different: for the forward pulse $D(0) = D_0$, while for the backward pulse $D_-(0) = D_+(L) = D_0 f(L)$. Therefore, the values of $\mathrm{d}\alpha/\mathrm{d}z$ and $\mathrm{d}\tau/\mathrm{d}z$ are different for these pulses after they are coupled into the fibre, which determines the nonreciprocity of the dynamics of these parameters.

Figure 4 shows the dependences of $\alpha(L)$ and $\tau(L)$ for $\rho(z) = 1$ and alternating dispersion profile: $f(z) =$

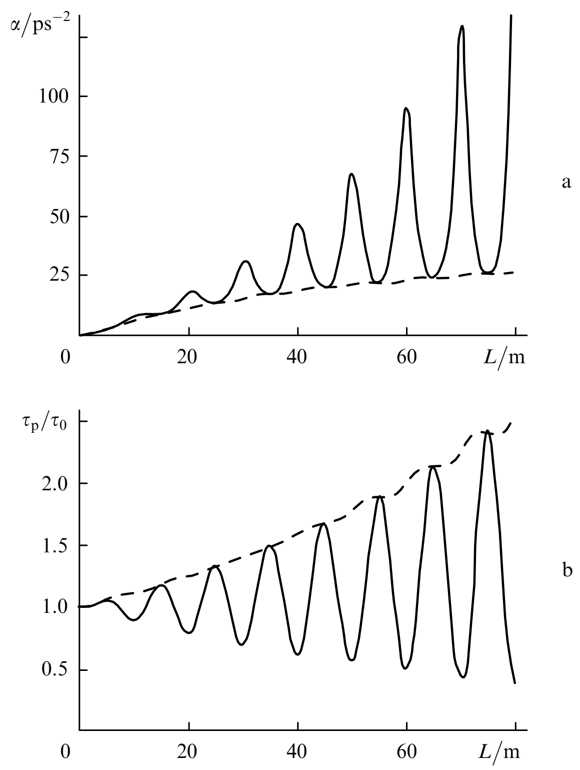

Figure 4. Dependences of the FM rate (a) and pulse duration (b) on the fibre length for $R(z) = R_0$, $D(z) = D_0 \sin(2\pi z/\Lambda)$ and $\Lambda = 10$ m. Solid curves correspond to the forward pulse and dashed curves – to the backward pulse.

$\sin(2\pi z/\Lambda)$, where the inhomogeneity period is $\Lambda = 10$ m. The case under study is interesting by the fact that for any lengths of the fibre the backward pulse is broadened at the output (despite the presence of weakly-pronounced regions with $d\tau_-/dz < 0$), while the forward pulse, depending on the fibre length, can be either broadened or compressed. Because for all L, the FM rate is $\alpha(z) > 0$, the pulse is broadened only at those regions where $D(z) > 0$ and is compressed at negative GVDs. Thus, the oscillation period $\tau(L)$ is equal to the inhomogeneity period of the GVD in the fibre. In the case under study, the average value of the effective nonlinearity is equal both for the forward and backward pulse, i.e. $\langle R_{\rm eff}^{+}\rangle = \langle R_{\rm eff}^{-}\rangle$. As a rule, the nonreciprocal dynamics of the pulse is caused by the difference in the above-mentioned values. Here, the nonreciprocity can be explained by different initial conditions for the forward and backward pulses, i.e. by different GVDs at the fibre ends. The nonreciprocity will be pronounced to the highest degree when the fibre length is related to the GVD period by the expression $L = n\Lambda$, where n is an integer. The backward pulse can propagate the first $\Lambda/2$ meters in a medium with the negative GVD, which provides its maximal compression. In this case, the forward pulse propagates the first $\Lambda/2$ meters in a medium with the positive GVD, and, hence, its compression is impossible. If $L = (1/2 + n)\Lambda$, an optical fibre symmetric with respect to both ends is realised, in which nonreciprocal effects do not appear.

According to (17), the nonreciprocity of the FM rate also leads to a difference in the spectral widths of counterpropagating pulses. In the case $\alpha_0 = 0$ and $\alpha_{\pm}\tau_0^2 \gg 1$, the expression for the spectral nonreciprocity takes the form $\eta_{\Delta\omega} \approx 0.44\eta_{\alpha}$. For the parameters used at the output of the fibre of length $L = 80$ m, the forward pulse is compressed, while the backward pulse is broadened by 2.7 times; in this case, $\eta_{\alpha} \approx 164$ and $\eta_{\Delta\omega} \approx 72$.

The example of the alternating inhomogeneity of the fibre represents a dynamic situation, which is exceptionally interesting and important for practical applications. In this case, the changes in the duration, amplitude and FM rate of a pulse propagating in one direction are very small (the pulse parameters can be considered almost constant), while the dynamics of these parameters for a counterpropagating pulse has a strongly pronounced oscillating character (with a periodic strong temporal compression of the pulse, increase in the FM rate and peak intensity).

6. Conclusions

By using the NSE solution in the 'dispersionless' approximation, we have obtained the general integral expression for the FM rate of the forward and backward Gaussian and hyperbolic secant pulses propagating in the fibre with the arbitrary distribution of the gain inhomogeneity and nonlinearity over its length. The nonreciprocity of the FM rate leads to the spectral nonreciprocity. The presence of the GVD substantially changes the character of the nonreciprocal FM dynamics of pulses. Based on the variation procedure of the NSE solution, equations have been derived, which determine the change in the main parameters of the pulse (duration, phase and FM rate) propagating in a fibre with a specified set of functional dependences of the gain, nonlinearity and GVD on the coordinate. The numerical analysis of these equations has shown that when the longitudinal profile of the GVD (for example, linear or exponential) changes monotonically, the nonreciprocity of the pulse parameters increases with increasing the fibre length. For the alternating periodic profile of the GVD, the nonreciprocity substantially depends on the fibre length and both total reciprocity (when the profile function is symmetric with respect to the fibre middle) and the strong difference in the output parameters of the forward and backward pulses.

Therefore, by changing the longitudinal profiles of the material parameters of the fibre, we can efficiently control the transformation of counterpropagating pulses. The plotted dependences of the nonreciprocity on the fibre length reflect only a small part of the variety of possible dynamic regimes in fibres inhomogeneous over their length.

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