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Diffraction phenomena in ring gas lasers

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***Abstract*. The frequency and amplitude diffraction nonreciprocity of counterpropagating waves in a ring gas laser is studied. The self-consistent eigenmode problem is solved within the framework of the model of Gaussian beams for a ring optical resonator containing a nonlinear gaseous medium and a model aperture. The expressions for frequency-dependent losses, frequencies and intensities of counterpropagating waves are obtained and analysed. Experimental data on the influence of diffraction phenomena on the frequencies and intensities of counterpropagating waves reported in the literature are analysed. Nonreciprocal effects related to diffraction are classified by comparing the experimental data with theoretical results. The mechanisms of the asymmetry of the lasing region with respect to the central frequency of the transition and asymmetry of the laser line are explained.**

Keywords: *ring laser, cavity eigenmodes, frequency-dependent losses, nonreciprocity.*

1. Introduction

Stable oscillation in both directions in a ring laser was first demonstrated in 1963 [1]. With the advent of a ring laser the unique possibilities opened up for precision investigations in various fields of fundamental physics. These are the studies of relativistic and gravitational effects in the theory of relativity [2, 3], the development of gravitational-wave detectors [4], the verification of quantum electrodynamics effects [5], and studies of other subtle phenomena [6, 7]. Even this short list shows that the investigation of physical processes determining the basic properties of a ring laser becomes increasingly urgent. It should be expected that new theoretical and experimental studies will not only increase the possibilities of existing devices but also expand the scope of applications of ring lasers.

One of the important applications of ring lasers is their use as sensors in laser gyros. Laser gyros available at present provide the accuracy allowing the measurement of the irregularity of the Earth's rotation [3]. Diffraction effects occupy a significant place among physical processes determining the limiting accuracy of laser gyros. These effects have been studied in many papers. Even an incomplete list of these works is quite impressive (see, for example, [8 – 31]). It was found that in a laser with an inhomogeneous resonator, the frequency difference of counterpropagating waves appears for different reasons even in the absence of rotation, i.e. the zero of the frequency characteristic is shifted. Effects resulting in the difference of frequencies and intensities of counterpropagating waves are called nonreciprocal effects, and the phenomenon itself is referred to as the frequency and amplitude nonreciprocity. A complete review of nonreciprocal effects in ring lasers is presented in [31].

Nonreciprocal effects give rise to a narrow frequency region (the so-called strong-coupling region) near the centre of the Doppler gain line within which one of the counterpropagating waves is completely or almost completely quenched. Under certain conditions, the frequency dependence of the wave intensity in this region has the form of a resonance peak. The presence of such intensity resonances stimulated studies on their applications to stabilise the frequency of a ring laser with an absorbing cell [32] and in ultrahigh-resolution spectroscopy [33].

In this paper, the dependence of the frequency and intensity difference of counterpropagating waves in a laser with nonreciprocal elements on the resonator frequency detuning from the line centre is theoretically analysed. It is shown that these dependences are qualitatively different when nonreciprocal elements produce the difference of losses or frequency difference of counterpropagating waves. Diffraction-related nonreciprocal effects are classified for the first time by comparing the experimental data accumulated for many years with our theoretical results. It is established that the predominance of one or another nonreciprocity mechanism determines the behaviour of the frequency and intensity difference of counterpropagating waves appearing in this case.

Having answered the question which physical reasons give rise to the amplitude or phase nonreciprocity of counterpropagating waves in a laser without nonreciprocal elements, one can explain the appearance of the frequency-dependent shift of the zero of the frequency characteristic of the ring laser used as a sensor in a laser gyro. Theoretical papers published earlier could not answer this question. In this paper, the answer is obtained by solving the self-consistent eigenmode problem for a ring optical resonator containing a nonlinear gaseous medium and a model aperture. It is shown that diffraction from apertures of such a

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resonator gives rise both to the nonreciprocity of frequency-independent losses and frequency-dependent phase shifts of counterpropagating waves. In a ring laser with an aligned resonator, the combined action of diffraction and the field-induced radial inhomogeneity of a nonlinear medium makes the role of the loss difference dominant. The misalignment of the resonator caused by the misalignment of mirrors, the application of apertures asymmetric with respect to the optical axis (or the aperture displacement perpendicular to the resonator plane) leads to the dominant role of the frequency difference.

In addition, we studied the behaviour of the lasing intensity and frequency near and within the strong-coupling region in the case of the amplitude and phase nonreciprocities. This is of current interest in problems of nonlinear spectroscopy and frequency stabilisation, where the high accuracy of measuring the quantum-transition frequency ω_{ab} by the power peak and nonlinear dispersion resonance is very important.

2. Study of generation in a ring laser in the plane-wave model

We analyse below the behaviour of frequencies and intensities of stationary counterpropagating waves of a ring laser in the plane-wave model developed by Lamb [34] for a linear laser. The generalisation of this theory for a ring laser was presented in many papers. However, the author is not aware of papers where a laser with different phase or amplitude conditions for counterpropagating waves in the resonator are systematically analysed, although such attempts have been made in some papers [35–37]. The results obtained in our paper differ from those reported in the above papers mainly because we calculated for the first time the polarisability of a medium in the fields of counterpropagating waves in the third order of the perturbation theory in the general form, not restricting ourselves to the second-order terms in the ratio of the natural linewidth to the Doppler width.

The modes of ring resonators are travelling waves. Consider a laser in which propagation in each direction occurs only on one of the eigenmodes ω_r and ω_l. And although the field in the resonator is a superposition of the fields of two travelling counterpropagating waves:

$$E(z) = E_r(z)\exp(-i\omega_r t) + E_l(z)\exp(-i\omega_l t) + \text{c.c}, \quad (1)$$

such a laser is called a single-mode laser because both waves have the same longitudinal index. The field of each of the waves satisfies the stationay wave equation

$$\left(\frac{\partial^2}{\partial z^2} + k_j^2\right)E_j(z) = -4\pi k_j^2 P_j(z), \quad k_j = \omega_j/c, \quad j = \text{r,l}. \quad (2)$$

Field (1) induces in the nonlinear medium a macroscopic polarisation of the active medium

$$P(z) = P_r(z)\exp(-i\omega_r t) + P_l(z)\exp(-i\omega_l t) + \text{c.c.}, \quad (3)$$

where

$$2\pi P_r(z) = [-(1/k_r)K(Z_r - \beta_r I_r - \theta_l I_l)]E_r(z) = 2\pi\varkappa_r E_r(z). \quad (4)$$

Hereafter, the equations for a counterpropagating wave are obtained by the subscript replacement r $\leftrightarrow$ l. The dimensionless wave intensities I_r and I_l and the gain K of the medium are determined in Appendix 1, where the dependences of Z_j, β_j, and θ_j on the detuning and transition parameters [the half-widths γ_a and γ_b of the levels, and the homogeneous (γ_{ab}) and inhomogeneous (ku) half-widths of the line [$k = (k_r + k_l)/2$)] are also given. The imaginary part of the function $Z_j = Z_j' + iZ_j''$ describes the dependence of the unsaturated gain on the detuning $\omega_j - \omega_{ab}$, while the real part is responsible for linear dispersion. The nonlinear gain saturation and nonlinear dispersion caused by the self-action of counterpropagating waves are described by the imaginary and real parts of the coefficients $\beta_j = \beta_j' + i\beta_j''$, while the same parameters caused by the interaction of counterpropagating waves are described by the imaginary and real parts of the coefficients $\theta_j = \theta_j' + i\theta_j''$, respectively. Figure 1 presents these functions for $\omega_r = \omega_l = \omega$ and $\lambda = 3.39$ μm, $\gamma_a = 18$ MHz, $\gamma_b = 27$ MHz, $\gamma_{ab} = 120$ MHz, and $ku = 300$ MHz, which are used in calculations below.

The solution of Eqn (2), which is supplemented by the conditions of periodicity of the fields and the conditions on a partially transmitting mirror with the reflectivity $\mathcal{R}$, is found in the form of plane waves:

$$E_r(z) = E_{0r}\exp\left[ik_r\int_0^z n_{zr}(z)dz\right], \quad (5)$$

$$E_l(z) = E_{0l}\exp\left[-ik_l\int_L^z n_{zl}(z)dz\right], \quad (6)$$

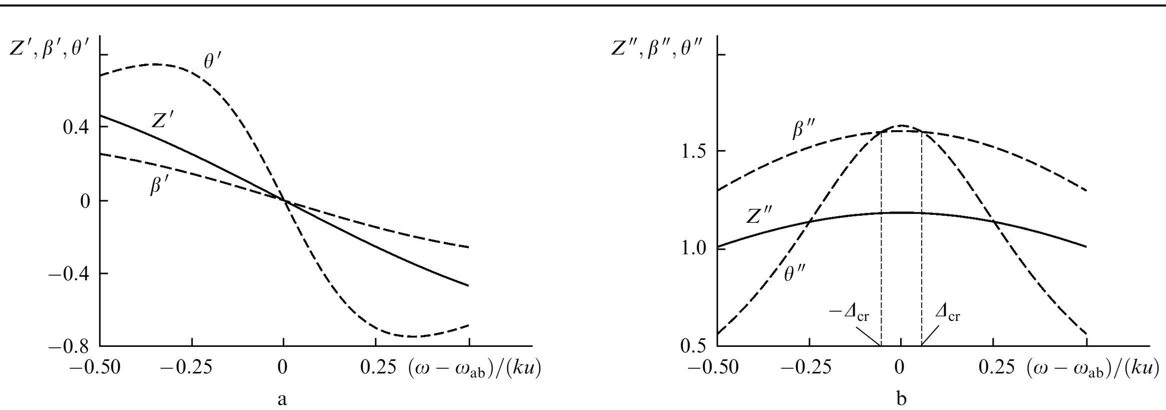

Figure 1. Dependences of the real (a) and imaginary (b) parts of coefficients Z, β, and θ on detuning.

where L is the resonator length. In the small-gain approximation, the refractive index $n_{zj}(z)$ for each of the waves over the gas-discharge tube length H is

$$n_{z\mathrm{r}} = 1 + 2\pi\varkappa_{\mathrm{r}} = 1 - \frac{1}{k_{\mathrm{r}}} K_0(Z_{\mathrm{r}} - \beta_{\mathrm{r}} I_{\mathrm{r}} - \theta_{\mathrm{l}} I_{\mathrm{l}}), \tag{7}$$

and $n_{z\mathrm{r}} = n_{z\mathrm{l}} = 1$ outside the tube. The field periodicity condition gives the system of complex equations for counterpropagating waves

$$-\varphi_j + \mathrm{i}\varepsilon_j + k_j \int_0^L n_{zj}(z)\mathrm{d}z = 2\pi\tilde{q}. \tag{8}$$

Here, $\tilde{q}$ is a large integer (the longitudinal index of a mode); $\varepsilon_{\mathrm{r,l}} = \varepsilon_0 \pm \delta\varepsilon$; $\varepsilon_0 = \ln(1/\sqrt{\mathcal{R}})$ are mirror losses; $\pm\delta\varepsilon$ and φ_j are losses and phase shifts of counterpropagating waves caused by the action of nonreciprocal elements, which are additional to the geometric-optical shift. By equating the imaginary parts of (8) to zero, which means that the saturated gain of each of the waves is equal to its losses, we obtain the expression for the wave intensities:

$$I_{\mathrm{r}} = \frac{\eta_{\mathrm{r}}\beta_{\mathrm{l}}'' - \eta_{\mathrm{l}}\theta_{\mathrm{l}}''}{\beta_{\mathrm{r}}''\beta_{\mathrm{l}}'' - \theta_{\mathrm{r}}''\theta_{\mathrm{l}}''}, \quad \eta_{\mathrm{r}} = Z_{\mathrm{r}}'' - \frac{\varepsilon_{\mathrm{r}}}{KH}. \tag{9}$$

The real parts of (8) give equations for lasing frequencies ω_{r} and ω_{l}:

$$\omega_{\mathrm{r}} = \Omega + \delta\omega_{\mathrm{r}} + (c/L)KH(Z_{\mathrm{r}}' - I_{\mathrm{r}}\beta_{\mathrm{r}}' - I_{\mathrm{l}}\theta_{\mathrm{l}}'), \tag{10}$$

where $\Omega = (c/L)2\pi\tilde{q}$ is the frequency of the empty resonator and $\delta\omega_{\mathrm{r}} = (c/L)\varphi_{\mathrm{r}}$ is the frequency base. When the laser resonator does not contain sources producing unequal losses or unequal frequencies of counterpropagating waves, both waves have equal frequencies and intensities over the entire lasing region. The dependence of the wave intensities $I_{\mathrm{r}} = I_{\mathrm{l}} = \eta/(\beta'' + \theta'')$, where $\beta = (\beta_{\mathrm{r}} + \beta_{\mathrm{l}})/2$, $\theta = (\theta_{\mathrm{r}} + \theta_{\mathrm{l}})/2$, and $\eta = (\eta_{\mathrm{r}} + \eta_{\mathrm{l}})/2$, on the detuning represents a curve with a dip near the central transition frequency. Such dependence is typical for single-mode normal lasers. The picture cardinally changes when a certain difference of losses or frequency difference is produced for counterpropagating waves with the help of special nonreciprocal devices, of which the most popular are elements based on the Faraday effect.

Due to different losses $\varepsilon_{\mathrm{r,l}} = \varepsilon_0 \mp \delta\varepsilon$ for counterpropagating waves, their intensities and frequencies are also different: $I_{\mathrm{r}} - I_{\mathrm{l}} = 2\Delta I$ and $\omega_{\mathrm{r}} - \omega_{\mathrm{l}} = 2\Delta\omega$. It follows from (9) and (10) that

$$I_{\mathrm{r,l}} = \eta\frac{1}{\beta'' + \theta''} \pm \frac{\delta\varepsilon}{KH}\frac{1}{\beta'' - \theta''} = I \pm \Delta I,$$
$$\Delta\omega = -\delta\varepsilon\frac{c}{2L}\frac{\beta' + \theta'}{\beta'' - \theta''}. \tag{11}$$

Because the dependences of coefficients β'' and θ'' on the detuning are even, while these dependences for coefficients β' and θ' are odd, the intensity difference is an even function, while the frequency difference is an odd function of the detuning $\omega - \omega_{\mathrm{ab}}$. It follows from (11) that nonreciprocal additions ΔI and $\Delta\omega$ change their signs when the average lasing frequency $\omega = (\omega_{\mathrm{r}} + \omega_{\mathrm{l}})/2$ passes through a value ω_{cr} at which the condition $\beta'' - \theta'' = 0$ is fulfilled. By using the expressions for β'' and θ'' presented in Appendix 1, we find that this condition is fulfilled for detunings

$$\Delta_{\mathrm{cr}} = \pm\frac{|\omega_{\mathrm{cr}} - \omega_{\mathrm{ab}}|}{ku} \approx \pm\left(\frac{\gamma_{\mathrm{ab}}}{ku}\right)^2\left[\frac{\gamma_{\mathrm{a}}\gamma_{\mathrm{b}}}{(\gamma_{\mathrm{a}} + \gamma_{\mathrm{b}})\gamma_{\mathrm{ab}}}\right]^{1/2}, \tag{12}$$

which are symmetrical with respect to ω_{ab}. The difference $|\omega_{\mathrm{cr}} - \omega_{\mathrm{ab}}|$ is about 16 MHz for the line at 3.39 μm and about 3 MHz for the line at 0.6328 μm. Near these detunings, the energy exchange occurs between counterpropagating waves due to their strong coupling. For frequency detunings $\xi = (\omega - \omega_{\mathrm{ab}})/(ku) < -\Delta_{\mathrm{cr}}$ and $\xi > \Delta_{\mathrm{cr}}$ (where $\theta'' < \beta''$), the intensity of the strong wave (i.e. the wave with lower losses) increases resonantly, whereas the intensity of the weak wave (the wave with higher losses) decreases. Within the frequency region $-\Delta_{\mathrm{cr}} < \xi < \Delta_{\mathrm{cr}}$ (where $\theta'' > \beta''$), the weak-wave intensity increases at the expense of the strong wave. If the difference of losses of counterpropagating wave is small, both waves can be generated in this frequency interval. In this case, due to the energy redistribution caused by a strong coupling, the intensity of the wave with higher losses exceeds the intensity of the wave with lower losses. As the loss difference increases, the wave with higher losses can completely quench the wave with lower losses. The appearance of intensity resonances for frequency detunings close to $\pm\Delta_{\mathrm{cr}}$ is accompanied by a drastic change in the difference frequency. The results of numerical calculations presented in Figs 2a and b confirm our estimates.

In the case when the phase nonreciprocity $2\delta\omega = \delta\omega_{\mathrm{r}} - \delta\omega_{\mathrm{l}}$ is produced between counterpropagating waves, in the low frequency region $\omega < \omega_{\mathrm{ab}}$ the wave of the higher frequency is stronger because it is stronger amplified being closer to the central transition frequency. For $\omega > \omega_{\mathrm{ab}}$, the situation changes, and the wave with the lower frequency is amplified stronger. Because outside the strong coupling region the energy is transferred from the weak wave to the strong one, the intensity difference of the counterpropagating waves

$$I_{\mathrm{r,l}} = I \pm \frac{F(\omega - \omega_{\mathrm{ab}})}{\beta'' - \theta''} = I \pm \Delta I \tag{13}$$

is an odd function of the detuning (here, F is the parameter that is an even function of the detuning) (see Fig. 2c). For this reason, the dependences of the intensities of counterpropagating waves within the strong-coupling region have the X-like shape. In this case, the frequency difference (Fig. 2d) is an even function of the detuning. The dependence of the frequency difference on the detuning within the strong-coupling region has the form of a resonance peak whose centre coincides with that of the gain line. Note that both for the amplitude and phase nonreciprocities, the dependences of the sum of intensities on the detuning are described by a symmetric curve with a dip near the gain-line centre.

3. Results of the experimental study of nonreciprocal effects and attempts to explain them

The inequality of frequencies of counterpropagating waves in a single-mode ring laser without nonreciprocal devices was first reported in [9]. The experiment was performed in a laser with a square resonator (with an arm of 92 cm)

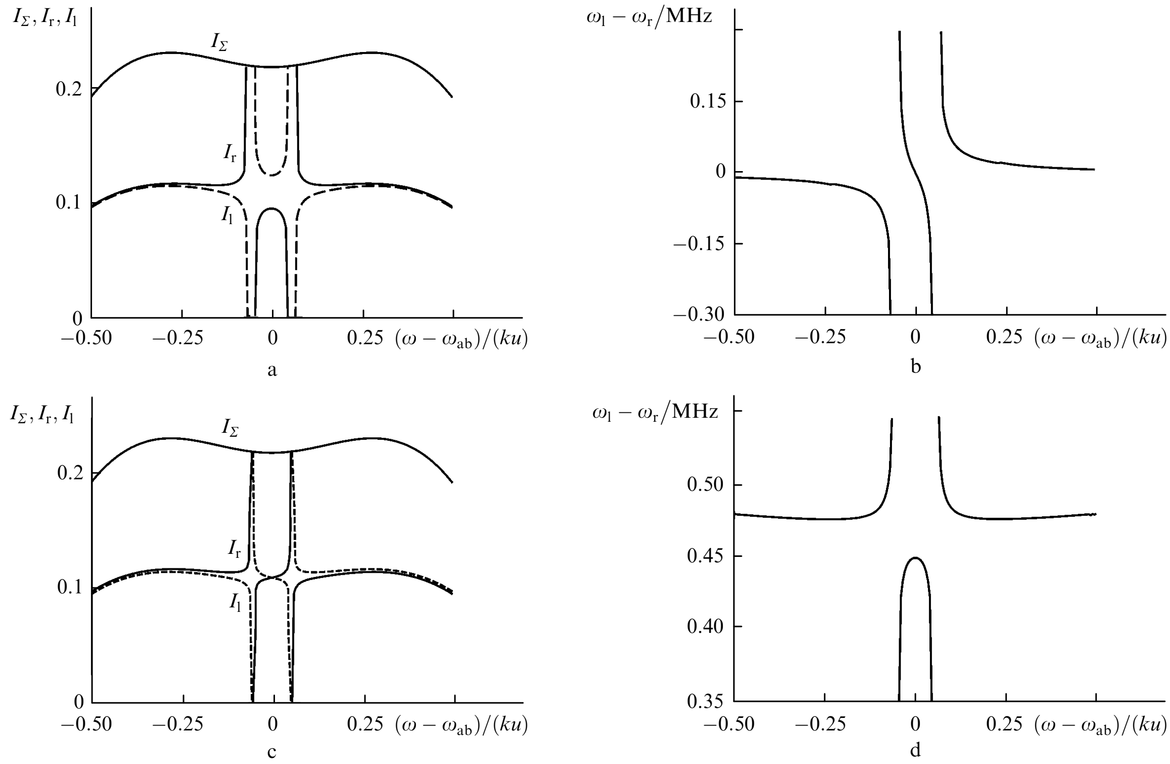

Figure 2. Dependences of the dimensionless intensities I_r and I_l of counterpropagating waves, their sum I_Σ (a, c), and the frequency nonreciprocity $\omega_l - \omega_r$ (b, d) on detuning calculated in the cases of using a device producing different losses for counterpropagating waves, for $\varepsilon_r = 1$, $\varepsilon_l = 1.002$, $KH = 0.12$, $L = 1$ m (a, b), and using a device providing the frequency difference for counterpropagating waves $\delta\omega = 0.5$ MHz for $KH = 0.12$ and $L = 1$ m (c, d).

emitting at 3.39 μm. By inserting a needle point or a razor blade into a laser beam (at a distance of about 1 mm from the beam axis), beats at a frequency of ~ 5 kHz were obtained. It was pointed out that this effect, which was later called the diffraction nonreciprocity, was stronger when the needle point was inserted into the beam near the tube window. The authors related the frequency difference $\Delta\omega$ to the nonreciprocity $\Delta n \sim 10^{-10}$ of the refractive indices of a medium for counterpropagating waves by assuming that the reason for this nonreciprocity is the intensity saturation. Thus, the nonlinearity of this phenomenon has been pointed out already in the first paper.

The coherent correlation between the loss difference produced by a nonreciprocal element and the behaviour of intensities of counterpropagating waves was studied experimentally in [11]. It was found that, when counterpropagating waves had equal losses, each of them was generated over the entire detuning region. When losses were different, one of the waves was suppressed near the centre of the gain line, whereas the intensity of the other wave increased. As loss difference increased, one of he waves was suppressed in a large detuning interval. The results obtained in this paper can be explained completely within the framework of the plane-wave model (see Figs 2a, b).

Such dependences were also observed in lasers without specially produced amplitude nonreciprocity [10]. The dependences of intensities of counterpropagating waves on the detuning presented in Fig. 3 suggest that a source of amplitude nonreciprocity exists in the laser resonator. In this case, the dependence of the beat frequency had the form of the dispersion curve. The authors of paper [10] assumed that the Q factors of the resonator for counterpropagating waves are different. By assuming that $|\Delta\eta|/\eta = 10^{-2}$ (where $\Delta\eta = (\eta_r - \eta_l)/2$), they could compare the obtained dependences with theoretical curves described by expressions (9). They have failed to explain the reason for the difference of losses for counterpropagating waves. Its existence was simply postulated in this paper and other papers (see, for example, [18, 36]). In [12], where one-directional lasing was first obtained in a ring laser, it was assumed that different losses for counterpropagating waves appear due to the combined action of diffraction and transverse inhomogeneity of the active medium. The concept of frequency-dependent losses for counterpropagating waves was introduced; however, the mechanism of nonreciprocity formation was not determined.

The influence of the phase nonreciprocity produced by a Faraday element on the lasing intensity was experimentally studied in [11]. The corresponding dependences obtained in this paper well agree with our calculations (see Figs 2c, d). Such dependences were also observed in resonators without nonreciprocal elements when the diaphragming of the resonator caused its misalignment. In [13], experiments were performed both for a pure ^{20}Ne isotope and the mixture of isotopes ^{20}Ne and ^{22}Ne at a wavelength of 3.39 μm. A discharge tube of length 0.18 m had a diameter of 3 mm and was fed from a dc supply. The use of a high-frequency discharge (~ 40 MHz) almost did not change the picture of the effect. The laser resonator ($L = 0.75$ m) was formed by two plane mirrors and one spherical mirror (with the radius of curvature $R = 1.2$ m), which could be replaced by a plane mirror. Two apertures were placed symmetrically from both sides of the active medium. Their position and diameter could be controlled. When one of the apertures of

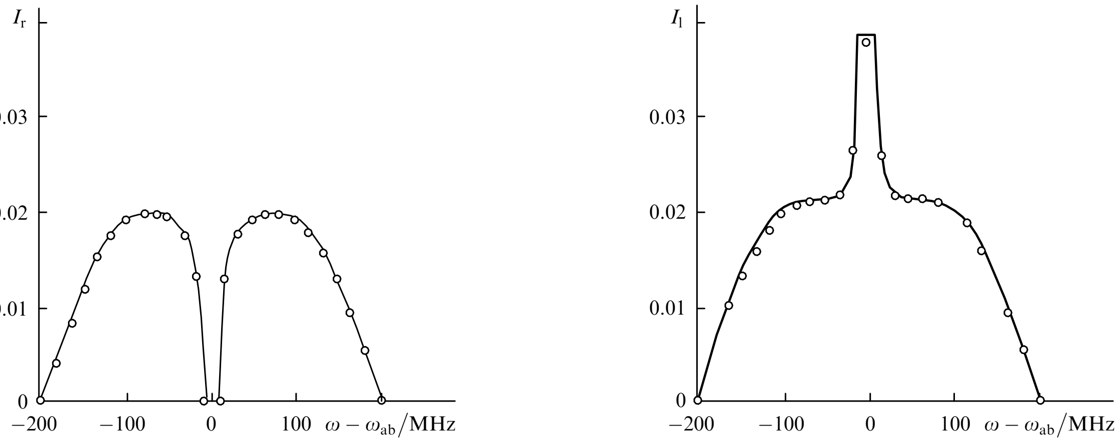

Figure 3. Dependences of the dimensionless intensities I_r and I_l of counterpropagating waves in a ring laser with a symmetric aperture [10] on detuning.

diameter ~ 1 mm was displaced to the resonator axis or from it by a distance of fractions of millimetre, the counterpropagating waves had different intensities I_r and I_l. The typical behaviour of these intensities for a pure isotope is shown in Fig. 4. The difference $I_r - I_l$ was determined by the degree of insertion of the aperture into the beam, and its sign changed after the passage of the resonator frequency Ω through the value ω_{ab}. Figure 5 shows the dependences of the lasing frequency difference on the detuning. One can see that the sign of the frequency difference did not change over the entire lasing region. An increase in the discharge current resulted in the increase in ΔI and $\Delta\omega$. If the plane mirror was replaced by a spherical one, the maximum frequency splitting increased by a factor of three (from 500 to 1500 kHz). The minimal frequency difference $\Delta\omega$ (0.8–1.2 kHz) observed in experiments was determined by the locking band.

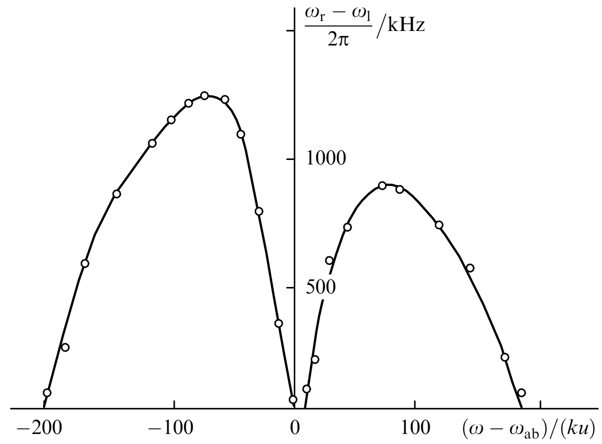

Figure 5. Dependences of the frequency difference $(\omega_r - \omega_l)/(2\pi)$ of counterpropagating waves in a ring laser with an asymmetric aperture [13] on detuning.

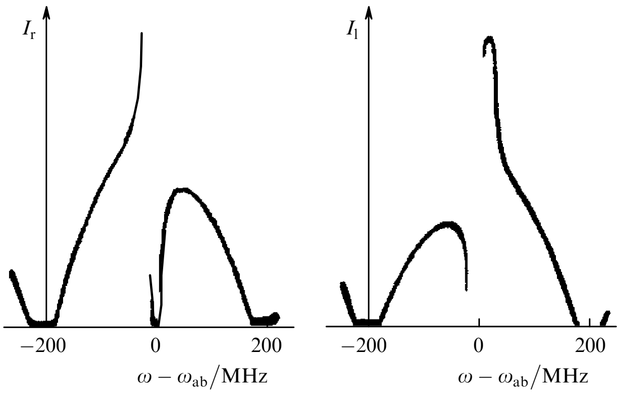

Figure 4. Dependences of the dimensionless intensities I_r and I_l of counterpropagating waves in a ring laser with an asymmetric aperture [13] on detuning.

It was assumed in [13] that the main reason for nonreciprocity is the radial inhomogeneity of the saturated refractive index of the medium. Due to nonreciprocal focusing related to the resonator geometry, the mutual action of counterpropagating waves on each other can be different. According to [13], due to incomplete overlap of radiation beams in the active medium, the paths of counterpropagating waves are different, their difference changing during frequency tuning due to refraction appearing upon the misalignment of the resonator with an asymmetric aperture. If radiation propagates in the medium not parallel to the tube axis, the beam path bends and deflects to one of the sides depending on the gradient of the refractive index of the medium. The gradient sign changes to the opposite after the frequency ω passes through the value ω_{ab}. The authors of [13] explain in this way the distortion of the lasing region: for frequencies $\omega > \omega_{ab}$, the beam is inclined to the tube axis, amplification increases and the lasing region expands, while the frequency region $\omega < \omega_{ab}$ narrows down.

The idea that the frequency and amplitude nonreciprocity appears due to different distributions of the fields of counterpropagating waves proposed in [13, 14] was developed in [20]. It was shown that the frequency splitting $\Delta\omega$ was maximal if the aperture was placed near a cell with the active medium. As the aperture was removed from the cell along the beam axis, the frequency splitting $\Delta\omega$ decreased monotonically and vanished when the aperture was in a plane located approximately at equal distances from the cell ends. The movement of the aperture through this plane was accompanied by a change in the sign of $\Delta\omega$, and the value of $\Delta\omega$ increased and achieved its maximum when the aperture approached the other end of the cell. The dependence on the resonator geometry was manifested in the fact that the frequency splitting increased with increasing the curvature of resonator mirrors.

The experiment performed in [13] was repeated later by French researchers [22] on a setup with different parameters. They used a three-mirror ring resonator of length $L = 1.2$ m with one spherical mirror with the radius of curvature $R = 2$ m. The ratio of these parameters provided the approximate equality of the g parameters of this resonator and that used in [13]. The diameters D_1 and D_2 of apertures were 2.6 mm. The laser also operated in the single-longitudinal mode regime at 3.39 μm. But unlike [13], the authors of paper [22] used discharge tubes of a larger internal diameter (6 mm), which, in their opinion, should reduce the influence of the lens effect. This resulted in a decrease in the maximum achievable frequency splitting after aperture misalignment approximately by 35 times compared to that observed in [13]. In this case, the dependence of the frequency splitting $\Delta\omega$ on the detuning $\omega - \omega_{ab}$ remained almost an even (somewhat asymmetric) function, as in Fig. 5. Note that the intensity profiles have a distinct asymmetry, which is the same for counterpropagating waves in the aligned resonator (for $\Delta\omega = 0$). When the aperture was displaced perpendicular to the beam, the frequencies and intensities of the waves became different. The intensity profiles acquired different asymmetry, the wave with a higher asymmetry having a higher frequency. The authors of [22] explained these results by different losses for counterpropagating waves, as was earlier assumed in [12]. They assumed that the reason for the difference of losses is the misalignment of the resonator. However, the authors of [22] attempted to substantiate the nonreciprocity of losses by using the plane-wave model, which is impossible in principle. Moreover, the dependences of the intensity and frequency difference on the detuning obtained in [22] suggest that the dominant mechanism of nonreciprocity in these experiments is the phase nonreciprocity rather than the difference of losses for counterpropagating waves.

Experiments with a laser emitting at 0.6328 μm [21] also confirm the fact that the use of symmetric or asymmetric apertures leads to qualitatively different results. It was shown that the replacement of a symmetric iris aperture by an aperture in the form of a half-plane resulted not only in the increase in the frequency difference by two orders of magnitude but also in the change of its sign. It was found earlier that the displacement of the aperture perpendicular to the beam was accompanied by oscillations of $\Delta\omega$ [9, 20].

By generalising the results of experimental studies [8 – 22] and comparing them with our calculations performed within the framework of the plane-wave model, the following conclusions can be formulated.

(i) Diaphragming of the radiation beam in a ring laser leads to the frequency and amplitude nonreciprocity of counterpropagating waves. In the case of symmetric diaphragms, the difference of wave intensities is described by an even function, while the frequency difference is described by an odd function of the frequency detuning from the gain line centre. This means that in the aligned resonator the dominating mechanism determining the behaviour of frequencies and intensities is the difference of losses for counterpropagating waves (the amplitude nonreciprocity).

(ii) When asymmetric apertures are used in ring lasers, the difference of intensities of counterpropagating waves is described by an odd function, while the frequency difference is described by an even function of the frequency detuning for the gain line centre, which indicates the dominant role of the phase nonreciprocity in the misaligned laser.

(iii) The loss nonreciprocity makes it possible to obtain one-directional lasing in a stable-resonator laser without using nonreciprocal elements. The elimination of nonreciprocity leads to the generation of both waves over the entire lasing region.

(iv) The existence of resonances of the wave intensities and their frequency difference near the strong-coupling region suggests that they have the amplitude and (or) phase nonreciprocity. The behaviour of the intensity and frequency of counterpropagating waves within this region is determined by the type of nonreciprocity dominating in the given setup.

(v) The frequency and intensity differences of counterpropagating waves depend considerably on the pump current and the resonator geometry: the curvature of mirrors, the mutual location of resonator elements and a cell with the active medium, its diameter, and parameters and position of the aperture.

In addition, it was established that the diaphragming of the resonator leads to the asymmetry of a dip in the total band of counterpropagating waves and distorts the lasing region as a whole [11, 13, 22, 23].

To understand the reasons for one or other mechanism of nonreciprocity in lasers without nonreciprocal elements and explain the obtained dependences, it is necessary to solve the eigenmode problem for the resonator containing a nonlinear medium, taking into account diffraction from its apertures. The theory of diffraction frequency and amplitude nonreciprocity developed earlier [24 – 30] has failed to describe correctly experimental dependences.

The characteristic feature of our approach [38] is the possibility of using the methods and results of the well-developed theory of open resonators [39 – 41] to study the characteristics of lasers with a weakly nonlinear medium. This approach, based on the standard asymptotic expansion [42], allows one to construct a simple solution of a system of integro-differential equations determining the eigenmodes of the resonator containing a model aperture and a weakly nonlinear radially inhomogeneous active medium. The use of the results of calculations of the eigenmodes of passive resonators formed by mirrors of finite size [39, 41] allowed us to relate variations in the losses and frequencies of the resonator mode with a change in its volume, which is caused in our case by the action of the lens and diaphragming properties of the medium.

4. Eigenmodes of a ring resonator with a weakly nonlinear medium

Because the problem of generation of counterpropagating waves is nonlinear and has no explicit solution, it can be solved only by the method of successive approximations. The success depends on a proper choice of the initial approximation, which should already contain most important information on the required solution. Mirrors used in gas lasers have typically quite large apertures, so that the mode structure in these lasers is well described by the Hermite – Gaussian approximation. The induced transverse inhomogeneity of the medium in the field of Gaussian beams [caused by the nonlinear part of the polarisability (A1.4)] is quadratic. For this reason, we consider a resonator with a quadratically inhomogeneous active medium linear in field as an unperturbed system, i.e. we assume that the dependence of the polarisability $\varkappa_j^{(1)}$ on the transverse coordinates is described by a quadratic function (see Appendix 2):

$$\varkappa_j^{(1)}(x,y) = \varkappa_{0j}\left(1 - \frac{x^2}{d_x^2} - \frac{y^2}{d_y^2}\right),$$
$$2\pi\varkappa_{0j} = -\frac{K_0 Z_j}{k_j}. \tag{14}$$

Here, $\varkappa_{0j}$ is the polarisability on the resonator axis; d_x and d_y are the half-widths of distributions along the x and y axes. The obtained results can be easily generalised to the case of the negative curvature of the gain profile (A2.2).

Consider a ring optical resonator in which a gas-discharge tube of length H with Brewster windows is placed and an aperture with the Gaussian transmission profile

$$T(x,y) = \exp\left(-\frac{x^2}{a_x^2} - \frac{y^2}{a_y^2}\right)$$

is mounted in the section z_t, where a_x and a_y are the half-width of the aperture along the x and y axes. This model is chosen because it gives a simple analytic description of resonator eigenmodes in the single-mode approximation since transverse mode losses increase with the mode index. Our numerical calculations of the resonator eigenmodes with a Gaussian aperture are in good qualitative agreement with calculations performed for resonators with mirrors of a finite size. We do not consider here frequency locking and assume that backward reflection from the resonator elements is absent. Depolarisation effects in such resonators are small and radiation proves to be plane polarised, which allows the use of the scalar model of the field in the resonator.

The problem of the eigenmodes of a resonator containing a Gaussian aperture and a linear quadratic inhomogeneous medium can be solved in the explicit form. The relevant equations have the form

$$(\Delta + k_j^2)E_j(x,y,z) = -4\pi k_j^2 P_j(x,y,z), \tag{15}$$

where Δ is the Laplace operator. These equations are supplemented by the conditions of transformation of the fields of the waves propagating through the resonator elements and by the conditions of their reproducibility after each round trip in the resonator:

$$E_j(x,y,z+L) = E_j(x,y,z). \tag{16}$$

We will seek the solution of Eqn (15) in the quasi-optical approximation in the form

$$E_j(x,y,z) = E_{0j}\psi_j(x,y,z)\exp[\mathrm{i}k_j\vartheta_j(z)] + \text{c.c.}, \tag{17}$$

where E_{0j} is the constant amplitudes of counterpropagating waves; $\vartheta_j(z)$ and $\psi_j(x,y,z)$ are slowly varying functions of coordinates. Let us substitute (17) into (15) and perform the corresponding transformations. Then, according to the parabolic equation method [42], we pass to the dimensionless variables $x \to x(k/L)^{1/2}$, $y \to y(k/L)^{1/2}$, and $z \to z/L$ and equate coefficients at the successive powers k to zero. In the principal order, we obtain the eikonal equation

$$\frac{\partial\vartheta_j}{\partial z} = \frac{\partial\vartheta_1}{\partial z} = 1 + 2\pi\varkappa_0 = n_z^{(1)}. \tag{18}$$

The first-order terms in k give equations for slowly varying wave amplitudes $\psi_j(x,y,z)$:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2\mathrm{i}n_z^{(1)}\frac{\partial}{\partial z} + n_x^{(1)}x^2 + n_y^{(1)}y^2\right]\psi_j(r) = 0. \tag{19}$$

Here,

$$n_p^{(1)} = -\pi\varkappa_0\left(\frac{2L}{d_p}\right)^2 = 2K_0 LZM_p \quad (p = x, y) \tag{20}$$

are the transverse components of the refractive index of the medium; $M_p = L/(kd_p^2)$ is the dimensionless parameter of the transverse inhomogeneity of the medium; $Z_\mathrm{r} = Z_\mathrm{l} = Z$. The real part $n_p^{(1)\prime}$ determines the focusing properties of the medium. The action of the imaginary part $n_p^{(1)\prime\prime}$ can be compared to the action of an aperture with the quadratic transmission law.

The solution of differential equation (19) with complex coefficients $n_z^{(1)}$ and $n_p^{(1)}$ is well-known Hermite–Gaussian functions. The fundamental $\mathrm{TEM}_{00\tilde{q}}$ resonator mode is described by the Gaussian beam with two symmetry planes:

$$\psi_j(x,y,z) = \psi_{xj}(x,z)\psi_{yj}(y,z). \tag{21}$$

By using the formalism of wave matrices, we represent the expression for $\psi_{xj}(x,z)$ in the form

$$\psi_{xj}(x,z) = \frac{1}{[m_{xj}(z)]^{1/2}}\exp\left[\frac{\mathrm{i}x^2}{2q_{xj}(z)}\right]. \tag{22}$$

A similar expression can be also written for the yz plane. Here, the dimensionless parameters $q_{pj}^{-1}(z) = S_{pj}(z) + \mathrm{i}W_{pj}(z) = s_{pj}(z)L + 2\mathrm{i}L/[kw_{pj}^2(z)]$ are introduced, which characterise the wave-front curvatures s_{pj} and half-widths of the counterpropagating Gaussian beams in the section z. These parameters are transformed during the propagation of beams in the resonator by the rules

$$q_{pj}(z) = \frac{a_{pj}(z) + b_{pj}(z)q_{pj}(0)}{c_{pj}(z) + d_{pj}(z)q_{pj}(0)}. \tag{23}$$

The parameters m_{pj} are transformed as

$$m_{pj}(z) = a_{pj}(z) + b_{pj}(z)q_{pj}^{-1}(0). \tag{24}$$

Here, $q_{pj}(0)$ are the values of the parameters q_{pj} in the reference section z_0; $a_{pj}(z)$, $b_{pj}(z)$, $c_{pj}(z)$, and $d_{pj}(z)$ are the elements of the matrix of transformation of the beam parameters of the optical system located between planes z_0 and z. The wave matrices describing the evolution of the Gaussian beam propagating in the resonator are presented in Appendix 3.

To find $q_{p\mathrm{r}}$ in the section z, it is necessary to find the cavity-round-trip matrix $A_pB_pC_pD_p$ as the product of the wave matrix of individual elements through which the light beam propagates. Because the refractive indices of the medium are identical for counterpropagating waves, we find the matrix of the resonator for the counterpropagating wave by interchanging elements A_p and D_p. Then, taking condition (16) into account, we find from relations (32) the expressions for parameters $q_{p\mathrm{r},p\mathrm{l}}^{-1}$ in the resonator section z:

$$q_{pr,pl}^{-1}(z) = \pm\frac{D_p(z) - A_p(z)}{2B_p(z)} + \mathrm{i}\left[\frac{1-G_p^2}{B_p^2(z)}\right]^{1/2}. \quad (25)$$

The G_p parameter of the resonator in (25) defined as $G_p = G_{pr} = G_{pl} = (A_p + D_p)/2$, in the ideal resonator, where the elements of the $A_{0p}B_{0p}C_{0p}D_{0p}$ matrix are real, is reduced to the real parameter $G_p = g_p$ of the resonator configuration. In our case, the G_p parameter is a complex frequency-dependent quantity [see expression (A3.7)]. The distributions of the wave fields also become frequency-dependent. By representing (25) in the form

$$q_{pr,pl}^{-1}(z) = \pm S_{0p}(1 + U_{p1}) + \mathrm{i}W_{0p}(1 + U_{p2}),$$

where the values of W_{0p} and S_{0p} correspond to the ideal resonator and quantities $U_{p1,p2} = U'_{p1,p2} + \mathrm{i}U''_{p1,p2}$ determine the perturbation of field distributions by the medium and aperture, we obtain that the reciprocal (the same for counterpropagating beams) deformation of field distributions is determined by the values $\delta W_p = W_{0p}U'_{p2}$ and $\delta S_p = S_{0p}U'_{p1}$, while the nonreciprocal components have the form

$$\Delta W_p = \frac{W_{pr} - W_{pl}}{2} = S_{0p}U''_{p1},$$
$$\Delta S_p = \frac{S_{pr} + S_{pl}}{2} = -W_{0p}U''_{p2}. \quad (26)$$

In real laser systems, where an aperture is used for mode selection, it is the aperture that mainly contributes to the nonreciprocity of the transverse distributions of the fields. The expressions for these nonreciprocity components can be written in the form

$$\Delta W_p(z) = N_p\frac{B_{0p}^2(z_t) - (z - z_t)^2}{B_{0p}^2(z)}$$
$$= N_p\left[\frac{w_{0p}^2(z_t)}{w_{0p}^2(z)} - \frac{(z - z_t)^2}{B_{0p}^2(z)}\right], \quad (27)$$

from which it follows that the deformation of field distributions is determined to a great extent by the resonator geometry. In this case, the nonreciprocity ΔW_p is maximal in the vicinity of the aperture and decreases with distance from the aperture. The transverse inhomogeneity of the active medium makes an additional contribution to the deformation of the field distributions of counter-propagating waves, and although this contribution is generally considerably smaller than distortions introduced by the aperture, the influence of the medium should not be neglected.

The wave amplitudes (22) are reproduced after the round trip in the resonator accurate to the propagation constants. The expressions for them are obtained by substituting (25) into (24):

$$\Lambda_p = \Lambda_{pr} = \Lambda_{pl} = [m_p(z+1)]^{-1/2}$$
$$= \exp\left\{-\frac{1}{2}\ln\left[G_p + (G_p^2 + 1)^{1/2}\right]\right\} = \exp\left(-\frac{\mathrm{i}}{2}\arccos G_p\right).$$

Taking this relation into account, the condition for the periodicity of field (16) has the form

$$\mathrm{i}\varepsilon_0 + \frac{1}{2}\sum_{p=x,y}\arccos G_p + kL\int_0^1 n_z^{(1)}(z)\,\mathrm{d}z = 0. \quad (28)$$

Because the replacement of one of the highly reflecting mirrors by a semitransparent mirror does not change the transverse structure of the fields, we introduced losses ε_0. The choice of the sign in the expansion

$$\arccos(G'_p + \mathrm{i}G''_p) = -\left[\arccos\left(\frac{G'_p}{\sigma_p}\right) + 2\pi\tilde{q} - \mathrm{i}\,\mathrm{arcosh}\,\sigma_p\right]$$

is determined by the fulfilment of the condition of the field weakening after the round trip in the resonator, where $\mathrm{arcosh}\,\sigma_p > 0$, and

$$\sigma_p = \frac{1}{2}\left\{\left[(1 + G'_p)^2 + (G''_p)^2\right]^{1/2} + \left[(1 - G'_p)^2 + (G''_p)^2\right]^{1/2}\right\}.$$

The imaginary part of Eqn (28) gives the threshold gain $K_{0\mathrm{th}}H$

$$K_{0\mathrm{th}}HZ'' = \varepsilon_0 + \frac{1}{2}\sum_{p=x,y}\mathrm{arcosh}\,\sigma_p. \quad (29)$$

The real part of (28) determines the frequency for which condition (29) is fulfilled:

$$\omega_{\mathrm{th}} = \frac{c}{L}\left[\frac{1}{2}\sum_{p=x,y}\arccos(G'_p/\sigma_p) + 2\pi\tilde{q} + K_{0\mathrm{th}}HZ'\right]. \quad (30)$$

Let us analyse the obtained dependences for a weakly perturbed resonator, by using the parameters of the Gaussian aperture (N_p) and transverse inhomogeneity of the medium ($M_p \sim n_p^{(1)}$) as small parameters, which are the quantities inverse to the effective Fresnel numbers. We assume that

$$|n_p^{(1)}| < N_p < 1. \quad (31)$$

In addition, we restrict out consideration to resonators in which conditions $(1 - G'_p)^2 \gg (G''_p)^2$ and $(1 + G'_p)^2 \gg (G''_p)^2$ are fulfilled simultaneously. Then, by using relation (A3.9), we obtain the expression

$$\varepsilon^{(1)} = \frac{1}{2}\sum_{p=x,y}\mathrm{arcosh}\,\sigma_p = \varepsilon_N + K_0H$$
$$\times\sum_{p=x,y}\frac{w_{0p}^2(z_0)}{4d_p^2}\left\{Z''\left[1 + \frac{w_{0p}^2(z_t)}{2a_p^2}\right]\mu_{1p} + Z'\frac{w_{0p}^2(z_t)}{2a_p^2}\mu_{2p}\right\} \quad (32)$$

for the wave losses, where μ_{1p} and μ_{2p} are the parameters of the resonator configuration, which are presented in Application 3. The first term is responsible for losses on the Gaussian aperture:

$$\varepsilon_N = \frac{1}{4}\sum_{p=x,y}\frac{N_p}{W_{0p}(z_t)}\left[1 + \frac{N_p}{2W_{0p}(z_t)}\right]$$
$$= \frac{1}{2}\sum_{p=x,y}\left[\frac{w_{0p}(z_t)}{2a_p}\right]^2\left\{1 + \left[\frac{w_{0p}(z_t)}{2a_p}\right]^2\right\}. \quad (33)$$

Two other terms in (32) determine the frequency-dependent change in the losses for the fundamental resonator mode caused by the action of the quadratic inhomogeneity of the active medium. The quantity K_0HZ'', which is proportional to amplification and repeats the symmetric gain profile, is responsible for the gain reduction compared to the gain in a medium layer with the homogeneous distribution of the gain K_0H. The reduction of the gain can be related to the increase in mode losses. The term proportional to Z' is responsible for a change in losses on the aperture caused by the deformation of the transverse distribution of the field by a lens in the medium, and its frequency dependence has the dispersion shape. For $\omega < \omega_{\rm ab}$, the scattering lens of the medium increases the width of the field distribution, thereby increasing aperture losses. At the line centre for $\omega = \omega_{\rm ab}$, this term is zero, and for $\omega > \omega_{\rm ab}$, the lens of the medium operates as a collecting lens, thereby reducing losses.

Taking (29) and (32) into account, the expression for the threshold gain can be written in the form

$$K_{0\rm th}H = (\varepsilon_0 + \varepsilon_N)\left\{Z'' - \sum_{p=x,y}\left[\frac{w_{0p}(z_0)}{2d_p}\right]^2 \times\left[\mu_{1p}Z'' + \mu_{2p}Z'\frac{w_{0p}^2(z_{\rm t})}{2a_p^2}\right]\right\}^{-1}. \quad (34)$$

By using expression (A3.10), we obtain from (30) the expression for detuning of the threshold frequency from the eigenmode of the ideal resonator $\Omega = (c/L)[(1/2)\times\sum_{p=x,y}\arccos g_p + 2\pi\tilde{q}]$:

$$\omega_{\rm th} - \Omega = \Omega_N + \frac{c}{L}K_{0\rm th}H \times\left\{Z'\left[1 - \sum_{p=x,y}\frac{w_{0p}^2(z_0)}{2d_p^2}\mu_{p1}\right] + Z''\sum_{p=x,y}\frac{w_{0p}^2(z_0)}{2d_p^2}\frac{w_{0p}^2(z_{\rm t})}{2a_p^2}\mu_{p2}\right\}. \quad (35)$$

The quantity

$$\Omega_N = \frac{c}{2L}\sum_{p=x,y}\left[\frac{w_{0p}(z_{\rm t})}{2a_p}\right]^4\frac{g_p}{(1-g_p^2)^{1/2}}$$

is responsible for the resonator frequency shift caused by a decrease in the mode volume due to diffraction from the aperture.

One can see from (35) that the dispersion lens of the medium reduces the linear frequency locking (which is determined by the term proportional to Z'). Indeed, for $\omega < \omega_{\rm ab}$, the medium lens operates as a diverging lens, increasing the mode volume and thereby decreasing the resonator frequency, by shifting it from the frequency $\omega_{\rm ab}$. For $\omega > \omega_{\rm ab}$, the medium lens operates as a collecting lens, decreasing the mode volume and thereby increasing the resonator frequency. At the gain line centre for $\omega = \omega_{\rm ab}$, the optical power of the lens is zero. The asymmetric shift of the active mode appears due to the combined action of two apertures: the Gaussian aperture and frequency-dependent aperture formed by the medium. This shift is proportional to Z''. This means that the shift is maximal when the resonator frequency Ω is tuned to the line centre.

Thus, we have constructed the solution of the problem: we have found the distributions of the fields of counter-propagating modes in an arbitrary section of the resonator and also losses and frequency detunings at the lasing threshold. We have established that the combined action of the transverse inhomogeneity of the medium and aperture distorts the lasing region [asymmetry of losses $\varepsilon^{(1)}$ (32) and threshold frequencies $\omega_{\rm th}$ (35) of the resonator mode], which was observed in [11, 13, 22, 23]. The asymmetry of the lasing region [proportional to the parameter μ_{2p} (A3.12)] can be minimised by choosing the optimal mutual arrangement of resonator elements and radii of curvature of resonator mirrors. In addition, the asymmetry depends on the half-width a_p of the aperture: as the half-width is decreased from a_{1p} to a_{2p}, the asymmetry increases approximately by a factor of $(a_{2p}/a_{1p})^2$. The asymmetry also depends on the transverse inhomogeneity of the medium, which is determined by the discharge geometry and current, and by pressure in the tube with the active medium.

Our results, obtained by using the theory of resonators, contradict the results obtained in [27, 29]. Let us present expressions (14) for threshold frequencies and gains from [27] [they follow from Eqns (16) in [25]]:

$$\omega^{(1)} = \Omega + \frac{c}{L}\chi', \quad \varepsilon^{(1)} = \varepsilon_N + \chi'', \quad (36)$$

$$\chi = Z\frac{\int K(x,y)\psi_{\rm r}(x,y,z)\psi_{\rm l}(x,y,z){\rm d}V}{\int\psi_{\rm r}(x,z)\psi_{\rm l}(x,z){\rm d}V} = \chi' + {\rm i}\chi'', \quad (37)$$

where V is the resonator volume. The dependence of the gain K on transverse coordinates is determined by function (A2.1). After corresponding algebraic transformations of (37), taking into account that the wave matrices are unimodular, we obtain

$$\chi = K_{0\rm th}HZ\left[1 + \frac{1}{h}\sum_{p=x,y}M_p\int_{-h/2}^{h/2}(q_{p\rm l}^{-1} + q_{p\rm r}^{-1})^{-1}{\rm d}z\right]. \quad (38)$$

By comparing expressions (36)–(38) with the corresponding formulas of our paper, it is easy to see the discrepancy between them. The dependences of threshold gains and frequencies on the transverse number of the mode are also different. The nonlinear problem was solved in papers [24–30] by multiplying the parabolic equation for the slowly varying amplitude of one of the waves by the amplitude of the other wave and then integrating over the resonator volume. In this case, the wave frequencies and intensities were determined by neglecting the periodicity of the field, the transverse field distributions in the resonator were found by neglecting the influence of the active medium, and losses for counterpropagating waves were assumed equal. It will be shown below that it is the additional deformation of the transverse field distributions of counterpropagating waves caused by the nonlinear medium that leads to different losses for counterpropagating waves.

5. Nonlinear theory

The eigenmode problem for a ring optical resonator with the active gas medium is solved in the weak-nonlinearity approximation. In this case, we can assume that the nonlinearity of the medium will not change the general type of the solution of Eqn (15) obtained in the first approximation. The nonlinear polarisation of the medium calculated in the weak-saturation approximation has the form

$$2\pi P_{\rm r}(x,y,z) = -\frac{1}{k_{\rm r}} K(x,y) \times [Z_{\rm r} - \beta_{\rm r}\mathcal{I}_{\rm r}(x,y,z) + \theta_1\mathcal{I}_1(x,y,z)]E_{\rm r}(x,y,z). \quad (39)$$

This expression shows that the spatial inhomogeneity of the nonlinear medium in the field of Gaussian beams is caused both by the transverse inhomogeneity of the unsaturated gain $K(x,y)$ in the medium and the spatial inhomogeneity of intracavity fields. Indeed, we have for Gaussian beams

$$\mathcal{I}_j(x,y,z) = I_{zj}(z)\exp\left[-\frac{2x^2}{w_{xj}^2(z)} - \frac{2y^2}{w_{yj}^2(z)}\right], \quad (40)$$

where $I_{zj} = I_j f_j(z)$ is the intensity of the jth wave on the resonator axis; I_j is defined by expression (A1.2); and the parameter

$$f_j = |m_{xj}(z)m_{yj}(z)|^{-1}\exp\left[-2k_j\int_0^z n''_{zj}(t)\mathrm{d}t\right]$$

describes the evolution of the beam along the resonator axis. By using the parabolic approximation, we represent the intensities of counterpropagating Gussian beams as

$$\mathcal{I}_j(x,y,z) = I_{zj}(z)\left[1 - \frac{2x^2}{w_{xj}^2(z)} - \frac{2y^2}{w_{yj}^2(z)}\right]$$

and write the expression for the nonlinear polarisation of the medium (39) in the form

$$2\pi P_{\rm r}(x,y,z) = -\frac{1}{k_{\rm r}}K_0\Big\{Z_{\rm r} - \beta_{\rm r}I_{z\rm r}(z) - \theta_1 I_{z1}(z) + \sum_{p=x,y}[Z_{\rm r}M_{p\rm r} - \beta_{\rm r}\Phi_{p\rm r}(z)I_{z\rm r}(z) - \theta_1\Phi_{p1}(z)I_{z1}(z)]p^2\Big\}, \quad (41)$$

where $\Phi_{pj}(z) = M_p + W_{pj}(z)$ are the dimensionless parameters of the transverse inhomogeneity of the nonlinear medium.

Let us substitute expression (41) into Eqn (15) and perform the corresponding transformation taking into account that the quantities I_{zj} to be determined are slowly varying functions of the coordinate z. Then, we obtain from (15)–(17) the equations for the refractive indices of the nonlinear active medium on the resonator axis

$$n_{z\rm r}(z) = 1 - \frac{1}{k_{\rm r}}K_0[Z_{\rm r} - \beta_{\rm r}I_{z\rm r}(z) - \theta_1 I_{z1}(z)] \quad (42)$$

and slowly varying amplitudes of counterpropagating waves

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \pm 2\mathrm{i}n_{zj}(z)\frac{\partial}{\partial z} + n_{xj}(z)x^2 + n_{yj}(z)y^2\right]\psi_j(r) = 0, \quad (43)$$

where

$$n_{p\rm r}(z) = 2K_0H[M_pZ_{\rm r} - \beta_{\rm r}\Phi_{p\rm r}(z)I_{z\rm r}(z) - \theta_1\Phi_{p1}(z)I_{z1}(z)]. \quad (44)$$

Equations for $n_{z1}(z)$ and $n_{p1}(z)$ are obtained from Eqns (42) and (44), respectively, by the index interchange $\rm r \leftrightarrow l$. Equation (43) with variable coefficients cannot be solved in the general case. However, when variations in the refractive indices $n_j(x,y,z) = n_{zj}(z) + n_{xj}(z)x^2 + n_{yj}(z)y^2$ of the medium at the wavelength are small, the propagation of optical beams in the inhomogeneous medium can be efficiently studied by the method of variable *abcd* matrices with slowly varying elements [43] (see Appendix 3).

Thus, we still seek the solution of Eqns (43) with coefficients slowly varying with z in the form of Gaussian beams (21) and (22). We repeat the procedure of solving the eigenmode problem for a resonator with a linear medium. To determine the eigenmodes of a ring resonator with a nonlinear medium, it is necessary to solve the system of six equations: two equations (43) for determining the transverse field distributions of counterpropagating waves and four real equations for lasing intensities and frequencies, which follows from the rate equations for the phases and amplitudes of counterpropagating waves:

$$\mathrm{i}\varepsilon_0 + \frac{1}{2}\sum_{p=x,y}\arccos G_{pj} + k_jL\int_0^1 n_{zj}(z)\mathrm{d}z = 0. \quad (45)$$

Because this system cannot be solved analytically in the general case, we used computer calculations. However, by using some approximations, we obtained basic expressions in the visible form and explained basic features of the behaviour of lasing frequencies and intensities obtained numerically, and elucidated thereby the physical mechanism of nonreciprocity. For this purpose, we used the 'short tube' approximation. This approximation assumes that variations in $n_{zj}(z)$, $W_{pj}(z)$, and $n_{pj}(z)$ over the tube length $h = H/L$ along the z axis are negligibly small, and these functions can be replaced by their values in the section z_0 and we can use the known matrix [44] for a transversely inhomogeneous medium. In addition, we restricted our consideration to the case of small parameters perturbing the resonator (31). In these approximations, by using (A3.9), we obtain expressions

$$\varepsilon_{\rm r} = \varepsilon_{\rm r}^{(1)} - K_0H\sum_{p=x,y}[(\beta''_{\rm r}I_{z\rm r}\Phi_{p\rm r} + \theta''_1I_{z1}\Phi_{p1})W_{0p}^{-1}\mu_{1p} + (\beta'_{\rm r}I_{z\rm r}\Phi_{p\rm r} + \theta'_1I_{z1}\Phi_{p1})W_{0p}^{-1}N_p\mu_{2p}] \quad (46)$$

for losses for counterpropagating waves, where $\varepsilon_{\rm r}^{(1)}$ is the frequency-dependent resonator losses calculated in the linear approximation.

From the conditions of the equality of the saturated gain of each of the wave to its losses $\varepsilon_{\rm r}$ (46), which follow from Eqns (45), we obtain the intensities of counterpropagating waves and represent them in the form $I_{z\rm r} = I_z + \Delta I_z$ and $I_{z1} = I_z - \Delta I_z$, where

$$I_z = \eta\frac{1}{(\beta'' + \theta'')\upsilon_1 - (\beta' + \theta')\upsilon_2}; \quad (47)$$

$$\Delta I_z = \eta\upsilon_1^{-2}\sum_{p=x,y}\frac{\Delta W_p}{W_{0p}} \times \frac{(\beta'' - \theta'')\mu_{1p} - (\beta' - \theta')\mu_{2p}N_p}{(\beta''\upsilon_1 - \beta'\upsilon_2)^2 - (\theta''\upsilon_1 - \theta'\upsilon_2)^2}; \quad (48)$$

$$\upsilon_1(z_0) = 1 - \sum_{p=x,y} \mu_{1p}\Phi_p(z_0)/W_{0p}(z_0);$$

$$\upsilon_2(z_0) = \sum_{p=x,y} \mu_{2p}N_p\Phi_p(z_0)/W_{0p}(z_0);$$

$$\eta = \frac{\eta_r + \eta_l}{2}; \quad \eta_j = \frac{\alpha_j - \varepsilon_j^{(1)}}{K_0 H}$$

is the relative excess of the unsaturated gain $\alpha_j = K_0HZ_j''$ over linear losses $\varepsilon_j^{(1)}$ (32); $\Phi_p = (\Phi_{pr} + \Phi_{pl})/2 = M_p + W_p$; and $W_p = (W_{pr} + W_{pl})/2$.

One can see from (47) that the total intensity $2I_z$ of counterpropagating waves has a dip near the gain line centre. The 'saturation aperture' reduces losses, thereby reducing the dip depth (through the quantity proportional to υ_{1p}). The term proportional to υ_{2p} is responsible for the asymmetry of the dip related to the asymmetric character of losses.

By substituting expressions for wave intensities into (46), we obtain that losses of the waves in a resonator with a nonlinear medium and an aperture are different in the general case: $\varepsilon_{r,l} = \varepsilon \pm \Delta\varepsilon$, where

$$\varepsilon = \varepsilon^{(1)} - K_0HI_z \sum_{p=x,y} \frac{W_p}{W_{0p}}[(\beta'' + \theta'')\mu_{1p} + (\beta' + \theta')\mu_{2p}N_p]; \quad (49)$$

$$\Delta\varepsilon = -K_0HI_z \sum_{p=x,y} \frac{\Delta W_p}{W_{0p}}(1 + \rho_p) \times [(\beta'' - \theta'')\mu_{1p} + (\beta' - \theta')\mu_{2p}N_p]; \quad (50)$$

$$\rho_p = \frac{W_p}{W_{0p}} \frac{(\beta'' - \theta'')\mu_{1p} + (\beta' - \theta')\mu_{2p}N_p}{(\beta'' - \theta'')\upsilon_{1p} - (\beta' - \theta')\upsilon_{2p}}. \quad (51)$$

The real parts of conditions (45) give equations for lasing frequencies, from which, taking (47) and (48) into account, we obtain the expression for their difference $\Delta\omega = \Delta\omega^{(1)} + \Delta\omega^{(3)}$, where the nonlinear component is

$$\Delta\omega^{(3)} = (\alpha - \varepsilon^{(1)})\frac{c}{2L}\sum_{p=x,y} N_p\mu_{2p}\upsilon_1\frac{\Delta W_p}{W_{0p}} \times \frac{(\beta' - \theta')^2 + (\beta'' - \theta'')^2}{(\beta''\upsilon_1 - \beta'\upsilon_2)^2 - (\theta''\upsilon_1 - \theta'\upsilon_2)^2}; \quad \alpha = (\alpha_r + \alpha_l)/2. \quad (52)$$

The violation of symmetry in the location of resonator elements gives rise to the amplitude and phase nonreciprocity, the amplitude nonreciprocity being dominant in a laser with the aligned resonator. Indeed, the difference of losses for counterpropagating waves (50) is the correction of the first-order smallness in N_p to the parameters of counterpropagating Gaussian beams unperturbed by diffraction: $\Delta\varepsilon = O(N_p)$ because $\Delta W_p = O(N_p)$ (27). The frequency difference of the waves (52) proves to be the quantity of a higher smallness order: $\Delta\omega^{(3)} = O(N_p^2)$. Note that in a passive resonator with a Gaussian aperture, the aperture losses for the resonator mode are represented by the first-order quantity, whereas frequency shifts caused by the aperture are represented by the quantities of the second-order smallness in the parameter N_p.

The correctness of estimates that can be made from expressions (47)–(50) is confirmed by numerous computer calculations, which were performed for resonators of different configurations (three- and four-mirror resonators). The choice of the radii of mirrors was dictated only by the fulfilment of the condition of the resonator stability.

The inequality of losses can be easily explained by considering the action of a system consisting of an aperture and a transversely inhomogeneous nonlinear medium. The nonlinear components of the optical power of a lens and diaphragming properties of the medium, which cause the additional nonreciprocal deformation of the fields of counterpropagating waves, are proportional to the parameter W_pj of the transverse inhomogeneity of the medium. The inequality of field distributions of counterpropagating waves over the length of a tube with the nonlinear medium caused by the action of the aperture is the reason for difference in the aperture and optical power of the medium lens for counterpropagating waves. The counterpropagating beams falling on the aperture after propagation through such an 'anisotropic' medium acquire different losses. Figure 6 demonstrates good correlation between the nonreciprocal change $\delta w_x = \Delta w_x(z_t + 0) - \Delta w_x(z_t - 0)$ of transverse field distributions of counterpropagating waves on the aperture (here, $\Delta w_x = [w_{xr}(z_t + 0) - w_{xl}(z_t - 0)]/2$) and the difference of the x components of their losses $\Delta\varepsilon_x = (\varepsilon_{xr} - \varepsilon_{xl})/2$ for different dimensionless detunings $\xi = (\omega - \omega_{ab})/(ku)$ and positions of the aperture. In this case, the nonreciprocity of losses appears mainly due to different diaphragming properties of the nonlinear medium. The corresponding term in (50), proportional to the difference $\beta'' - \theta''$, is an even function of the frequency detuning $\omega - \omega_{ab}$. The quantity proportional to $\beta' - \theta'$ determines the asymmetric dependence of the nonlinear component of losses on the detuning from the central frequency of the transition: the 'saturation lens' changes the transverse field distributions of counterpropagating waves according to the dispersion law, and as a result, the aperture losses acquire the additional component, which is an odd function of the detuning.

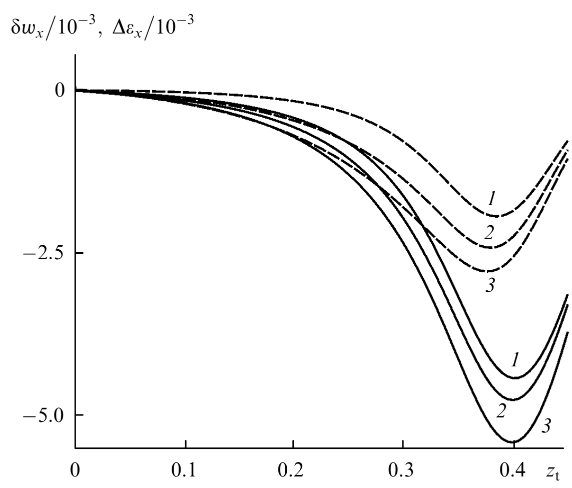

Figure 6. Dependences of the nonreciprocal changes δw_x of the transverse distributions of fields of counterpropagating waves (dashed curves) and the difference $\Delta\varepsilon_x$ of their losses (solid curves) on the position z_t of the aperture in the resonator for dimensional detunings $\xi = 0.1$ (*1*), 0 (*2*), and −0.1 (*3*); $g_x = -0.99$, $K_0H = 0.5$, $z_0 = 0.5$, $d_x = d_y = 1$ mm.

In this case, losses depend in a complicated way on the resonator geometry, parameters of the active medium, and detuning of the lasing frequency from the central transition frequency. When the role of diffraction is negligible ($N_p = 0$), losses are a symmetric function of detuning. As the influence of diffraction increases, the asymmetry of nonlinear components of losses also increases. The sign and value of the asymmetry of losses are determined by the relation between the linear and nonlinear parts of transverse components of the refractive index of the medium (44).

The dominant role of the mechanism of loss nonreciprocity determines the behaviour of the amplitude and frequency nonreciprocities of counterpropagating waves (Fig. 7): the intensity difference is an even function of the laser frequency detuning from the transition frequency and changes its sign only within the strong-coupling region (Fig. 7a); the frequency difference is an odd function of detuning, as shown in Fig. 7b.

The amplitude ($\Delta\mathcal{I}$) and frequency ($\Delta\omega$) nonreciprocities are maximal if the aperture is located near a cell with the active medium. Figure 8 presents the dependences of losses and intensities of counterpropagating waves when the aperture is mounted directly behind the cell. In this case, the increase in the loss nonreciprocity leads to the suppression of one of the waves in a broad detuning range, as was observed experimentally [11]. For certain values of parameters, a drastic change in saturated losses at the lasing region boundary can lead to the jump-wise increase in the radiation intensity at this boundary. Such an asymmetric jump is determined by the resonator geometry and increases with increasing the ratio $W_{pj}/W_{0p} = (w_{0p}/w_{pj})^2$. Such effects were experimentally observed in ring lasers in [13]. Distortions of the lasing region and the formation of the start jump of the radiation intensity in two-mirror lasers were investigated in [45]. By varying the aperture parameters and the resonator geometry, we can both decrease and increase the difference of losses of counterpropagating waves, thereby decreasing or increasing the region of unidirectional lasing. As the aperture is removed from the cell (along the beam), the loss nonreciprocity decreases monotonically and vanishes at some point. The passage through this point is accompanied by a change of signs of $\Delta\mathcal{I}$ and $\Delta\omega$, and their values again increase if the aperture approaches the other end of the cell. Due to the elimination of nonreciprocity, the two-wave regime exists over the entire lasing region, which was demonstrated experimentally in [11].

The calculations showed that the type of dependences of the radiation intensity and frequency difference on detuning is preserved upon varying the radius of curvature R of a spherical mirror and the aperture size a_p. Because the frequency and amplitude nonreciprocity of counterpropagating waves is a nonlinear effect, it is clear why this effect is small in the case of small apertures – in this case, losses are high and nonlinearity is correspondingly small. As the aperture is increased up to a certain value, nonreciprocity rapidly increases, and then, when the aperture exceeds the Gaussian beam width, it begins to decrease.

The dependences of radiation intensities on detuning are

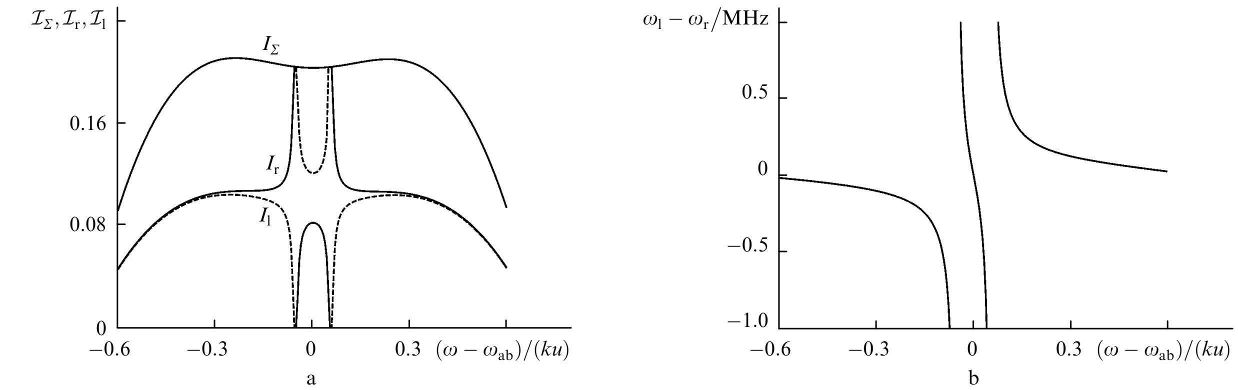

Figure 7. Dependences of the dimensionless intensities $\mathcal{I}_r$, $\mathcal{I}_l$, their sum $\mathcal{I}_\Sigma$ (a) and the frequency difference $\omega_l - \omega_r$ (b) of counterpropagating waves on detuning for $g_x = 0.038$, $a_x = a_y = 1.2$ mm, $d_x = d_y = 1$ mm, $z_0 = 0.125$, $z_t = 0.625$, $\varepsilon_0 = 0.1$, and $KH_0 = 0.3$.

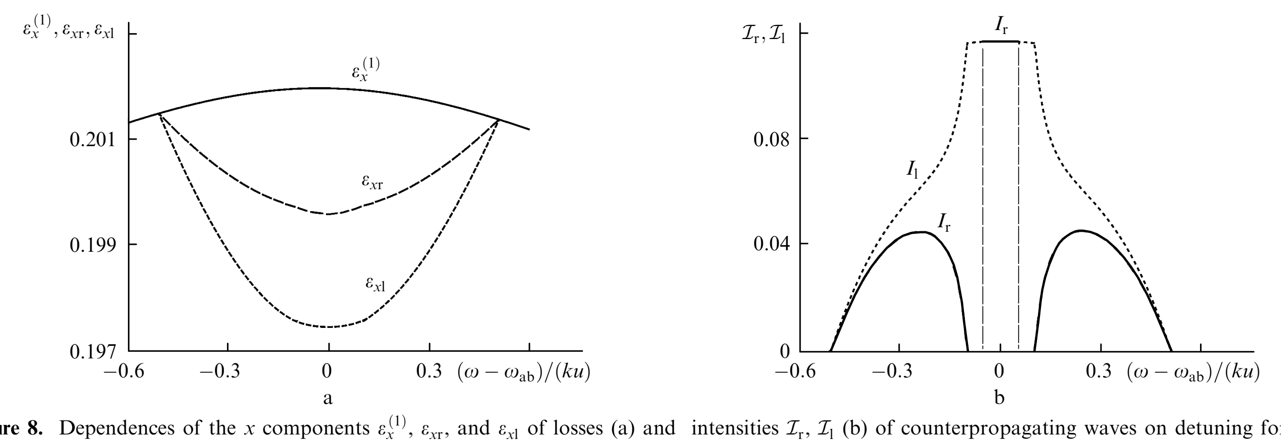

Figure 8. Dependences of the x components $\varepsilon_x^{(1)}$, ε_{xr}, and ε_{xl} of losses (a) and intensities $\mathcal{I}_r$, $\mathcal{I}_l$ (b) of counterpropagating waves on detuning for $g_x = 0.038$, $a_x = a_y = 1$ mm, $z_0 = 0.125$, $z_t = 0.15$, $\varepsilon_0 = 0.3$, and $KH_0 = 0.6$.

considerably different for the positive and negative gain profiles $K(x, y)$ [see expressions (A2.1) and (A2.2)], this is manifested most distinctly for small values of d_p. For these profiles, corrections to unsaturated losses due to the transverse inhomogeneity of the medium have opposite signs. Saturated losses are also different. In the case of the positive profile $K(x, y)$, saturation is proportional to the sum of parameters of the transverse inhomogeneity $W_{pj} + M_p$, while for the negative profile – to their difference $W_{pj} - M_p$. This leads to different dependences of losses, phase shifts, and, therefore, wave intensities on detuning. As a rule, due to refractive-index saturation, losses at the boundary of the lasing region decrease jump-wise. The exception is resonators with very narrow gas-discharge tubes for which the curvature of the gain profile $K(x, y)$ of the medium is negative for some or other reasons. Here, the resonator-mode losses can increase at the lasing-region boundary. Note that, as follows from the data presented in Appendix 2, the dependence $K(x, y)$ can change during lasing. This can change the type of asymmetry of the radiation intensity profile. Calculations performed for a laser operating at the $^{20}Ne - ^{22}Ne$ isotope mixture showed that the type of nonreciprocity is preserved in this case as well.

The behaviour of the intensities of counterpropagating waves within the strong-coupling region will be studied elsewhere. Here, we point out that the shape of intensity peaks of counterpropagating waves within the strong-coupling region is determined to a great extent by the difference of losses for these waves. When the loss difference is small, the boundaries of the regions of lasing for both waves virtually coincide with the boundaries of the strong-coupling region (Fig. 7a). As the loss difference increases, the wave with higher losses is still generated in the entire strong-coupling region; in this case, the frequency interval in which the wave with lower losses is generated within the strong-coupling region can decrease to zero (Fig. 8b).

6. Phase nonreciprocity of counterpropagating waves

Let us calculate numerically the intensities and frequencies of counterpropagating waves in a resonator similar to that used in experiments [13] and [22]. A tube with the active medium was placed symmetrically with respect to a spherical mirror into a three-mirror resonator with one spherical and two plane mirrors. Two apertures with coordinates z_{t1} and z_{t2} were located symmetrically with respect to the ends of the gas-discharge tube whose centre was located at the section $z_0 = 0.5$. When the apertures had the same size, the frequencies and intensities of counterpropagating waves were equal. Figure 9 presents the dependences of the sum $\mathcal{I} = 2\mathcal{I}_j$ of intensities $\mathcal{I}_r = \mathcal{I}_l$ of counterpropagating waves on detuning in a ring laser. The upper curve corresponds to a large width $d = d_x = d_y$ (A2.1) of the radial distribution of the unsaturated gain. One can see that the dependence of the radiation intensity on detuning is asymmetric. The intensity of the waves in the low-frequency detuning region ($\omega - \omega_{ab} < 0$) is higher than that in the high-frequency region ($\omega - \omega_{ab} > 0$). In paper [22], where a wide gas-discharge tube was used, so that the width of the transverse distribution of the unsaturated gain was rather large, the dependence of the radiation intensity on detuning was similar. Here, the asymmetry of losses (49) and, hence, the asymmetry of intensity (47) appears mainly due to the field-induced transverse inhomogeneity of the nonlinear medium. As d decreases, the dependence of the radiation intensity on detuning becomes virtually symmetric with respect to frequency ω_{ab} (dashed curve). In this case, the action of the transverse inhomogeneity of the linear gain $K(x, y)$ is compensated by the field-induced inhomogeneity. Such a situation was realised experimentally in a laser with a sufficiently narrow gas-discharge tube [13]. As d further decreases, the inhomogeneity of the gain $K(x, y)$ becomes dominant. In this case, the intensity of the waves in the low-frequency detuning region ($\omega - \omega_{ab} < 0$) is smaller than that in the high-frequency region ($\omega - \omega_{ab} > 0$) (lower curve). Figure 9 demonstrates the asymmetry of the lasing region with respect to ω_{ab}, which increases with increasing the inhomogeneity parameter of the medium (with decreasing d).

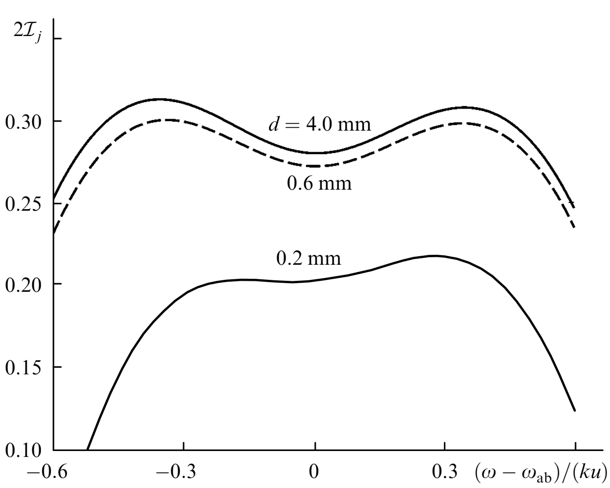

Figure 9. Dependences of the sum of intensities $\mathcal{I}_r = \mathcal{I}_l$ of counterpropagating waves on detuning for the positive gain profile; $a_{1p} = a_{2p} = 1$ mm, $z_0 = 0.5$, $z_{t1} = 0.2$, $z_{t2} = 0.8$, $R = 1.2$ m, $\varepsilon_0 = 0.1$, $K_0 H = 0.3$.

When the resonator was misaligned either by displacing the aperture [13, 22] or rotating mirrors [13], the dominant role of the phase nonreciprocity was manifested (see Figs 4 and 5). Because such a misalignment does not change significantly the type of nonreciprocity of transverse field distributions of counterpropagating waves [46], it is necessary to find other sources of the phase nonreciprocity appearing due to misalignment.

Theoretical calculations [47] confirmed experimentally [47–50] have shown that the main source of the phase nonreciprocity is the Langmuir drift of neutral excited neon atoms. The mechanism of the drift in a dc discharge is described in [47], where the drift velocity of neon atoms at low concentrations in the active medium was calculated. It was found that the atomic beam directed to an anode was localised near the gas-discharge tube walls, while the atomic beam directed to a cathode was localised near the tube axis. The drift velocities achieve 100 cm s^{-1} [50]. There also exist mechanisms competing with the drift such as cataphoresis, the dissociative recombination of molecular neon ions, etc. The authors of paper [49] determined the values of the difference frequency for counterpropagating waves in a ring laser caused separately by the atomic drift due to the Langmuir effect and the dissociative recombination of molecular neon ions. However, the fact that the type of nonreciprocity is also preserved upon high-frequency pumping of the discharge plasma [13] suggests the existence of other mechanisms producing the directional movement of excited atoms. In this connection it is of interest to consider the problem of separation of working gases in a gas-discharge tube due to thermal diffusion.

The mass transfer in a mixture of two different gases occurs both due to the concentration gradient of gases (concentration flow) and due to the temperature gradient (thermal-diffusion flow) [51]. The radial temperature distribution of the He – Ne mixture in a gas-discharge glass tube is usually described by the zero-order Bessel function of the first kind [52]. The experimental verification of this description [53] has shown that the temperature in the central part of the gas discharge was higher than that predicted by the theoretical distribution. Thus, the contraction of the discharge takes place in small-radius tubes, which results in a considerable increase in the temperature gradient. In a non-isothermal system containing a binary gas mixture, this leads to the separation of components, so that the heavier gas moves in the direction of the thermal flow, whereas the lighter gas moves in the direction of the temperature gradient [54]. This produces the concentration gradient in the initially homogeneous mixture [51, 55], resulting in the appearance of directional flows of the mixture components. The influence of the transverse atomic flows can be compensated by the preliminary alignment [13, 20 – 22], but it appears in the misaligned resonator.

The displacement of the aperture perpendicular to the resonator plane or the misalignment of mirrors in a plane-mirror resonator causes the displacement of the laser beam, and in a resonator with spherical mirrors – to the rotation of the resonator axis [41, 46]. In this case, the optical axis makes an angle with the tube axis (according to estimates of the authors of paper [13], this angle in their experiments was from 1 to 3 angular minutes), which results in the appearance of the nonzero component of the projection of velocity υ_z on the optical axis of the resonator, i.e. the appearance of a longitudinal flow. The direction of the flow is determined by the sign of the misalignment angle and the value – by the value of this angle and the temperature gradient in the tube. This explanation is confirmed by the difference between experimental data obtained in [13] and [22]. The increase in the diameter of a gas-discharge tube from 3 mm in [13] to 6 mm in [22], other parameters of resonators being the same, resulted in a decrease in the frequency splitting more than by a factor of thirty. The authors of paper [22] explain this difference by the fact that the radial inhomogeneity of the unsaturated gain in narrow tubes is manifested much stronger than in wide tubes (effect of a linear gas lens). However, our theoretical analysis showed that this effect cannot lead to the dominating role of the phase nonreciprocity. Measurements performed in [49] show that the difference frequency changes inversely proportionally to the discharge tube radius, i.e. in our case it can change only by a factor of four. We assume that the phase nonreciprocity decreases because the temperature gradient increases with decreasing the discharge tube cross section, and as result, the optical axis in a narrow tube will stronger deviate from the tube axis.

To take into account the misalignment of the resonator caused by the displacement of the aperture perpendicular to the resonator plane, we related it to certain values of the projection υ_z of the velocity of atoms on the optical axis z of the resonator. Now, by calculating the polarisation of a medium (see Appendix 1), we should make the replacement $\upsilon \to \upsilon - \upsilon_z$ in the Maxwell velocity distribution of atoms, by reducing it to the form $\mathcal{W}(\upsilon) = [1/(u\sqrt{\pi})]\exp[-(\upsilon - \upsilon_z)^2 \times u^{-2}]$, and transformations $\omega_j - \omega_{\rm ab} \to \omega_j \pm k\upsilon_z - \omega_{\rm ab}$ should be performed in the expressions determining the dependence of the polarisation of the medium in the fields of counterpropagating waves on detuning.

The degree of misalignment and, hence, the velocity υ_z depend on the position of the asymmetric aperture in the resonator and the degree of asymmetry of the aperture with respect to the optical axis, and more exactly – on the ratio $[\delta a_p w_{0p}^{-1}(z_{\rm t})]^2$, where δa_p is the displacement of the aperture centre from the optical axis. Let us explain this by the example of a three-mirror resonator with one spherical mirror, in which a tube with the active medium is located so that its centre coincides with the waist of the beam caustic in the resonator. If an asymmetric aperture is placed near the gas-discharge tube in the resonator, the inclination angle χ of the optical axis to the tube axis will be maximal. As the aperture is removed from the tube, the angle χ decreases and vanishes when the aperture is located on the spherical mirror (symmetrically with respect to the centre of the medium). As the aperture is further displaced, the angle changes its sign and its value increases as the aperture approaches the other end of the tube. If the discharge parameters and the inclination angle χ (i.e. the degree of the resonator misalignment) are known, we can calculate the value of υ_z.

Figure 10 presents the dependences of the intensity and frequency difference of counterpropagating waves on detuning calculated numerically for different values of $\bar{\upsilon} = \upsilon_z/u$. It is easy to see that for the values of $\bar{\upsilon}$, close to zero, the amplitude nonreciprocity mechanism dominates which is related to the loss difference. The phase nonreciprocity is negligibly small, which provides the symmetry of dependences of the intensities and frequency difference of counterpropagating waves with respect to $\omega_{\rm ab}$. As $\bar{\upsilon}$ increases, the asymmetry of the curves with respect to the transition frequency also increases, and then, when the role of the phase nonreciprocity becomes dominant, the type of the asymmetry changes. The frequency difference is now an even function, whereas the intensity difference of counterpropagating waves is an odd function of detuning.

An increase in the role of the phase nonreciprocity leads to a qualitative change in the behaviour of the intensity and frequency difference within the strong-coupling region. This is confirmed by experiments [19] in which the X-like dependence of the output radiation intensity on detuning was obtained upon modulation of the resonator perimeter, which is similar to that in Fig. 10c.

Note that the phase nonreciprocity in four-mirror resonators is manifested stronger than in three-mirror resonators because the resonator with an odd number of mirrors is subjected to misalignment to a lesser degree than the resonator formed by an even number of mirrors [41].

7. Conclusions

Nonreciprocal diffraction effects have been classified based on the analysis of experimental data on the influence of diffraction phenomena on the frequencies and intensities of counterpropagating waves and their comparison with theoretical results. The basic conclusions are as follows:

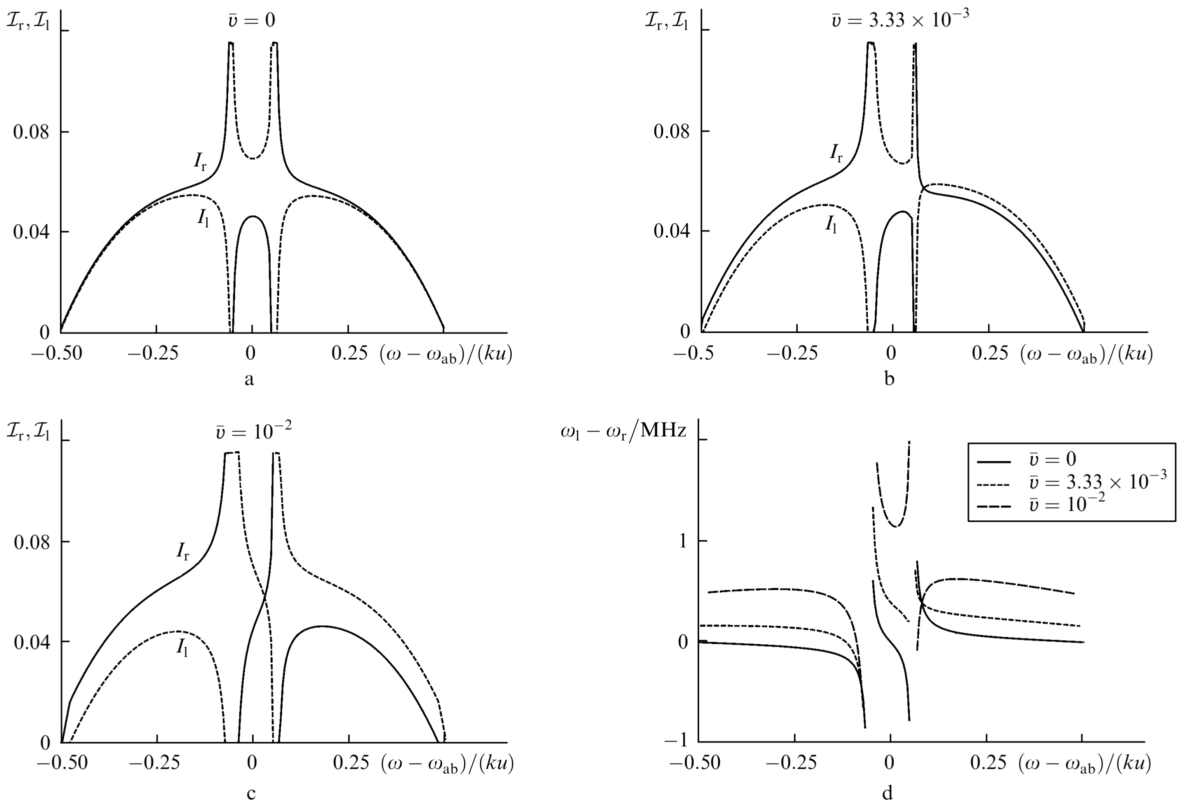

Figure 10. Dependences of the dimensionless intensities $\mathcal{I}_r$, $\mathcal{I}_l$ (a–c) and the frequency difference $\omega_l - \omega_r$ (d) of counterpropagating waves on detuning for different values of $\bar{v} = v_z/u$, $a_{1p} = 2$ mm, and $a_{2p} = 1.6$ mm. The values of other parameters are same as in Fig. 9.

(i) The combined action of the transverse inhomogeneity of the active medium and diffraction produce the asymmetry of the lasing region and intensity profiles of counterpropagating waves with respect to the central frequency of the transition. This effect can be revealed even in the case of the symmetric arrangement of elements in the resonator.

(ii) An asymmetric aperture-nonlinear medium system placed into a ring resonator forms a nonreciprocal device producing the inequality of losses and frequencies of counterpropagating waves.

(iii) The dominance of the mechanism of different losses over the phase nonreciprocity mechanism in a laser with the aligned resonator determines the type of the dependences obtained: the difference of intensities of counterpropagating waves is an even function of the detuning $\omega - \omega_{ab}$ of the mean frequency, while the frequency difference is an odd function.

(iv) The inequality of losses makes possible unidirectional lasing in a ring laser without nonreciprocal devices. The frequency region of unidirectional lasing can be considerably broader than the strong-coupling region. If the reasons producing different conditions for the propagation of counterpropagating waves are eliminated, both waves exist within the entire lasing region and their intensities are equal.

(v) The misalignment of the resonator can lead to the dominant role of the phase nonreciprocity when the intensity difference of counterpropagating waves is described by an odd function of detuning, and the frequency difference is described by an even function.

(vi) In real situations, these nonreciprocity mechanisms can exist simultaneously, resulting in the asymmetric behaviour of the frequency and intensity difference of counterpropagating waves with respect to ω_{ab}.

(vii) The frequency and (or) phase nonreciprocity existing in the resonator of a gas laser operating on a pure isotope causes the appearance of the intensity and frequency-difference resonances of counterpropagating waves. They are not related to the resonances n_j'' of the saturated gains in the medium (whose values are determined by the value of losses). They appear due to the nonlinear interaction of waves in the medium. In this case, the intensity resonances are accompanied by the resonances of the refractive indices n_j' of the medium for counterpropagating waves.

(viii) To increase the accuracy of measurements performed by using ring gas lasers, it is necessary to study thoroughly the gas-discharge plasma in narrow tubes. Probe and resonator measurements of plasma parameters during lasing and theoretical studies performed so far can give only qualitative estimates of contributions of these parameters to the relevant phenomena.

Appendix 1

It is assumed that the dependence of the polarisability $P(x, y, z)$ of the active medium on the field $E(x, y, z)$ only weakly differs from linear. Therefore, the relation between these quantities can be described in the lower orders of the perturbation theory [34] in a small saturation parameter (dimensionless intensity)

$$\mathcal{I}_j(x, y, z) = \frac{|d_{ab} E_j(x, y, z)|^2}{\hbar^2} \frac{\gamma_a + \gamma_b}{\gamma_a \gamma_b \gamma_{ab}}, \tag{A1.1}$$

and the polarisability can be represented as a sum of linear ($\varkappa_j^{(1)}$) and nonlinear ($\varkappa_j^{(3)}$) components: $P_j = \varkappa_j E_j = (\varkappa_j^{(1)} + \varkappa_j^{(3)})E_j$. In the case of plane waves, expression (A1.1) takes the form

$$I_j(z) = \frac{|d_{\rm ab}E_j(z)|^2}{\hbar^2}\frac{\gamma_{\rm a}+\gamma_{\rm b}}{\gamma_{\rm a}\gamma_{\rm b}\gamma_{\rm ab}}. \quad (A1.2)$$

For rarefied gases, in which spectral lines have the Doppler profile caused by the thermal motion of atoms, $\varkappa_j^{(1)}$ is described by the expression

$$2\pi\varkappa_j^{(1)} = -\frac{KZ_j}{k_j}, \quad (A1.3)$$

where

$$Z_j = Z(\zeta_j) = 2{\rm i}\int_0^\infty \exp(-\rho^2 + 2{\rm i}\rho\zeta_j){\rm d}\rho$$

is the plasma function of the complex argument $\zeta_j = (\omega - \omega_{\rm ab} + {\rm i}\gamma_{\rm ab})/(ku)$; $K = 2\pi d_{\rm ab}^2 N/(\hbar u)$ is the gain per unit length; $d_{\rm ab}$ is the transition dipole moment; N is the stationary unsaturated excess of the density of active atoms (excitation density); and u is the root-mean-square velocity of atoms. The nonlinear part of the polarisability of the r wave calculated in the third order of the perturbation theory can be found from the expression

$$2\pi\varkappa_{\rm r}^{(3)} = \frac{1}{k_{\rm r}}K(\beta_{\rm r}\mathcal{I}_{\rm r} + \theta_{\rm l}\mathcal{I}_{\rm l}). \quad (A1.4)$$

Here,

$$\beta_{\rm r} = {\rm i}Z_{\rm r}'' + 2{\rm i}\frac{\gamma_{\rm ab}}{ku}(1+\zeta_{\rm r}Z_{\rm r}); \quad \theta_{\rm l} = \sum_{t=1}^{4}\theta_{{\rm l}t}; \quad (A1.5)$$

$$\theta_{\rm l1} = \frac{1+{\rm i}\varDelta}{2(1+\varDelta^2)}(Z_{\rm r}+Z_{\rm l}); \quad \theta_{\rm l2} = -\frac{\rm i}{2\varDelta}(Z_{\rm r}+Z_{\rm l}^*); \quad (A1.6)$$

$$\theta_{\rm l3} = \frac{f\gamma}{\gamma_{\rm ab}}\sum_{n={\rm a,b}}|v_n|^{-2}\left[Z(\zeta_{\rm rl}^{(n)}) - \frac{Z_{\rm r}-Z_{\rm l}^*}{2} - \alpha_n\theta_{\rm l2}\right]; \quad (A1.7)$$

$$\theta_{\rm l4} = \frac{f\gamma}{\gamma_{\rm ab}}\sum_{n={\rm a,b}}v_n^{-2}\left\{Z_{\rm r}\left[1+2\left(\frac{\gamma_{\rm ab}}{ku}\right)^2 v_n v\right] - Z(\zeta_{\rm rl}^{(n)}) + \frac{2v_n\gamma_{\rm ab}}{ku}\right\}; \quad (A1.8)$$

$$\varDelta = \frac{\omega-\omega_{\rm ab}}{\gamma_{\rm ab}}; \quad \omega = \frac{\omega_{\rm r}+\omega_{\rm l}}{2}; \quad \zeta_{\rm rl}^{(n)} = \frac{\omega_{\rm r}-\omega_{\rm l}+{\rm i}\gamma_n}{2ku};$$

$$v = \varDelta + {\rm i}; \quad v_n = \varDelta + {\rm i}\alpha_n; \quad \alpha_n = 1 - \frac{\gamma_n}{2\gamma_{\rm ab}};$$

$$f = \frac{\gamma_{\rm a}\gamma_{\rm b}}{4\gamma^2}; \quad \gamma = \frac{\gamma_{\rm a}+\gamma_{\rm b}}{2}.$$

Appendix 2

The dependence of $K(x,y)$ on various physical parameters in narrow gas-discharge tubes has not been studied yet. The radial inhomogeneity of the polarisability is caused by the inhomogeneity of the gain $K(x,y)$, which is determined by many factors: the discharge geometry, the gas mixture composition, pump parameters [56–60], the working transition, etc. Figures 1A2a and b show the radial dependences of the gain in a wide enough gas-discharge tube placed into the resonator of a 1.15-μm He–Ne laser [56], which illustrate the fact that the dependence of the gain on the discharge current is determined by pressure in the tube. At a low pressure (1 Torr), the gain increases with increasing discharge current (Fig. 1A2a) and the distribution width decreases. At a pressure of 3 Torr, the gain decreases with increasing the pump current and changes to absorption (Fig. 1A2b).

Measurements performed in a narrow gas-discharge tube at the optimal pressure at the transitions at 0.6328 and 3.39 μm showed [57] that the dependences of the gain $K(x,y)$ on the pump current at the same pressure are different for different transitions. In the case of the 0.6328-μm transition, a dip was observed at the centre of the gain band at high pump currents, whereas the current dependence of $K(x,y)$ for the 3.39-μm transition was rather weak and a dip was not observed (Fig. 1A2d).

In [56, 57], a discharge was excited by a direct current. In [58], the distribution $K(x,y)$ was studied in the gas-discharge tube (with the inner diameter ~ 3 mm) of a 0.6328-μm He–Ne laser upon high-frequency excitation. It was found that upon excitation by an alternating current, the gain increased with distance from the cell walls much faster than upon excitation by a direct current. However, the type of dependences was preserved: the gain at the cell centre begins to decrease with increasing pump current and finally changes to absorption.

By generalising the experimental data, we can say that the radial dependence of the gain K is well described within rather broad discharge-current and pressure ranges of the working mixture of He–Ne lasers by one of he dependences

$$K(x,y) = K_0\left(1 - \frac{x^2}{d_x^2} - \frac{y^2}{d_y^2}\right), \quad (A2.1)$$

$$K(x,y) = K_0\left(1 + \frac{x^2}{d_x^2} + \frac{y^2}{d_y^2}\right). \quad (A2.2)$$

Here, K_0 is the gain on the resonator axis; d_x and d_y are the distribution half-widths in the directions of the transverse axes x and y. The curvature of distribution profile (A2.1) is assumed positive and that of (A2.2) – negative.

Appendix 3

To determine the complex parameters $q_{xj}(z)$ and $p_{xj}(z)$, we substitute (22) into (43) and equate coefficients at the same powers of x. As a result, we obtain that $q_{xj}(z)$ satisfies the Riccati equation

$$n_{zj}(z)\frac{{\rm d}q_{xj}}{{\rm d}z} = 1 - n_{xj}(z)q_{xj}^2, \quad (A3.1)$$

and the parameter $p_{xj}(z)$ is expressed in terms of $q_{xj}(z)$ by the quadrature

$$p_{xj}(z) = -\frac{1}{2}\int_0^z \frac{{\rm d}t}{n_{zj}(t)q_{xj}(t)}. \quad (A3.2)$$

The Riccati equation with variable coefficients cannot be

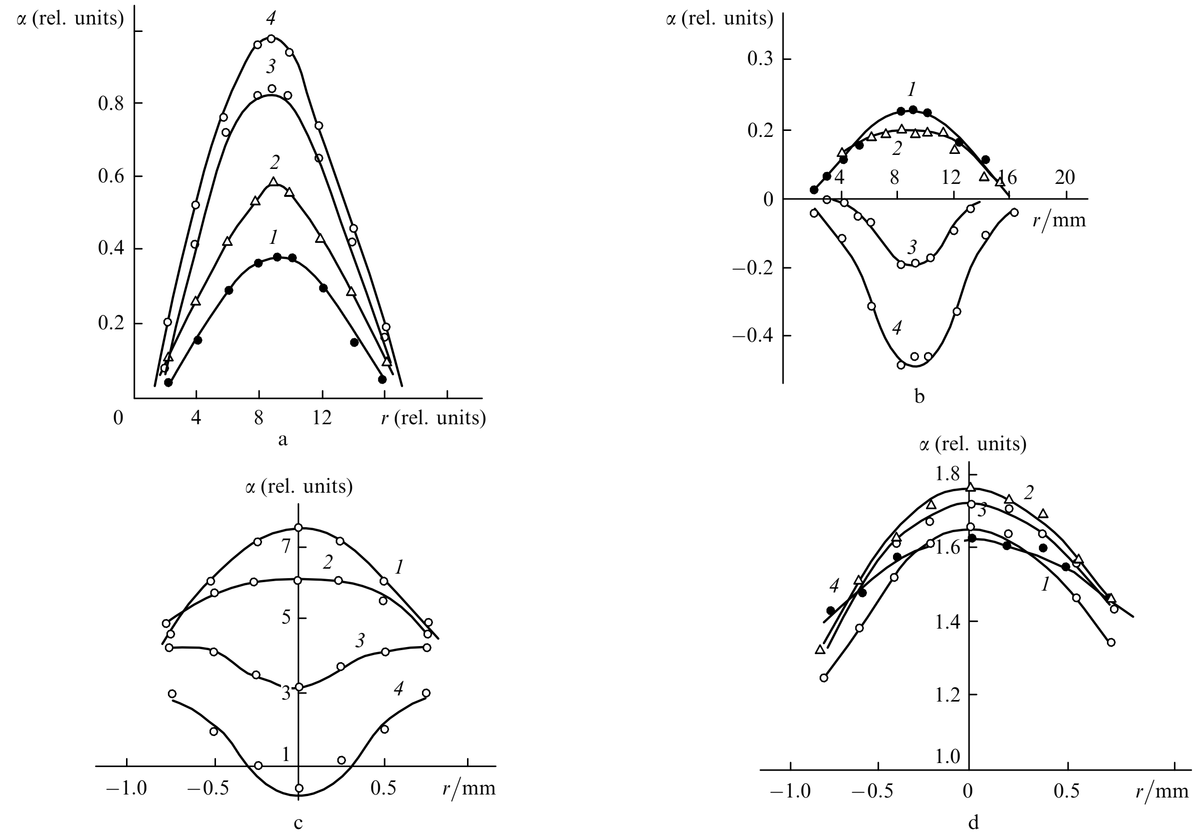

Figure 1A2. Radial dependences of the gain α for $\lambda = 1.15$ (a, b), 0.6328 (c), and 3.39 μm (d) for pressures in the tube of 1 (a) and 3 Torr (b), the pump current 5 (*1*), 10 (*2*), 20 (*3*), and 45 mA (*4*) [56] (a, b) and 10 (*1*), 15 (*2*), 20 (*3*), and 40 mA (*4*) [57] (c, d).

solved in the general case. It was shown in [43] that, if the coefficients in the Riccati equation are represented by slowly varying functions of coordinates, a plane–parallel layer of a quadratic inhomogeneous active medium can be associated with the matrix

$$a_{pj}(z) = \left[\frac{n_{pj}(0)}{n_{pj}(z)}\right]^{1/4} \cosh U_{pj}(z) + \frac{n'_{pj}(0)}{4n_{pj}(0)} \times \frac{1}{[n_{pj}(0)n_{pj}(z)]^{1/4}} \sinh U_{pj}(z),$$

$$b_{pj}(z) = \frac{1}{[n_{pj}(0)n_{pj}(z)]^{1/4}} \sinh U_{pj}(z),$$

$$c_{pj} = [n_{pj}(0)n_{pj}(z)]^{1/4} \sinh U_{pj}(z) - \frac{n'_{pj}(z)}{4n_{pj}(z)} \left[\frac{n_{pj}(0)}{n_{pj}(z)}\right]^{1/4} \times \cosh U_{pj}(z) + \frac{n'_{pj}(0)}{4n_{pj}(0)} \left[\frac{n'_{pj}(z)}{n_{pj}(0)}\right]^{1/4} \cosh U_{pj}(z) \quad \text{(A3.3)}$$
$$- \frac{n'_{pj}(0)}{16n_{pj}(0)} \frac{n'_{pj}(z)}{n_{pj}(z)} \frac{1}{[n_{pj}(0)n_{pj}(z)]^{1/4}} \sinh U_{pj}(z),$$

$$d_{pj} = \left[\frac{n_{pj}(z)}{n_{pj}(0)}\right]^{1/4} \cosh U_{pj}(z) - \frac{n'_{pj}(z)}{4n_{pj}(z)} \times \frac{1}{[n_{pj}(0)n_{pj}(z)]^{1/4}} \sinh U_{pj}(z),$$

$$U_{pj}(z) = \int_0^z n_{pj}^{1/2}(t)\,\mathrm{d}t.$$

Here, the prime denotes the derivative with respect to z. In the case when the complex refractive index n_{pj} of the medium (20) is independent of the longitudinal coordinate z, this matrix takes the known form [44]

$$T_{pj}^{\mathrm{m}}(z) = \begin{pmatrix} \cosh\sqrt{n_{pj}}\,z & (1/\sqrt{n_{pj}})\sinh\sqrt{n_{pj}}\,z \\ \sqrt{n_{pj}}\sinh\sqrt{n_{pj}}\,z & \cosh\sqrt{n_{pj}}\,z \end{pmatrix}. \quad \text{(A3.4)}$$

As the initial section z_0, we will take the middle of the active medium layer and place an aperture in the same arm of the resonator at a distance of $-z_{\mathrm{t}}$ from the layer end. This allows us to separate explicitly the matrix $T_{0pr} = A_{0p}B_{0p}C_{0p}D_{0p}$ of the ideal resonator without relation to its configuration. The resonator matrix for the r wave has the form

$$T_{pj} = T_{pj}^{\mathrm{m}}(h/2)T^{\mathrm{fs}}(z_{\mathrm{t}})T_p^{(N)}T^{\mathrm{fs}}(-z_{\mathrm{t}})T_{0pj}T^{\mathrm{fs}}(-h/2)T_{pj}^{\mathrm{m}}(h/2)$$
$$(h = H/L). \quad \text{(A3.5)}$$

To the free interval of length Δz and a Gaussian aperture, matrices

$$T^{\mathrm{fs}}(\Delta z) = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix},\ T_p^{(N)} = \begin{pmatrix} 1 & 0 \\ 2\mathrm{i}N_p & 1 \end{pmatrix}$$
$$\left(N_p = \frac{L}{ka_p^2}\right) \quad \text{(A3.6)}$$

correspond. Matrix (A3.4) corresponds to a layer of a quadratic inhomogeneous active medium.

Let us calculate the elements of the $A_pB_pC_pD_p$ matrix (A3.5) when parameters perturbing the resonator are small ($|n_{pj}| < N_p < 1$), then find the G_{pj} parameter of the resonator and reduce it to the form $G_{pj} = (A_{pj} + D_{pj})/2 = g_p + \delta G_{pj}$, where $g_p = (A_{0p} + D_{0p})/2$;

$$\delta G_{pj} = \mathrm{i}N_p B_{0p}(z_t) + \frac{1}{2} n_{pj} h[B_{0p}(z_0) - (h/2)^2 C_{0p}]$$

$$+ \mathrm{i}N_p n_{pj} h[B_{0p}(z_t)z_t - (h/2)^2 A_{0p}(z_t)]. \tag{A3.7}$$

The values of matrix elements $A_{0p}(z_t)$ and $B_{0p}(z_t)$ of the unperturbed resonator at the section $x = z_t$ of the aperture are related to the matrix elements in the reference (initial) section by the expressions

$$A_{0p}(z_t) = A_{0p}(z_0) - z_t C_{0p}, \ B_{0p}(z_t) = B_{0p}(z_0) - z_t D_{0p}.$$

Let us present the wave propagation constants in the form

$$\Lambda_{pj} = \Lambda_{0p} \exp(\mathrm{i}\delta\Gamma_{pj}), \ \Gamma_{pj} = \delta\Gamma'_{pj} + \mathrm{i}\Gamma''_{pj}. \tag{A3.8}$$

This allows us to write the quantities Γ''_{pj} and $\delta\Gamma'_{pj}$ in the form

$$\Gamma''_{pj} = n''_{pj} h[2W_{0p}(z_0)]^{-1}\mu_{1p} + n'_{pj} hN_p[2W_{0p}(z_t)]^{-1}\mu_{2p}$$

$$+ N_p[2W_{0p}(z_t)]^{-1}\mu_{3p}, \tag{A3.9}$$

$$\delta\Gamma'_{pj} = n'_{pj} h[2W_{0p}(z_0)]^{-1}\mu_{1p} - n''_p hN_p[2W_{0p}(z_t)]^{-1}\mu_{2p}$$

$$- N_p^2[2W_{0p}(z_t)]^{-2}\mu_{4p}, \tag{A3.10}$$

where

$$\mu_{1p} = \frac{1}{2}\left[1 - \left(\frac{h}{2}\right)^2 \frac{C_{0p}}{B_{0p}(z_0)}\right][1 + N_p W_{0p}^{-1}(z_t)]; \tag{A3.11}$$

$$\mu_{2p} = \frac{g_p}{2(1-g_p^2)^{1/2}}\left[1 - \left(\frac{h}{2}\right)^2 \frac{C_{0p}}{B_{0p}(z_0)}\right]$$

$$+ W_{0p}(z_0)\left[z_t - \left(\frac{h}{2}\right)^2 \frac{A_{0p}(z_t)}{B_{0p}(z_t)}\right]; \tag{A3.12}$$

$$\mu_{3p} = 1 + N_p(2W_{0p}^{-1})(z_t); \ \mu_{4p} = \frac{g_p}{(1-g_p^2)^{1/2}}; \tag{A3.13}$$

$$W_{0p}(z) = \left[\frac{1-g_p^2}{B_{0p}^2(z)}\right]^{1/2} = \frac{2L}{kw_{0p}^2(z)}. \tag{A3.14}$$

Because, as a rule, $h < 1$, we can neglect the terms proportional to $(h/2)^2$ in expressions (A3.7), (A3.11), and (A3.12).

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