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Nuclear gamma-ray laser: the evolution of the idea

L.A. Rivlin

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***Abstract*. The evolution of the foreign and native search for solving the problem of a nuclear gamma-ray laser (NGL), which has been attracting attention for almost half a century despite the absence at present of any convincing data about its experimental solution, is considered. It is shown that the key conflict inherent in any conception of the NGL is the antagonism between the necessity to accumulate a sufficient amount of excited nuclei and the requirement to narrow down the emission gamma-ray line to its natural radiative width. The critical analysis of different approaches for solving this conflict (Mössbauer scheme, deeply cooled ensembles of free nuclei with the hidden inversion, nuclear inversionless amplification, two-quantum gamma emission in counter-propagating photon beams, hypothetical amplifying medium of long-lived isomers in a Bose – Einstein condensate) shows that this search is important not only due to the expected result, which could stimulate the development of quantum nucleonics as a new branch in physics, but also is of interest due to a variety of physical disciplines and experimental approaches used in this search.**

L.A. Rivlin Applied Physics Laboratory, Moscow State Institute of Radio Engineering, Electronics and Automatics (Technical University), prosp. Vernadskogo 78, 119454 Moscow, Russia;
e-mail: lev_rivlin@mail.ru

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1. Introduction

1.1 Quantum nucleonics

The idea of the possibility of creation of a nuclear gamma-ray laser (NGL) and the extension of the concepts and methods of the quantum electronics of atoms and optical photons on nuclei and gamma-ray quanta appeared almost immediately after the advent of the first optical lasers. This direction in the research activity was soon called *quantum nucleonics* (it seems that this term was first coined in [1]).

Of course, the possibility of the stimulated emission of gamma-ray photons by nuclei was obvious as early as a hundred years ago, as following directly from the Einstein radiation laws. Thus, Eddington has written at the beginning of the last century: 'The decay of radium is a spontaneous event if a radium atom is an isolated system. However, this decay can be stimulated by the gamma-ray field, which has the same frequency as that of gamma rays emitted during the decay of radium ... the value of this effect can be calculated from Einstein formulas' [2].

However, between the understanding of the possibility and realisation of the idea there exists sometimes a great distance. Thus, optical lasers were created only half a century after the discovery of stimulated emission laws. To realise in practice even an attractive idea, which does not contradict to the laws of nature, a combination of several conditions is required: the conceptual and technological preparedness, as well as the demand, of which the first condition seems to be the most important. However, such a good fortune favours far from all good ideas.

This concerns in full measure the NGL problem. The importance of solving this problem was not by chance emphasised by V.L. Ginzburg, who included it time and again into the list of the most important problems of modern physics [3–6] and again recalled it recently in his Nobel lecture [7].

Nevertheless, the experience of the last half century demonstrates the absence (as far as we know) of not only any serious experimental attempts to solve the NGL problem but also a complete understanding of the NGL problem even at the conceptual level. V.L. Ginzburg pointed out as early as 1971 that the NGL problem is waiting for 'a new breakthrough' – a new idea or the discovery of new phenomena [3]. Therefore, the discussion of numerous attempts in our country and abroad to achieve such a breakthrough, in particular, based on new phenomena and experimental tools, is undoubtedly of interest.

A seemingly attractive direct way of solving this problem based on the attempt to apply one of the developed schemes of an optical laser (of course, with other values of parameters) to the higher-frequency range is actually dead-end. As we will see below, the vast quantitative difference between the energies of optical and gamma-ray photons (by four–five orders of magnitude) makes such a direct approach unpromising.

Thus, the aim of this paper is to analyse the evolution of various NGL concepts. Not restricting ourselves to a scope of concepts inherent conventionally in optical laser practice, we consider the relevant studies performed in quite remote fields of physics.

1.2 Brief chronology

The possibility of solving the NGL problem was first discussed in the early 1960s, when numerous papers and patents were published in our country and abroad ([8–21], et al.), in which the physical foundations of the problem, discussed below in section 2, were considered at different levels of understanding and development. Almost all these papers considered the so-called Mössbauer NGL scheme (see section 3).

The second surge of interest in the NGL was observed in the 1970s, mainly in connection with papers published in our country ([22–47], et al.), in which attempts were made to improve the Mössbauer scheme and, in particular, to narrow down drastically the emission gamma-ray line, to increase the pump efficiency (for example, by neutrons), etc.*

The results of these two research stages and their critical discussion are considered in detailed reviews [48–54] and later papers [55, 56] et al. and there is no need to analyse them again. Nevertheless, the basic ideas considered at these stages are very briefly mentioned below in corresponding sections.

The modern, third stage of the evolution of the NGL concept, which is mainly discussed in the present paper, began in the last decade of the past century and in the early present century. This stage is characterised in fact by the abandonment of the direct Mössbauer scheme, which seems now unpromising, and a search for various alternative approaches both to the problem of producing the required nuclear population inversion and the problem of eliminating the excess broadening of the emission line.

2. Peculiarities of stimulated gamma-ray emission

Leaving aside important, but secondary technological details, we consider the basic physical factors caused by the above-mentioned vast increase in the frequency scale, which is the main obstacle to a simple realisation of the NGL.

The known standard expression for the gain

$$g = \sigma\beta\left(n_2 - \frac{2J_2+1}{2J_1+1}\,n_1\right) - \chi n \quad (1)$$

of the flux of photons of wavelength λ in a medium with the inverted concentration of emitters n_2 and n_1 at the upper and lower levels of the laser transition with angular momenta J_2 and J_1, respectively, depends on the maximum stimulated emission cross section

$$\sigma = \frac{\lambda^2}{2\pi} \quad (2)$$

on the ratio

$$\beta = \frac{\Delta\omega_\gamma}{\Delta\omega_{\rm tot}}\,\frac{1}{1+\alpha} \quad (3)$$

of the natural width $\Delta\omega_\gamma$ of the transition line to its full width $\Delta\omega_{\rm tot}$ taking into account all the types of the excess homogeneous and inhomogeneous broadening (α is the internal electron conversion coefficient), and on all losses of photons in the medium, which are proportional to the total concentration n of atoms of different types and are described by the averaged cross section χ for losses.

It follows from (1) that:

* Note, by the way, that the surge of activity in the 1970s has found unexpectedly somewhat euphoric reflection in the scientific popular literature: thus, interviews with leading scientists, in particular, with some of the authors [22–47] were published in the Science and Life journal under the encouraging common title 'The breakthrough is being planned' [57].

(i) The increase in the frequency of gamma-ray quanta by four – five orders of magnitude compared to the optical range results in the increase in the spontaneous transition probability by 12 – 15 orders of magnitude and in the energy released upon the transition by 16 – 20 orders of magnitude, so that it is difficult to find real methods for producing and maintaining the noticeable inverse population for such a short-lived excited state and propose efficient pumping. This circumstance was pointed out already at the dawn of optical quantum electronics by Townes and Schawlow, who indicated the high-frequency UV boundary for lasers [58].

(ii) The inhomogeneous broadening $\Delta\omega_{\text{tot}}$, which is proportional to the radiation frequency and is related to the chaotic motion of atoms (for example, the Doppler broadening in gases or the line broadening caused by the emission of phonons in solids) increases by four – five orders of magnitude, thereby reducing the linewidth ratio β (3) by the same factor.

(iii) The maximum possible value of σ (2) decreases by 8 – 10 orders of magnitude, and this reduction cannot be prevented for the given transition energy. As a result (even for $\beta \to 1$), the product $\sigma\beta$, corresponding to the acting stimulated emission cross section, is reduced by 8 – 10 orders of magnitude.

The first two factors severely contradict each other. Indeed, the estimate presented above and the Townes – Schawlow prohibition are related to dipole transitions, which are typical for optical lasers. The use of multipole transitions from metastable states with long radiative lifetimes τ (as in almost all NGL versions considered below) can reduce this estimate by many orders of magnitude, but at the same time, according to the uncertainty relation, this means that the natural linewidth $\Delta\omega_\gamma$ also decreases. Therefore, to obtain sufficiently high values of β (3), it is necessary now to reduce the excess total width $\Delta\omega_{\text{tot}}$ by the same orders of magnitude, which is also a rather complicated problem.

As a result, a combination of these frightening competing factors looks like an almost absolute prohibition of the possibility of observing stimulated emission of gamma rays by nuclei (here, we mean amplification of the lasing type rather than the experiment at the level of photon counting, which hardly could be considered as the required confirmation of the validity of the Einstein laws in the nuclear region).

Therefore, the evolution of the NGL concept considered below is directed first of all to the simultaneous elimination of the first two key obstacles mentioned above. The problem consists in the accumulation of the sufficient concentration n_2 of excited nuclei and in the complete elimination or compensation of the excess line broadening ($\beta \to 1$).

3. Mössbauer scheme of a NGL

3.1 Initial proposals

The necessity of overcoming the above-mentioned negative factors was obvious to the authors of the first papers at the initial stage [8 – 21] based on the use of metastable nuclei under conditions of the Mössbauer effect. At first glance, this approach successfully combined the lowering of requirements for pumping relatively long-lived metastable states with the possibility of narrowing down the zero-phonon emission line to the limiting linewidth ratio $\beta \to 1$. However, the obtained quantitative estimates soon proved to be disappointing.

Mössbauer spectroscopy have shown that gamma-ray lines with the natural linewidth cannot be observed in nuclei with the metastable-state lifetime noticeably longer than 10^{-5} s and the corresponding radiative linewidth smaller than 4×10^{-4} μeV due to various line-broadening mechanisms existing in real crystals. For this reason, it was difficult to propose a rational method for preparing in advance a Mössbauer solid matrix with the sufficient amount of metastable states spontaneously decaying for a few microseconds and, therefore, it was necessary to pump nuclei in a crystal sample already prepared for the observation of stimulated emission. However, it was found that the intensity of pumping of any type (incoherent X-rays, neutron fluxes, etc.) required to produce the population inversion virtually instantly violate the conditions necessary for emission of zero-phonon lines, in particular, due to the heating of the crystal and production of defects in it. In addition, apart from the violation of these conditions for the crystal as a whole, each excited nucleus can leave an ensemble of nuclei capable of emitting gamma-ray lines with the natural linewidth due to the recoil energy imparted to the nucleus upon pumping (for example, by neutrons).

At the same time, the concentration of metastable states should be high enough at least by two reasons. First, because of the spectral coincidence of the zero-phonon Mössbauer emission and absorption lines, to produce inversion, it is necessary to excite, roughly speaking, more than half the nuclei. Second, because the total concentration n of atoms in the Mössbauer matrix considerably exceeds the concentration of metastable nuclei ($n \gg n_2$), the nonresonance losses in the matrix can be overcome and the threshold condition $g \geqslant 0$ can be fulfilled only at a high enough concentration n_2 even for the usual small cross-section ratio $\chi/\sigma \ll 1$. These considerations, which were called the unsolvable 'dilemma of the gamma-ray laser' in review [56] of the Mössbauer version of the NGL, were revised in the studies at the second stage.

3.2 Improvement of the Mössbauer NGL scheme

Interest in the NGL problem was rekindled in 1972 after the publication of paper [22], which was followed by numerous proposals ([23 – 47] et al.) to obviate or alleviate obstacles considered above.

Thus, it was proposed to eliminate the inadmissible overheating of a crystal breaking the conditions for emission of a zero-phonon line upon pumping nuclei with $\tau \leqslant 10^{-5}$ s due to the radiative capture of neutrons with integrated fluxes of the order of $10^{20}\tau^{-1}$ cm^{-2} required to produce inversion (which can be achieved at present probably only in a nuclear explosion), by using a light solid beryllium matrix in the form of thin needles of micron diameter [33, 34].

In turn it was proposed to reduce the above-mentioned unacceptable requirements on the intensity of a neutron source by using the so-called two-stage neutron pumping [35, 37] involving the radiative capture of neutrons in a separate auxiliary converter target; then, the resonance gamma-ray quanta emitted by this target are absorbed by the nuclei of the main target serving as the active medium of the laser. It was expected that the pumping efficiency will increase because the absorption cross section for resonance photons considerably exceeds the cross section for neutron capture, while the number of produced defects will be smaller and the Mössbauer matrix, which is not irradiated by neutrons, will be heated weaker.

The attractiveness of pumping by synchrotron radiation by using the standard laser scheme [29] was limited by the insufficient spectral brightness of available setups. It was proposed to increase the X-ray pump efficiency by selecting the characteristic X-ray lines coinciding with the absorption lines of nuclei [38] and also by using the giant dipole resonance [59].

The use of anti-Stokes transitions, as in atomic optics, for pumping NGLs [60–65], when radiative transitions are excited by incoherent X-rays from long-lived metastable nuclear states at a higher level, could noticeably reduce the pump energy because the absorbed energy of exciting quanta would be only a part (and possible small) of the upper level energy, being only added to the energy of the initial metastable state. However, the anti-Stokes pumping is inherently contradictory because it does not allow one to combine efficiently a great difference between the angular momenta of the ground and metastable states providing the long lifetime of the latter, and the requirement that the difference between the angular momenta of the metastable and higher-lying states and between the momenta of the latter and ground state should be small because both these transitions should be rapid enough. The anti-Stokes method also did not eliminate other obstacles to the development of the NGL, first of all related to the requirement of the line narrowing ($\beta \to 1$) and to the limited brightness of X-ray sources, the overheating of a matrix, etc. In addition, the results of many experiments on the observation of nuclear anti-Stokes transitions in isomers ^{180m}Ta and ^{178m}Hf ([66–68], et al.) still remain controversial [69, 70].

To reduce the pump power in a cardinal way, it was proposed [45] to use long-lived metastable states with $\tau \gg 10^{-5}$ s, which required the elimination of limitations on the minimal width of the zero-phonon Mössbauer line. First of all this concerns the efficient reduction of the role of magnetic dipole–dipole interactions by averaging them in radiofrequency fields, as in the NMR method, when nuclei are found in each of the three possible orientations with equal probabilities during a finite time interval [24, 25, 39]. This averaging method suggested that the line broadening could be similarly reduced due to the interaction of nuclear quadrupole moments with the gradient of an electric field in crystals with defects [22–24]. The line broadening could be also decreased in sufficiently perfect single crystals, in which the line broadening caused by local chemical shifts is also smaller [23, 40, 41].

The success of such an approach to long-lived nuclei would allow one to separate in time all preliminary procedures (preparation of a crystal matrix without defects with the required concentration of operating nuclei, its cooling, pumping, etc.) and stimulated emission in itself.

The compromise approach to nuclei with intermediate lifetimes $\tau \approx 0.01 - 10$ s [36] involved the successive creation of excited states by using available pump sources, the rapid separation and collection of metastable states by the known methods used for laser isotope separation, and growing of perfect crystals.

In [26–28, 30–32], the observation of the nuclear Dicke superfluorescence was considered as a method eliminating the hindrances of gamma rays by nuclei.

Beginning from papers [8–10, 13] at the first stage, the development of a gamma-ray resonator with Bragg reflections from separate single-crystal blocks and of a resonator with one single crystal was considered ([44, 71], et al.) under the condition that the Ewald propagation sphere in the reciprocal crystal lattice space intersects with no less than two lattice points, except for the point at the coordinate origin (the intersection of the sphere with one point is sufficient if this point and the coordinate origin lie at the ends of the sphere diameter).

The possibility of reducing the nonresonance losses of gamma-ray photons in a crystal by two orders of magnitude under the conditions of the Borrmann effect [72] for nuclear transitions of higher multiplicity than E1 [43, 44] was discussed in many papers ([13, 14, 44], et al.) (the zero electric field on nuclei inherent in electric dipole transitions excludes probably for them the combination of conditions for stimulated emission and Borrmann effect).

Note finally that neither of the proposals considered in this section promised the solution of the NGL problem as a whole, first of all taking into account simultaneously the first two key problems pointed out above. It seems that for this reason no serious experimental attempts to observe stimulated nuclear emission by using these schemes have been made so far.

4. Modern stage of the evolution of the NGL concept

'... nothing is perfect in human inventions ... so we have to strive for improvements if we do not want to be asses'.
Roger Bacon 'Opus Majus'

4.1 'Arithmetic' of the NGL

The middle of the last decade of the past century may be thought of as the beginning of the modern stage of the development of the NGL concept. As mentioned above, this stage is characterised by the abandonment of the direct Mössbauer schemes at the first and second research stages and by a search for alternative approaches for solving the key problems formulated above. Some of these approaches are based on attempts to solve these problems simultaneously by using experimental advances in adjacent fields of physics such as the superdeep laser cooling of atoms, optical amplification without population inversion, the observation of the Bose–Einstein condensation of atoms, the development of Doppler-free optical spectroscopy, the development of a two-photon laser, etc.

In particular, the aim of some proposals (see sections 4.2, 4.3, and 4.5) was to eliminate the necessity of population inversion by excluding by one or another method the interaction of the lower level of the laser transition with radiation. It should be emphasised that, unfortunately, even the positive result of these efforts leading to the zero resonance losses of gamma-ray photons does not remove the strict requirement for a high concentration of excited nuclei and, hence, for the pump intensity because it is obvious that the noticeable gain can be achieved only if the concentration of excited nuclei is high enough.

The latter follows from elementary estimates at the 'arithmetic' level. Taking into account the above-mentioned possibility of excluding the resonance absorption of gamma rays and neglecting other losses of gamma-ray photons in a medium, we obtain the local gain (1) $g = \sigma\beta n_2$. Here, it is convenient to introduce the artificial test parameter $G_A = \exp(gL) = \exp(\sigma\beta n_2 L)$, which characterises the upper bound of the single-pass exponential gain in the nuclear medium of length L. Then, for the optimal condition $\beta \to 1$ (3), the value of G_A acceptable for the observation of stimulated emission of gamma rays is achieved if the total number of excited nuclei per unit cross section of the medium is $n_2 L \geqslant \sigma^{-1} \ln G_A$. Thus, for the moderate value $G_A = 1.5$ and 20-keV photons, the total number of nuclei per unit cross section of the medium will be not too small ($n_2 L \approx 0.8 \times 10^{18}$ cm^{-2}). As a matter of fact, instead of this value the total number N_A of the acting excited nuclei in the volume of the medium with a transverse size $\Delta S \gg \lambda^2$ should be considered. For the limited value of ΔS, the total number $N_A \equiv n_2 L \Delta S$ is already not too great, but is still considerable, for example, $N_A \approx 10^{10}$ for $\Delta S = 10^{-8}$ cm^2.

This scholar arithmetic shows that the possible NGL concepts can be divided into two alternative classes [73]: NGLs with a condensed medium of small length (for example, $n_2 = 10^{18}$ cm^{-3} and $L = 1$ cm) and with a rarefied medium of long length (for example, $n_2 = 10^{15}$ cm^{-3} and $L = 10^3$ cm). The advantages and disadvantages of each of these classes, which are partially obvious from the estimates presented, are considered below for particular NGL schemes.

4.2 Stimulated gamma rays in cooled ensembles of free nuclei with the hidden inversion

This concept [74–81] is based on the suppression of the Doppler broadening of gamma-ray lines of deeply cooled ensembles of free nuclei to achieve $\beta \to 1$ and the simultaneous use of the recoil of free nuclei following any gamma-ray photon absorption or emission event.

Known advances in the laser manipulation by free atoms resulted in cooling atoms down to temperatures in the micro- and even nanokelvin ranges. This means that in the case of free nuclei (gas, atomic beam, etc.), when the gamma-ray line is mainly broadened due to the random motion of atoms containing nuclei, and the corresponding Doppler width of the gamma-ray line is

$$\hbar\Delta\omega_D = 2E\left[2\ln 2\left(\frac{k_B T}{Mc^2}\right)\right]^{1/2} \approx 0.7E\left(\frac{T}{A}\right)^{1/2} \text{ meV} \quad (4)$$

(E is the transition energy, M is the atom mass, T is the absolute temperature, k_B is the Boltzmann constant, c is the speed of light; in the numerical expressions E is measured in keV, T – in kelvins, and A is the mass number of the atom), the narrowed nuclear lines can be observed providing the condition $\beta \to 1$ (in the absence of other sources of broadening) for a transition with the radiative lifetime $\tau \approx 2\pi/\Delta\omega_D$. Thus, for $E = 10$ keV, $A = 100$, $T = 10^{-6}$ K, $\hbar\Delta\omega_D \approx 0.7$ μeV and $\alpha \ll 1$, the condition $\beta \to 1$ can be achieved for a transition with the radiative lifetime $\tau \approx 6$ ns.*

*Note that expression (26) in [78] and estimates based on it contain considerable misprints.

4.2.1. Mutual kinematic shift of emission and absorption lines due to nuclear recoil and appearance of the hidden inversion

A significant specific feature of radiative processes in free nuclei (unlike processes in nuclei in the case of the Mössbauer effect) is the recoil effect due to the laws of conservation of energy and momentum during emission or absorption of a gamma-ray photon with a relatively large momentum. Thus, a 10-keV photon can impart a noticeable additional recoil velocity of 30 m s^{-1} to a free nucleus with $A = 100$. Thus, the nucleus acquires the kinetic recoil energy

$$E_{rec} = \frac{E^2}{2Mc^2} \approx 0.5\,\frac{E^2}{A} \text{ meV} \quad (5)$$

during any radiative transition (E is measured in keV), which is taken from the nuclear state energy upon emission or from the photon energy upon absorption:

$$\hbar\omega_{e,a} = E \mp E_{rec}. \quad (6)$$

This means that the centres of the emission and absorption lines are shifted with respect to the transition energy E by the energy $\mp E_{rec}$ and with respect to each other by the energy $2E_{rec}$. We will call this phenomenon and its more complicated forms considered below the kinematic line shift and splitting.

The kinematic shift of emission and absorption lines leads to the important result pointed out as early as 1963 in [82]: if the shift $2E_{rec}$ is comparable to the total excess linewidth $\Delta\omega_{tot}$, the so-called hidden or spectrally local population inversion can appear and the total inversion becomes unnecessary, i.e. the excess of the number of excited nuclei over that of unexcited nuclei becomes unnecessary [55].

Recall that there exist lasers with the hidden population inversion emitting in the visible range, for example, semiconductor injection lasers, in which a relatively narrow gain band and absorption band are spectrally separated [74]. Another type of the hidden inversion was proposed in the method of forced Doppler modulation of emission and absorption of nuclei [83]. The hidden inversion produced due to the nuclear recoil in a two-temperature quasi-equilibrium gas upon radiative transitions involving very hard gamma-ray quanta was considered in [84].

The quantitative criteria for the appearance of the hidden inversion in an ensemble of free nuclei with concentrations n_2 and n_1 at the upper and lower levels of the transition are two inequalities in which the kinematic splitting $2E_{rec}$ of the line is compared with the natural Lorentzian radiative width $\Delta\omega_\gamma$

$$\frac{n_2}{n_1} > \left[1 + \left(4\,\frac{E_{rec}}{\hbar\Delta\omega_\gamma}\right)^2\right]^{-1} \approx \left(\frac{\hbar\Delta\omega_\gamma}{4E_{rec}}\right)^2, \quad (7)$$

or with the Doppler width

$$\frac{n_2}{n_1} > \exp\left(-\frac{E_{rec}}{k_B T_\leftrightarrow}\right), \quad (8)$$

where $T_{\leftrightarrow}$ is the 'temperature' of the translational degree of freedom of nuclei parallel to the photon flux being amplified.

These inequalities are fulfilled for any reasonable choice of parameters. For example, the right-hand side of (7) is in fact always much smaller than unity, and the hidden inversion between levels appears, according to (8), already upon moderate cooling of nuclei with $E = 10$ keV and $A = 100$ for $n_2 = n_1/2$ if $T_{\leftrightarrow} < 8$ K, and for $n_2 = 10^{-7} n_1$ if $T_{\leftrightarrow} < 0.5$ K.

In the case of the hidden population inversion, the concentration difference in (1) is replaced by the concentration of excited nuclei n_2, and the gain g depends neither on the concentration n_1 of unexcited nuclei nor on the ratio of spins J_1 and J_2, while the first term in (1) is always positive for any finite value of n_2. The latter does not mean at all that amplification is possible for arbitrarily low concentrations n_2: the positive gain $g > 0$ appears only when

$$n_2 > n_{\text{thr}} \equiv \frac{\chi}{\sigma\beta}\, n\,. \tag{9}$$

In many cases, $\chi/\sigma \ll 1$ and $n_{\text{thr}} \ll n$; however, not for a small absolute value of n_2 required for the noticeable complete single-pass amplification (see section 4.1). By assuming that the linewidth ratio in (1) is $\beta \to 1$ and $n_2 = 5 \times 10^{13}$ cm^{-3}, we estimate the exponential single-pass gain for 10-keV photons in the medium with $L = 500$ cm as $\exp(\sigma n_2 L) = 1.87$. This gain is high enough to obtain lasing in a standard scheme by using available mirrors with the reflectivity $\sim 50\,\%$ [85].

Note that the kinematic splitting of emission and absorption gamma-ray lines can be observed not only by the methods of gamma-ray spectroscopy but also by the optical spectra of atoms containing nuclei in which radiative processes occur [86, 87].

4.2.2. Two-level pump scheme and threshold conditions

All the most promising pump methods are based on two elementary processes – the resonance absorption of an X-ray photon and the radiative capture of a neutron. The latter process could be quite efficient for few nuclei with the anomalously high capture cross section, but its use in a medium with free nuclei is complicated because the dissipation of the produced excess energy and nucleus momentum is impossible. In the case of X-ray pumping, another positive consequence of the kinematic splitting of the lines of free nuclei is that a nuclear ensemble can be pumped in the two-level scheme [80], which is partially similar to the standard three-level scheme of visible lasers, but uses only two levels of the laser transition, the third auxiliary level being unnecessary. This considerably simplifies the situation compared to the standard three-level scheme and, in particular, spares the necessity of a search for nuclei with the three-level transition scheme. The role of the third level of the standard scheme is played in this case by the upper level of the kinematically split doublet, whereas the lower level of the doublet acts as the upper level of the laser transition. Incoherent pump radiation is, as a rule, broadband and its width considerably exceeds $\Delta\omega_\gamma$ and $\Delta\omega_{\text{tot}}$, but it should be bounded below by the value of the order of kinematic splitting $2E_{\text{rec}}/\hbar$ to avoid the stimulation of the undesirable radiative decay of the lower doublet level reducing the concentration n_2 of excited nuclei.

The stationary nuclear concentration n_2 on the lower level of the doublet produced by X-ray pumping with the spectral density $(\mathrm{d}\Psi/\mathrm{d}\omega)_{\text{x}}$ is determined by the rate equation [88]

$$n_2 = 2\pi\sigma n_1 \frac{2J_2+1}{2J_1+1}\left(\frac{\mathrm{d}\Psi}{\mathrm{d}\omega}\right)_{\text{x}}(1+\sigma\beta\tau\Phi)^{-1}, \tag{10}$$

where Φ is the density of the stimulated gamma-ray photon flux. In the case of a weak gamma-ray signal, when $\Phi \ll (\sigma\beta\tau)^{-1}$ and for $n_2 \ll n_1$, we have

$$n_2 \approx 2\pi\sigma n \frac{2J_2+1}{2J_1+1}\left(\frac{\mathrm{d}\Psi}{\mathrm{d}\omega}\right)_{\text{x}}. \tag{11}$$

This gives the threshold spectral density of pump radiation when $g > 0$ in a medium with the hidden inversion (1):

$$\left(\frac{\mathrm{d}\Psi}{\mathrm{d}\omega}\right)_{\text{x}} > \left(\frac{\mathrm{d}\Psi}{\mathrm{d}\omega}\right)_{\text{thr}} = \frac{\chi}{2\pi\sigma^2\beta}\,\frac{2J_1+1}{2J_2+1}. \tag{12}$$

This yields the estimate $(\mathrm{d}\Psi/d\omega)_{\text{thr}} \approx 6 \times 10^{14}$ phot$\times$ cm^{-2} s^{-1} Hz^{-1} for $\lambda = 10^{-9}$ cm, $\chi/\sigma = 10^{-5}$ and $\beta \to 1$, which exceeds 10^{31} phot cm^{-2} s^{-1} keV^{-1} in standard units. This value can be achieved only in relativistic-electron X-ray sources of the modern generation.

Any of the pump schemes for free nuclei assumes irradiation of an already prepared cooled ensemble because the short (nanosecond) lifetime of the laser transition excludes in fact the reverse sequence of cooling and pump procedures. Thus, along with other qualities, the pump should be nonperturbing, i.e. it should not disturb the monokinetic nature of a cooled nuclear ensemble.

This is provided by the resonance absorption of pump photons and does not cause any additional heating of the atomic ensemble.

Indeed, in a simple case [78], when a cooled beam of unexcited nuclei propagates towards a directed pump beam of incoherent photons, each nucleus absorbing a photon of energy E acquires its momentum and the nucleus velocity acquires the negative addition

$$\delta v = -\frac{E}{Mc} \approx -2c\,\frac{E_{\text{rec}}}{E}\,. \tag{13}$$

Because we assume that $\beta \to 1$ and, hence, the absorption linewidth is $\Delta\omega_\gamma$, this addition to the velocity can have the dispersion $\hbar\Delta\omega_\gamma/Mc$, and the nuclear ensemble acquires the additional inhomogeneous Doppler broadening of the order of $\Delta\omega_\gamma E/Mc^2$. The latter can be neglected if it is small compared to the linewidth $\Delta\omega_\gamma$, i.e. if $E/Mc^2 \ll 1$, which is obviously always fulfilled. The perturbing action related to the divergence of the pump photon beam restricts the solid angle of the beam by the value of the order of a few hundredths of steradian [78].

Ionisation introduces much larger pertrubations into a beam of cooled nuclei contained in atoms pumped by inntense X-rays. This process excludes produced ions from the acting nuclear beam, from which they can be removed by applying the transverse electric field. The relative nuclear losses estimated as the ratio of ionisation cross section to the cross section for excitation of the nuclear level can be a few percent [78].

It is important to emphasise that the resonance interaction of the pump with nuclei and the absence of any absorbing centres except active nuclei completely eliminates the problem of the overheating of the amplifying medium, which is one of the central problems in solid-state gamma-ray lasers.

4.2.3. Anisotropy of the hidden inversion and the local gain

As pointed out above, a nucleus acquires negative addition (13) to its velocity in each X-ray photon absorption event. Therefore, the resonance frequency ω_e of the excited nucleus in the coordinate system connected with the nuclear flux proves to be Doppler shifted:

$$\omega_e \approx \frac{1}{\hbar}\left(E - E_{rec} \pm \frac{E^2}{Mc^2}\right)$$

$$= \frac{E}{\hbar}\left(1 - \frac{E}{2Mc^2} \pm \frac{E}{Mc^2}\right), \quad (14)$$

where the sign at the third, Doppler term depends on the observation direction. For gamma rays propagating towards the pump photon beam, we have

$$\omega_e \approx \frac{1}{\hbar}\left(E - E_{rec} - \frac{E^2}{Mc^2}\right) = \frac{E}{\hbar}\left(1 - \frac{3}{2}\frac{E}{Mc^2}\right), \quad (15)$$

and for radiation propagating in the opposite direction,

$$\omega_e \approx \frac{1}{\hbar}\left(E - E_{rec} + \frac{E^2}{Mc^2}\right) = \frac{E}{\hbar}\left(1 + \frac{E}{2Mc^2}\right). \quad (16)$$

Thus, taking the Doppler shift into account, the resonance frequency ω_e (16) of excited nuclei with respect to gamma rays copropagating with pumping X-rays proves to be equal to the kinematically shifted absorption frequency ω_a of unexcited nuclei (6). This produces the anisotropy of the hidden inversion, which appears only with respect to radiation directed towards the pump photon beam, but not to radiation directed oppositely [78].

As a result, the local gain in the case of the anisotropy of the hidden inversion has the form

$$g \approx n\chi\left[\frac{d\Psi/d\omega}{(d\Psi/d\omega)_{thr}} - 1\right]. \quad (17)$$

One can see that for the reasonable values of n and χ and a slight excess over the threshold, the local gain is small ($g \ll 1$), although positive ($g > 0$).

4.2.4. Configuration of a possible experiment

The configuration of a possible experiment is described in [78]. An atomic beam cooled by the methods of laser manipulation by atoms down to the temperature at which the condition $\beta \to 1$ is fulfilled propagates along a quantum trap towards a directed X-ray pump beam. If inequalities (7) and (8) are fulfilled, the hidden inversion appears in the direction towards the pump radiation, which upon the excess over the threshold (12) facilitates the anisotropic gain with $g > 0$. This process as a whole can be considered as stimulated coherent backscattering of pump photons accompanied by the decrease in energy by $-3E^2/2Mc^2$ (15).

Quantitative estimates (without attempts to perform optimisation) are presented for isotopes $^{40}_{19}$K ($E = 29.83$ keV, $\tau = 4.24$ ns, $\chi \approx 10^{-21}$ cm^2) and $^{134}_{55}$Cs ($E = 11.24$ keV, $\tau = 46.6$ ns, $\chi \approx 2 \times 10^{-20}$ cm^2). The condition $\beta \to 1$ (3) requires the cooling of the atomic ensemble of both isotopes below temperatures 10^{-7} and 10^{-8} K, respectively, at which inequalities (8) and (7) are simultaneously fulfilled with a great margin. The threefold excess over the pump threshold (12) $(d\Psi/d\omega)_{thr} = 5 \times 10^{30}$ and 2×10^{30} phot cm^{-2} s^{-1} keV^{-1} [the level achievable in relativistic-electron X-ray sources and in free-electron lasers such as LCLS/SLAC and Advanced Photon Source (Argonne National Lab., USA) and HASY-Lab/DASY (Germany)] gives the estimates of the gain $g \approx 2 \times 10^{-7}$ and 2×10^{-6} cm^{-1}, respectively, where $n = 10^{14}$ cm^{-3}. Both nuclides are convenient for laser cooling and manipulation of atoms.

For the values of g presented above and a long enough medium (for example, ~ 10 m), the exponential single-pass gain G can achieve a few fractions of percent, which suggests that, having highly reflecting Bragg single-crystal mirrors [85], it is possible to construct a scheme with a ring (for example, three-mirror) resonator and multiple transits of a unidirectional travelling wave through the amplifying medium. The latter is necessary because the condition $g > 0$ is fulfilled only for one propagation direction of gamma-ray photons in the resonator, while backward propagation only introduces losses, including resonance losses.

The obtained results suggest that the NGL concept with the amplifying medium containing deeply cooled free nuclei is inherently consistent, which allows one to propose experiments on the observation of stimulated emission of gamma-ray photons by nuclei, although it seems unlikely that quantitative estimates (in particular, of the quantum yield of this process considered as stimulated coherent backscattering of pump photons) can arouse the enthusiasm of experimenters. Nevertheless, the positive results of such an experiment would be an important stage not only on the way to the operating NGL but also in the general progress of quantum nucleonics.

4.3 Stimulated emission of nuclear gamma rays without population inversion

4.3.1. Inversionless amplification in the optical range

The induced emission of photons in a medium without inversion [89–91] is based on the atomic coherence and interference in atomic systems in which one of the levels of the laser transition is a doublet. The simplest example is the so-called three-level Λ-scheme with the lower-level splitting [89, 91], in which resonance absorption is suppressed by the interference of transitions upward from the doublet sublevels due to the preparation of the coherent superposition of their states and population trapping [92, 93]. Therefore, stimulated emission becomes possible without the excess of the upper-level population over the lower-level population [as in the scheme with the hidden inversion (see section 4.2), although for another physical reason].

The observation of inversionless stimulated emission in the optical region [94–98] aroused great interest in this phenomenon, and at present many schematic modifications of this phenomenon exist (see, for example, reviews [99, 100]). Of course, this interest was soon extended to the NGL problem [101–110]. The possibility of inversionless amplification in NGLs was first pointed out in papers [89–91]. In this case, the absence of necessity in the production of the population inversion was considered as the main encouraging circumstance [102]. (However, as pointed out in section 4.1, the main difficulty in the solution of the NGL problem is not the creation of inversion but a simple accumulation of a sufficient amount of excited nuclei even in the case when the real inversion is not required by some or other reasons.)

4.3.2. Possibility of nuclear inversionless amplification of gamma rays

Considerable fundamental and experimental difficulties encountered in producing coherent nuclear states were pointed out already in the first papers discussing the possibility of observing inversionless stimulated emission of gamma rays by nuclei [101, 110] and related phenomenon of induced nuclear gamma-ray transparency [111, 112].

Initial proposals for observing inversionless stimulated emission of gamma rays by nuclei [101, 110] are based on the application of microwave fields to the hyperfine structure of Mössbauer nuclei in an external magnetic field or without it, which requires the deep cooling of samples (possibly considerably lower than liquid helium temperature), that is prevented by their considerable heating by intense hard pumping needed to populate sufficiently the upper level of the laser transition. In addition, the conditions for producing the inversionless amplification required the fast relaxation of the microwave transition compared to the limited decay time of the laser nuclear level ($\tau \leqslant 10^{-6}$ s), which would allowed the use of the Mössbauer zero-phonon line with $\beta \to 1$ (3).

It is possible that the so-called double Λ-scheme [102] with coherent pumping by X-ray lasers and the P-scheme with the use of the Zeeman splitting in an external magnetic field and of the hyperfine splitting are more attractive for experiments. The possibility of the latter approach was indirectly confirmed by the observation of coherent and interference phenomena in the hyperfine structure of a classical Mössbauer nucleus ^{57}Fe with the transition energy 14.4 keV [113, 114].

The possibilities of using optical lasers for preparing coherent states, for example, by using the Λ- and V-schemes or the double Λ-scheme were considered in [104, 106–108]. Advances in the laser-manipulated gamma-ray spectroscopy [106] create in principle the necessary prerequisites for observing the suppression of the resonance absorption of gamma rays, i.e. the inversionless amplification of gamma rays. However, certain difficulties are encountered on this way because the hyperfine structure is usually overlapped by the inhomogeneous phonon broadening of lines at rather high temperatures ~ 100 K [109] due to heating produced by the intense hard pumping. It should be taken into account that the requirement that the optical-transition lifetime should be shorter than that of the gamma-ray transition means that it is necessary to use allowed electric dipole optical transitions [109].

These contradictions are partially removed in the degenerate double Λ-scheme [107, 115], but the Zeeman degeneracy of hyperfine sublevels produces additional decay channels of optical states, resulting in the decease in the level populations [109].

Of special interest is nonselective optical pumping [109] at which the inversionless amplification of gamma rays can occur in the polarised pump field of the proper intensity in the case of a sufficiently large hyperfine splitting and a comparatively small radiative width of the gamma-ray transition, even when the optical lines overlaps the hyperfine splitting. In this case, the radiative width of the gamma-ray line of oriented nuclei should be smaller than the hyperfine splitting if the pump field has circular polarisation.

The typical conditions for producing inversionless amplification upon incoherent optical pumping were estimated in [109]. According to this estimate for the Mössbauer gamma-ray transition with the lifetime 10^{-7} s for the atomic transition with energy 1 eV, the radiative linewidth 3×10^7 s^{-1}, the inhomogeneous linewidth of the allowed electric dipole optical transition in a solid matrix equal to 10^{12} s^{-1}, and the relaxation time 10^{-3} s between the sublevels of the hyperfine splitting, the population inversion corresponding to 0.005 % of the number of excited nuclei appears if the optical pump intensity exceeds the threshold equal to 160 W cm^{-2}.

According to [109], a strong suppression of the nuclear resonance absorption of gamma-ray quanta (by 3–5 orders of magnitude), creating the necessary prerequisites for inversionless stimulated emission can be achieved upon sufficiently hard pumping without the violation of the Mössbauer and Borrmann effects due to coherent optical pumping of electronic transitions in the presence of coupling between the nuclear and electronic degrees of freedom. This can be achieved by two methods.

In the first method, the frequency-selective optical pumping of hyperfine sublevels performed by using one of the schemes considered above requires a considerable hyperfine splitting, exceeding the broadening of the optical transition. This means that the lifetime of the optical transition should be considerably shorter than the gamma-ray transition lifetime, which limits the optical lifetime for the allowed electric dipole transition by the value less than 10^{-6} s when Mössbauer isotopes are used. Such a transition proves to be strongly inhomogeneously broadened at comparatively high temperatures of a crystal matrix caused by hard pumping. As a whole, the simultaneous fulfilment of all these conditions looks problematic.

The second method involves the suppression of gamma-ray absorption upon coherent optical pumping (even when the hyperfine structure is overlapped by the optical broadening) by using the inversionless degenerate Λ-scheme or the depopulation of the ground state due to polarisation-selective optical pumping, however, this is difficult due to additional decay channels.

It seems that frequency-nonselective optical pumping is more promising for the suppression of the resonance gamma-ray absorption and producing inversionless gamma-ray amplification.

4.3.3. Experimental attempts

By concluding the consideration of this very attractive direction in the study of the NGL problem, an important remark should be made. Formally speaking, the nuclear inversion in the case of the exclusion of resonance interaction with the lower level (inversionless amplification), as in the case of the so-called hidden inversion considered in section 4.2, appears in the presence of at least a single excited nucleus. Nevertheless, as pointed out in section 4.1, it is obvious that the noticeable gain and its excess over nonresonance losses can be achieved only if the concentration of excited nuclei is sufficient. Therefore, problems are remained that have not been solved so far in any Mössbauer NGL version, including its inversionless variant: the inversionless amplification does not eliminate the necessity in the intense pumping of nuclei with short lifetimes ($\tau \leqslant 10^{-5}$ s) and the simultaneous elimination of the excess broadening of the emission line of isomers ($\beta \to 1$).

It is possible that just for this reason no full-scale attempts to develop NGLs based on inversionless amplification have been reported so far. However, successful experiments are known which can be considered as preliminary steps in this direction.

Thus, the nuclear gamma-ray transparency was observed by the method of level mixing [116, 117] in the classical Mössbauer nucleus ^{57}Fe in a $FeCO_3$ crystal [111, 118]. In this case, the hyperfine splitting of nuclear levels was controlled by the two types of interaction: magnetic dipole (Zeeman) in the internal or external magnetic field and electric quadrupole in a gradient external electric field.

In other experiments, the laser-induced modification of the Mössbauer spectrum was observed in $Fe^{2+}:MgO$ [119, 120] and $Eu^{3+}:CaF_2$ crystals [121].

4.4 Two-quantum stimulated emission of gamma rays in counterpropagating photon beams

The alternative to the solution of the NGL problem by eliminating the excess broadening of the emission gamma-ray line ($\beta \to 1$) is based on the known advances of optical sub-Doppler spectroscopy within an inhomogeneously broadened line and the development of two-photon lasers.

This radiative process consists in the following [75, 76, 79, 122]. According to the laws of conservation of energy and momentum upon a two-photon radiative transition with the total energy E in a nucleus of mass M accompanied by emission of two photons with frequencies ω_1 and ω_2 in the opposite directions, the sum of photon energies is

$$\hbar\omega_1 + \hbar\omega_2 = E + \hbar\delta\omega\,\frac{u}{c} - \frac{(\hbar\delta\omega)^2}{2Mc^2}, \qquad (18)$$

where $\delta\omega \equiv \omega_1 - \omega_2$ is the detuning of photon frequencies; u is the projection of the nucleus velocity on the direction of the wave vector of the first photon. If the natural radiative width $\Delta\omega_\gamma$ of the transition line is small enough, it follows from (18) that all the nuclei, independent of their individual random velocities, are involved in stimulated emission only for $\delta\omega = 0$ and with frequencies

$$\omega_1 = \omega_2 = \frac{E}{2\hbar} \qquad (19)$$

(accurate to a small shift $-\hbar(\delta\omega)^2/2Mc^2$ caused by the nucleus recoil).

This circumstance, on which sub-Doppler spectroscopy is based, allows one to eliminate the negative action of the Doppler line broadening of the induced transition on the gain without the necessity of suppressing the random spread of nuclear velocities [75, 76, 79, 81]. For this purpose, the inverted nuclear ensemble should be irradiated by two counterpropagating photon fluxes with frequencies (19) from an external source. Therefore, a strongly forbidden one-photon radiative decay of the metastable state with a great difference of angular momenta with respect to the lower level is replaced by the two-photon stimulated transition with smaller differences of the angular momenta of each of the two simultaneous one-photon transitions involved in the two-photon transition. Unfortunately, estimates [122] showed that such a replacement can compete even with not too strongly forbidden one-photon process only in the case of very intense external stimulating photon fluxes.

A positive property of the scheme under study is the dynamic feedback inherent in the induced two-photon emission, creating, in particular, the prerequisites for the effective development of lasing without mirrors, which application involves difficulties in the gamma-ray range. However, advances in the development of efficient single-crystal Bragg reflectors for hard radiation [85] deprived this scheme of this advantage, but created simultaneously the potential prerequisites for reducing the required intensity of external counterpropagating photon beams and, possible, even for the abandonment of their constant stimulating action [123, 124].

Indeed, it could be expected that for a high enough Q factor of a single-crystal resonator, both counterpropagating photon beams produced during two-photon stimulated emission would play, after reflection from mirrors, the role of external photons stimulating the two-photon process in fact without the involvement of the latter ones. In this case, external photons would be necessary only for initiating (ignition) lasing at the initial stage.

This possibility was analysed [123, 124] for a simplified model of a standard laser configuration. The model represented a nuclear amplifying medium of length L placed between two reflectors with reflectances R_0 and R_L. External photons were injected through a semitransparent mirror with the reflectivity R_0, and the counterpropagating beam was formed due to reflection from the second mirror with the reflectance R_L. Analysis showed that in the stationary regime the output radiation intensity from the mirror with R_L

$$\Phi_{\text{out}}(L) = \frac{1 - R_L}{1 + R_L} \times \frac{PL + 2\Phi_{\text{ign}}(0)(1 - R_0)(1 + R_0)^{-1}}{\chi n L + 2(1 - R_L R_0)(1 + R_L)^{-1}(1 + R_0)^{-1}} \qquad (20)$$

achieves the maximum upon continuous supply of excited nuclei uniformly over the volume at the rate P ($cm^{-3}\,s^{-1}$) and at the density of the external flux of stimulating photons supplied through the mirror with R_0 equal to

$$\Phi_{\rm ign}(0) = \frac{PL}{2}\,\frac{1+R_0}{1-R_0}. \qquad (21)$$

Contrary to expectations, the stationary regime in the absence of ignition [$\Phi_{\rm ign}(0) = 0$] proved to be impossible due to its instability. The ratio $G^* = \Phi_{\rm out}(L)\Phi_{\rm ign}^{-1}(0)$ (in a sense equal to the gain of the external flux) is

$$G^* = \frac{1-R_L}{1+R_L} \times \frac{2(1-R_0)(1+R_0)^{-1} + PL/\Phi_{\rm ign}(0)}{2(1-R_L R_0)(1+R_L)^{-1}(1+R_0)^{-1} + \chi n L}. \qquad (22)$$

It is obvious that the useful effect is achieved for $G^* > 1$, which imposes the condition

$$\frac{PL}{\Phi_{\rm ign}(0)} \geqslant \frac{1+R_L}{1-R_L} \times \left[\chi n L + 2\,\frac{1-R_L R_0}{(1+R_L)(1+R_0)}\right] - 2\,\frac{1-R_0}{1+R_0} \qquad (23)$$

additionally to (21) and restricts photon losses in the medium by the inequality

$$\chi n L \leqslant 2\,\frac{1-2(R_L+R_0)+3R_L R_0}{(1+R_L)(1+R_0)}. \qquad (24)$$

The pulsed regime is achieved in the absence of the supply of new isomeric nuclei ($P = 0$) (which is, possibly, more realistic from the point of view of experimental realisation than cw lasing) with a gradual exhaustion of the initial amount of isomers; if the output radiation intensity $\Phi_{\rm out}(L)$ is high enough, it can further increase even after the switching off of the external photon flux $\Phi_{\rm ign}(0)$ producing the initial ignition of lasing.

In conclusion, we consider the isomeric nucleus ${}^{242}_{95}$Am with the metastable-state energy $E = 48.6$ keV, the lifetime 141 year, the multiplicity of the transition to the ground state E4 for the atomic absorption cross section $\chi \approx 10^{-20}$ cm^2 for photons with energy $E/2 = 24.3$ keV. This isomer is attractive because of its relative availability as a side product of nuclear reactors. In a medium of length $L = 100$ cm with the concentration of isomeric nuclei $n_2 = 10^{16}$ cm^{-3} and a constant inflow of new isomeric nuclei at the rate of about 10^{15} cm^{-3}s^{-1}, it is possible to observe a continuous two-photon process at the ignition intensity exceeding 10^{20} phot cm^{-2}s^{-1}. These estimates seem not too reliable mainly because of the scarce experimental data about the coefficient of two-photon stimulated emission, which we set equal to 10^{-40} cm^4 s based on rather rough theoretical considerations.

It follows from the above discussion that it is unlikely that the method of two-photon stimulated emission of gamma rays in counterpropagating photon beams can be promising.

4.5 Bose–Einstein condensate (BEC) as a hypothetically promising gain medium for NGLs

4.5.1. Inherent inconsistency of NGLs based on long-lived isomers with a ultranarrow gamma-ray line

The analysis of all the concepts considered above has shown that they are hardly capable of solving the central NGL problem of accumulation of excited nuclei in the amount sufficient for the observation of the noticeable and useful amplification of the gamma-ray flux by eliminating simultaneously the excess broadening of the gamma-ray line to achieve the maximum stimulated emission cross section $\sigma = \lambda^2/2\pi$ (2) for $\beta \to 1$.

The first part of the problem is related to difficulties encountered in achieving the efficient pumping, which have been predicted already in fundamental paper [58] in1958 and have not been eliminated so far. The obvious solution of this problem would be the use of long-lived isomeric states, which do not require any pumping in the limiting case of natural isomers. Such an approach was discussed as early as 1961 and applied to the Mössbauer NGL scheme [8, 10].

Recall that for any and arbitrarily small probability of spontaneous emission $\tau_\gamma{}^{-1}$ (i.e. in long-lived isomers), the total stimulated emission cross section is $\sigma\beta$ (2), (3) and depends neither on the matrix-element value nor the multiplicity and the degree of the transition forbiddeness. The contribution of these factors is completely determined by the value of the Einstein coefficient $A_{21} = \tau_\gamma{}^{-1}$, which can be very small for isomeric states. The independence of the stimulated emission cross section $\sigma\beta$ of these factors is obvious and follows directly from the thermodynamic (and, hence, the most general and applicable to transitions of any type) derivation of the Einstein radiation laws with coefficients A_{21} and B_{21} in the form of a chain of equalities [125, 126]

$$\hbar\omega\,\frac{4B_{21}}{c\Delta\omega_{\rm tot}} = \frac{A_{21}}{2\pi}\,\frac{\lambda^2}{\Delta\omega_{\rm tot}} = \frac{\lambda^2}{2\pi}\,\frac{1}{\Delta\omega_{\rm tot}\tau_\gamma} = \sigma\beta. \qquad (25)$$

However, the solution of the first half of the problem by using long-lived isomers severely contradicts the outlook for solving its second half by eliminating the excess broadening of the ultranarrow line of a long-lived isomeric state down to $\beta \to 1$.

These circumstances stimulate a search for new approaches to the radical reduction of the excess broadening of the emission gamma-ray line of long-lived isomers which now look like hypothetical possibilities rather than recommendations to the crucial experiment.

4.5.2. Hypothetical giant reduction of the excess broadening of gamma-ray lines of nuclei in a BEC

Advances in the manipulation of neutral atoms and in their laser-induced deep cooling introduced in fact into experimental studies another aggregate state of matter – a BEC [127, 128]. Such an atomic ensemble with the overlapping wave functions of individual boson atoms can be conditionally treated as a megaatom in which the differences between the states of atoms become minimal due to their mutual quantum coherence. This circumstance can cause the elimination of the main factors producing the inhomogeneous (in particular, Doppler) broadening of the gamma-ray line. The quantum coherence of the BEC considerably reduces the dispersion of random atomic velocities compared to the dispersion calculated from the thermodynamic temperature of the atomic ensemble. It seems that this should lead to the radical suppression of the inhomogeneous broadening of the gamma-ray lines of nuclei in atoms in the BEC down to $\beta \to 1$.

This hypothesis [125, 126] is used in the NGL scheme with the working substance consisting of isomeric nuclei belonging to BEC atoms. This scheme opens up the possibility to solve the conflict between the two above-mentioned antagonistic features of the NGL problem: the low probability of the spontaneous decay of isomeric nuclei, allowing their accumulation in a sufficient amount, and the quantum coherence of the BEC atoms creating the prerequisites for reducing the gamma-ray linewidth down to $\beta \to 1$ (3) and achieving the maximum value of the total stimulated emission cross section $\sigma\beta \to \sigma = \lambda^2/2\pi$.

Because the bond energy between BEC atoms in a gas is many orders of magnitude smaller than the nuclear recoil energy $E_{\rm rec}$ (5), radiative gamma transitions in the BEC are accompanied (as in the NGL scheme with deeply cooled free nuclei, see section 4.2) by the kinematic splitting of emission and absorption lines and the appearance of the hidden inversion. By the way, this also illustrates the cardinal difference of the BEC situation from the conditions of the Mössbauer effect: gamma-ray lines in the BEC narrow down in fact due to a decrease in the dispersion of atomic velocities and, hence, the suppression of the Doppler broadening rather than due to zero-phonon recoilless gamma transitions.

4.5.3. Quantum coherence of a BEC

The quantum coherence of a BEC is not, of course, absolute, and this circumstance determines the minimal inhomogeneous broadening of gamma-ray lines achievable within the framework of the accepted hypothesis. In the absence of external factors breaking the quantum coherence of the condensate, its fundamental value is determined by the natural lifetime $\Theta_{\rm BEC}$ of BEC atoms, which are in the stationary case in the dynamic equilibrium with the rest of the gas atoms [129]. This lifetime, which characterises the rate of continuous dynamic exchange by atoms between both fractions, has the same meaning as the natural lifetime of the excited states of quantum oscillators (atoms) with respect to spontaneous decay determining their natural radiative linewidth.

Studies of the dynamics of the condensed BEC fraction and various types of perturbations of the BEC coherence affecting the value of $\Theta_{\rm BEC}$ (see, for example, [130–135]) are considered in review [136].

Phenomena related to the deviation from the assumption about the ideal gas of noninteracting atoms can be used as an illustrative example, which however by no means characterises the variety of these perturbations [136]. This assumption for a gas in a quantum trap is based on two inequalities, which are fulfilled in many cases: $n|s|^3 \ll 1$ and $\varepsilon_{\rm int}/\varepsilon_{\rm kin} \ll 1$, where $|s| \sim 3 \times 10^{-7}$ cm is the scattering length; $\varepsilon_{\rm int}$ and $\varepsilon_{\rm kin}$ are the interaction energy of atoms in the ground state and the kinetic energy of an atom, respectively. In a special case, when the scattering length is negative ($s < 1$) and the energy $\varepsilon_{\rm int}$ considerably exceeds the distance between levels in the trap, a long-lived metastable BEC appears.

Unfortunately, at present the reliable theoretical or experimental estimates of $\Theta_{\rm BEC}$ are absent. Some theoretical attempts [137] to estimate the natural width of the BEC line cannot be considered reliable enough. And, of course, the confinement times of a BEC in traps measured in experiments at present characterise only the degree of perfection of the experimental method rather than the value of $\Theta_{\rm BEC}$.

Thus, the question about the value of $\Theta_{\rm BEC}$ remains open, whereas this value is fundamental for the problem under study. Despite this uncertainty and by following the accepted hypothesis, our analysis is based on the important assumption that the fundamental lifetime $\Theta_{\rm BEC}$ of a BEC can exceed the lifetime τ of the metastable state of an isomer with respect to its spontaneous radiative decay,

$$\Theta_{\rm BEC} > \tau, \qquad (26)$$

and the experimental method can be so perfect that the condensate confinement time in a trap is not shorter than $\Theta_{\rm BEC}$.

However, the elimination of the excess broadening of the gamma-ray emission line does not mean at all that the total single-pass gain $G > 1$ of gamma-ray quanta in an isomeric medium will exceed unity.

To obtain the quantitative notion about the situation, it is useful, similarly to section 4.1, to estimate the test parameter $G_{\rm A}$, which determines in fact the upper bound of the maximum achievable real value of the total gain $G < G_{\rm A}$:

$$G_{\rm A} = \exp(\sigma n L). \qquad (27)$$

For example, for typical values $\sigma = 10^{-18}$ cm^2, $L = 10^3$ cm, and $n = 10^{13}$ cm^{-3}, the test parameter $G_{\rm A}$ only slightly exceeds unity ($G_{\rm A} = 1.01$). This is the obvious consequence of the smallness of σ in the short-wavelength range and the limited concentration of nuclei n in the BEC (see below section 4.5.4). Thus, it is very difficult to achieve $G > 1$ in the isomeric BEC (if not impossible at all).

Note also that the test parameter was estimated by neglecting factors that can make the inequality $G < G_{\rm A}$ much stronger. Thus, instead of the concentration n^* of active isomeric nuclei directly involved in amplification, the total concentration $n > n^*$ was used in the expression for $G_{\rm A}$; we also neglected a decrease in the isomer concentration n^* due to the spontaneous decay of metastable states, a decrease in the stimulated emission cross section caused by the so-called laser lethargy (see below section 4.5.5), the nonresonance losses of gamma-ray photons in the medium, etc.

Thus, the real value of the total gain G is always lower than $G_{\rm A}$. Moreover, the neglect of negative factors listed above can make the achievement of $G > 1$ impossible at all. The factors causing this alarm are considered below.

4.5.4. Some external factors restricting the quantum coherence of a BEC

Along with the above-mentioned factors breaking the quantum coherence of a BEC, the coherence time is also limited by obvious external perturbations related to the inevitable simultaneous presence of two gas fractions at the gas temperature T not equal to the absolute zero [129].

An atom of the BEC undergoes a transition from the lower state and is evaporated from the condensed fraction if a finite momentum and energy are imparted to it after a collision with another atom of the gas. The average time between collisions in a simple model of gas of an unbounded volume is

$$\Delta t_{\rm col} = \frac{[\sigma_{\rm col} u(T)]^{-1}}{n - n_{\rm BEC}} = 0.32\,\frac{(2J_{\rm a}+1)^{1/3} M}{\hbar\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2$$

$$\approx 500\,\frac{(2J_{\rm a}+1)^{1/3} A}{\sigma_{\rm col} n^{4/3}}\left(\frac{T_0}{T}\right)^2, \qquad (28)$$

where $\sigma_{\rm col}$ is the collision cross section; $u(T) = (3k_{\rm B}T/M)^{1/2}$ is the root-mean-square thermal velocity of gas atoms of mass M;

$$n_{\rm BEC} = n\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right] \qquad (29)$$

is the concentration of BEC atoms; $J_{\rm a}$ is the angular momentum of the atom;

$$T_0 = 3.3\,\frac{\hbar^2 n^{2/3}}{k_{\rm B} M (2J_{\rm a}+1)^{2/3}} \qquad (30)$$

is the degenerate and condensation temperature of atoms. Let us present a numerical example: $\Delta t_{\rm col} \approx 10^5$ s for $n = 10^{12}$ cm^{-3}, $A = 100$, $T_0/T = 1.3$ and $\sigma_{\rm col} = 10^{-16}$ cm^2.

Therefore, if even $\Theta_{\rm BEC} > \tau$ (26), the requirement $\Delta t_{\rm col} > \tau$, which should be fulfilled to obtain $\beta \to 1$ (3), restricts the total concentration of gas atoms:

$$n < 0.43\left(\frac{M}{\hbar\sigma_{\rm col}\tau}\right)^{3/4}(2J_{\rm a}+1)^{1/4}\left(\frac{T_0}{T}\right)^{3/2}$$

$$\approx 107\left(\frac{A}{\sigma_{\rm col}\tau}\right)^{3/4}(2J_{\rm a}+1)^{1/4}\left(\frac{T_0}{T}\right)^{3/2} \qquad (31)$$

and its temperatutre

$$T < 1.9\hbar^{3/2} k_{\rm B}^{-1}[(2J_{\rm a}+1)M\sigma_{\rm col}\tau]^{-1/2}$$

$$\approx 3.6\times10^{-13}(\sigma_{\rm col}A\tau)^{-1/2}(2J_{\rm a}+1)^{-1/2}. \qquad (32)$$

For the numerical example presented above and $\tau = 60$ s, this gives $n < 2.3\times10^{14}$ cm^{-3} and $T < 0.45$ μK.

4.5.5. 'Laser lethargy'

The long lifetime of isomeric metastable states and the extremely narrow natural radiative width of transitions introduce significant features into the transient process of stimulated emission, which is called the *laser lethargy* [11, 138–141].

The time dependence of the transient process is determined by the kinetics of the transition cross section $\sigma(t)$ and the asymptotic nature of the establishment of the spectrum of an acting electromagnetic wave, which changes from the infinite width and zero amplitude at the zero instant to a finite stationary value at infinity. The latter dependence is in fact the consequence of the classical Fourier uncertainty relation (see, for example, [142]). Such asymptotics can be conveniently represented by a simple exponential model of the time dependence of the current value of the stimulated emission cross section [143]:

$$\sigma(t) = \frac{\lambda^2}{2\pi}\left[1 - \exp\left(-\eta\,\frac{t}{\tau}\right)\right], \qquad (33)$$

where $\eta = $ const. Such a behaviour of $\sigma(t)$ leads to negative phenomena in the form of the delay in the appearance of stimulated emission, which is especially large for metastable states with large τ and should be taken into account in the analysis of the dynamics of the amplification process.

4.5.6. Some examples of isomeric nuclei belonging to boson atoms

The choice of an isomeric nucleus is determined by the lifetimes of metastable states, which vary from fractions of microseconds to many thousands of years. The choice should take into account several mutually contradictory considerations.

On the one hand, it is attractive to use long-lived isomers, which can be simply prepared (pumped), while in the case of natural isomers, the pumping is not requires at all. However, the laser lethargy mentioned above, which delays the onset of amplification by the time close to the metastable-state lifetime τ, although does not produce the absolute theoretical prohibition, makes long-lived isomers unattractive for experimenters.

On the other hand, very short lifetimes τ require very intense and rapid pumping to prepare an isomer and, of course, exclude the use of natural isomers.

It seems that isomers with moderate lifetimes τ can be most efficiently produced in the reaction of radiative capture of neutrons. Thus, in the typical case, thermal neutrons with the flow density $\sim 10^{13}$ cm^{-2} s^{-1} can produce isomers on the linear region of the capture reaction at a rate of 10^6 cm^{-2} s^{-1} from parent isotopes with the capture cross section $\sigma(n,\gamma) = 10^{-22}$ cm^2 and concentration $n_{\rm m} = 10^{15}$ cm^{-3} up to achieving the saturated isomer concentration of the order of $10^6\tau$ cm^{-3}. These estimates can be increased by several orders of magnitude taking into account the possibilities of new promising sources of thermal neutrons with the flow density up to 10^{20} cm^{-2} s^{-1} in a pulse of duration ~ 100 μs [144]. The passage from the linear region to saturation, when the reaction efficiency decreases due to the spontaneous decay of metastable states of the isomer, occurs for the time $\sim\tau$ during which the isomer concentration increases, approaching the saturated value. This means that it is undesirable to use short-lived isomers produced in the reaction of radiative capture of neutrons.

The estimate of the schemes of incoherent X-ray excitation, which could be of interest for the formation of rapidly decaying metastable states ($\tau < 1$ s), shows that these schemes are hopeless because the spectral density of radiation sources available at present is many orders of magnitude lower than that required for pumping. Thus, at present the compromise is the choice of isomers with

Table 1.

Isomer	$\hbar\omega$/keV	τ	Parent isotope in the (n,γ) reaction and its lifetime	$\sigma(n,\gamma)/10^{-24}$ cm^2 for the neutron energy 0.0253 eV	$\sigma/10^{-20}$ cm^2	$\chi/10^{-23}$ cm^2
$^{91}_{39}$Y	**555**	49.7 min	$^{90}_{39}$Y, 64.1 h	< 6.5	0.8	~ 1.4
$^{95}_{41}$Nb	**235**	86.6 h	$^{94}_{41}$Nb, 2×10^4 years	14.9	4.5	~ 2.5
$^{105}_{45}$Rh	**129**	45 s	$^{104}_{45}$Rh, 42.3 s	40	14.5	~ 10
$^{111}_{47}$Ag	**60**	64.8 s	$^{110\mathrm{m}}_{47}$Ag, 250 days	82	68.5	~ 100
$^{135}_{55}$Cs	**781**, 840	53 min	$^{134}_{55}$Cs, 2 years	140	0.405	~ 2
$^{178}_{72}$Hf	**89**, 313, 326, 426	4.0 s	$^{177}_{72}$Hf, stable	365	31.2	~ 40
$^{180}_{72}$Hf	**58**, 215, 333, 444	5.5 h	$^{179}_{72}$Hf, stable	45	73	~ 90
$^{190}_{76}$Os	**187**, 361, 502, 616	9.9 min	$^{189}_{76}$Os, stable	23	7.1	2
$^{193}_{77}$Ir	**80**	10.6 days	$^{192}_{77}$Ir, 74.2 days	1100	38.5	300

Note: The energy values $\hbar\omega$ in bold were used to calculate σ and χ.

moderate values of τ, lying between a few tens of seconds to a few tens of minutes.

Table 1 presents the parameters of isomers [145] contained in boson atoms, which can be produced in the reaction of radiative capture of thermal neutrons. The table demonstrates wide ranges of metastable-state lifetimes τ, energies $\hbar\omega$ of gamma rays, cross sections $\sigma(n,\gamma)$ for the radiative capture of neutrons, stimulated emission cross sections σ, averaged cross sections χ for photon losses, etc.

These parameters are often antagonistic with respect to the outlook for using the corresponding isomers in experiments on stimulated emission of gamma rays. If we also add atomic characteristics which are important for the efficient formation of the BEC, it seems unlikely that the optimal combination of atomic and nuclear parameters can be found.

It is obvious from the above discussion that the choice of the object for estimates is quite arbitrary. Below, we performed estimates (without pretensions for any optimisation) for silver $^{111}_{47}$Ag and cesium $^{135}_{55}$Cs isomers representing both limits for acceptable lifetimes τ.

4.5.7. Experimental configuration ('quantum conveyer' for atoms)

To discuss the outlook for realising an NGL based on isomers contained in a BEC, it is necessary to specify a possible experiment involving the preparation of the amplifying nuclear medium (including the accumulation of the isomer and BEC formation) and amplification of a gamma-ray flux. One of the possible versions of such an experiment is the construction of the so-called quantum conveyer for atoms [146], representing an extended quantum well (trap) with the transverse potential over coordinates x and y (in particular, parabolic)

$$U(x,y) = a\left(x^2 + y^2\right) \qquad (34)$$

holding atoms (a is a coefficient), in which a flow of Bose atoms propagates along the coordinate z. Such an atomic ensemble is described by the wave function of the travelling wave type with the eigenvalues

$$E_{km} = \hbar\left(\frac{2a}{M}\right)^{1/2}(k+m+1) \qquad (35)$$

of the Schrödinger equation over transverse coordinates and the longitudinal component of the wave vector $p_{km}/\hbar$

$$p_{km} = \pm[2M(E_a - E_{km})]^{1/2}, \qquad (36)$$

where $E_a = \text{const}$ is the total energy of the atom and k and m are integer subscripts. Such waveguide-like channels (traps) were used for deep cooling of neutral atoms [147].

According to the scenario of the experiment, atoms with nuclei moving along the channel are subjected to a number of successive operations taking place in certain regions: the preparation of the isomer (region I), the enrichment of the atomic ensemble with the isomer (region II), the deep laser-induced cooling and formation of the directed atomic flow (region III), the deceleration of the atomic flow and increasing its concentration (region IV), and, finally, the amplification of the flux of gamma-ray photons upon their stimulated emission by isomers (region V) (Fig. 1) [129].

The fourth operation performed in the quantum conveyer with the potential-well depth increasing with z [146]

$$\frac{\mathrm{d}a}{\mathrm{d}z} > 0, \quad \frac{\mathrm{d}U}{\mathrm{d}z} > 0 \qquad (37)$$

plays an important role in this scenario.

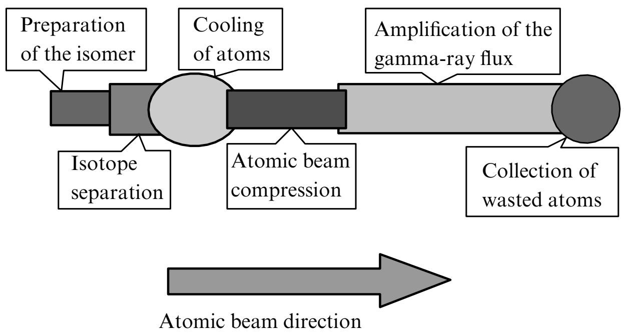

Figure 1. Principal scheme of the quantum conveyer.

If this increase occurs slowly enough, the general type of the solution in the form of a travelling wave is retained, but with gradually increasing energy eigenvalue E_{km} (35) and decreasing component of the wave vector $p_{km}/\hbar$ (36)

$$\frac{dE_{km}}{dz} > 0, \quad \frac{dp_{km}}{dz} < 0. \tag{38}$$

Simultaneously, the effective transverse size D of the potential well decreases ($dD/dz < 0$); for example, for the lower state with $k = m = 0$, we have

$$D = 2\left(\frac{2\hbar^2}{aM}\right)^{1/2} = 2\hbar\left(\frac{2}{ME_{00}}\right)^{1/2}. \tag{39}$$

Thus, in the case of a constant value of the continuous total atomic flow, the slowing down of the flow occurs (the transport velocity $V = p_{km}/M$ decreases) and the concentration n increases:

$$\frac{dV}{dz} < 0, \quad \frac{dn}{dz} > 0. \tag{40}$$

For a flow of atoms of the condensed fraction in the lower state with $k = m = 0$ and moving from the coordinate z_1 to z_2, it is convenient to characterise this procedure by kinematic coefficients [129, 143]: the drag coefficient

$$\Xi^V_{00}(z_2, z_1) = \frac{V(z_2)}{V(z_1)} = \frac{1 - E_{00}(z_2)/E_a}{1 - E_{00}(z_1)/E_a} < 1\,, \tag{41}$$

the coefficient of compression of the flow cross section S_{00}

$$\Xi^S_{00}(z_2, z_1) = \frac{S_{00}(z_2)}{S_{00}(z_1)} = \frac{E_{00}(z_1)/E_a}{E_{00}(z_2)/E_a} < 1 \tag{42}$$

and the resulting coefficient of the flow compression

$$\Xi_{00}(z_2, z_1) = \frac{n(z_2)}{n(z_1)} = \left[\Xi^V_{00}(z_2, z_1)\Xi^S_{00}(z_2, z_1)\right]^{-1}$$

$$= \frac{E_a/E_{00}(z_1) - 1}{E_a/E_{00}(z_2) - 1} > 1. \tag{43}$$

Thus, in the ideal case of noninteracting atoms, the gas concentration would increase indefinitely with approaching a critical cross section of the channel with the coordinate $z \to z_{km}$, by reproducing the phenomenology of a shock wave, which, of course, does not occur in reality. The obvious appearing limit is the transition of gas to the usual condensed phase.

However, the Bose condensation of a certain part of gas atoms can also occur before this transition under certain conditions if for a constant temperature T before approaching $z = z_{km}$, the increase in the concentration n leads to the increase in the degeneracy temperature so that the inequality $T_0/T > 1$ required for the formation of a BEC is fulfilled. Such a phase transition only formally differs from the usual phase transition: the condition $T_0/T > 1$ is achieved not by decreasing the gas temperature for $n = \text{const}$ but by increasing T_0 due to an increase in the concentration n for $T = \text{const}$.

4.5.8. Some numerical estimates

The numerical estimates of the possibility of achieving the total single-pass amplification $G > 1$ for isomers in a BEC by using the scenario proposed above were performed for the $^{111}_{47}Ag$ and $^{135}_{55}Cs$ isomers [129] taking into account the laser lethargy, spontaneous decay, and the requirement to eliminate the time-of-flight broadening, which is necessary to satisfy condition $\beta \to 1$ and consists in the restriction of the length L of the region of interaction with the gamma-ray field by the inequality

$$L > V\tau. \tag{44}$$

Isomeric nuclei in region I are prepared by irradiating parent isotopes with concentrations $n_m \sim 10^{15}$ cm^{-3} by a flux of thermal neutrons of density $\sim 10^{13}$cm^{-2} s^{-1}. The someric nuclei are produced in the reactions $^{110}_{47}Ag\,(n,\gamma)^{111}_{47}Ag$ or $^{134}_{55}Cs\,(n,\gamma)^{135}_{55}Cs$ of the radiative capture of neutrons at a rate of $\sim 10^6$ cm^{-3} s^{-1}. The neutron exposure time is specified by the residence time Δt_n of atoms in region I, which is assumed small compared to the isomer lifetime τ ($\Delta t_n \ll \tau$), which allows us to neglect the influence of the spontaneous decay of metastable states. Then, the resulting concentrations n^*_I of isomers $^{111}_{47}Ag$ and $^{135}_{55}Cs$ in this region are 0.82×10^6 cm^{-3} for $\Delta t_n = 1$ s and 0.84×10^8 cm^{-3} for $\Delta t_n = 60$ s.

Then, the gaseous isomeric mixture and remnants of paternal isotopes enter region II, where the laser separation of the parent isotope from the isomer is performed (for example, by the method of two-step selective photoionisation [148, 149]) and ionised parent isotopes are extracted by an external electric field (in [149], the 100 % extraction was performed – up to 10^{13} ions from 1 cm^3). In region III, the further laser-induced deep cooling of the ensemble is performed. These procedures, based on the laser isotope separation, repeat partially one of the NGL proposals [36, 150]. If these operations are performed for the time much shorter than τ, the spontaneous decay of isomers can be neglected, as earlier.

The directed beam of cold atoms with isomeric nuclei is produced by the laser manipulation of neutral atoms {thus, in [151] a directed cooled sodium flow was obtained (10^9 atoms per second, $V = 5 \times 10^4$ cm s^{-1})}.

Then, the atomic beam is loaded into a trap with the depth of the transverse potential well increasing with increasing coordinate z (see section 4.5.7). Here, in region IV, the atomic beam is transformed with the kinematic coefficients $\Xi^V_{00} = 5 \times 10^{-5}$, $\Xi^S_{00} = 10^{-4}$, and $\Xi_{00} = 2 \times 10^8$ for silver and $\Xi^V_{00} = 6.6 \times 10^{-3}$, $\Xi^S_{00} = 10^{-3}$, and $\Xi_{00} = 1.5 \times 10^5$ for cesium.

The evolution of the atomic beam in region IV continues until the high concentration of atoms is achieved in the beam at the point z_{BEC} and atoms form a BEC with concentrations $n = 1.8 \times 10^{14}$ and 1.3×10^{13} cm^{-3} for silver and cesium, respectively. This is the final event in region IV. Then, the mixture of atoms with isomeric and other nuclei, both in the BEC and usual gas, is directed into amplification region V. Here, the atomic beam propagates at a constant velocity in the quantum trap with a constant depth of the potential well along the z axis. This atomic mixture contains isomers and other nuclei and represents the active medium for amplification of gamma-ray photons, which has the following properties.

It is assumed that the fulfilment of conditions (31), (32), and (44) means the complete elimination of the excess broadening of the gamma-ray emission line ($\beta \to 1$) and the equality of the total value of σ to $\lambda^2/2\pi$.

The resonance amplification frequency for isomers moving in the laboratory coordinate system with the transport velocity V proves to be shifted with respect to the nuclear transition energy due to the Doppler effect.

Amplification occurs under the conditions of hidden inversion, when the mutual kinematic shift of gamma-ray emission and absorption lines by the doubled recoil energy of the nucleus excludes the resonance absorption of photons by unexcited nuclei, which is reflected in the absence of the corresponding term with their concentration in the rate equation (45) for photons. This equation takes into account the asymptotic behaviour of the current value of stimulated emission in the exponential model (33), which begins to manifest itself with penetrating nuclei into the region of interaction with the photon field in amplification region V.

As a result, the rate equation for the photon flux of density $\Phi(z)$ takes the form

$$g(z) \equiv \frac{1}{\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}z} = \sigma n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})\left[1-\exp\left(-\eta\,\frac{z-z_{\mathrm{BEC}}}{V(z_{\mathrm{BEC}})\tau}\right)\right]$$
$$\times\exp\left(-\frac{z-z_{\mathrm{BEC}}}{V(z_{\mathrm{BEC}})\tau}\right) - \chi n(z_{\mathrm{BEC}}), \tag{45}$$

where $g(z)$ is the local gain; $n(z_{\mathrm{BEC}})$ and $n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})$ are the total concentrations of atoms and isomers, respectively; and $V(z_{\mathrm{BEC}})$ is their transport velocity at the input to amplification region V at the point $z = z_{\mathrm{BEC}}$. The first exponential factor in (45) reflects the laser lethargy (33), and the second one – the spontaneous decay of metastable states.

The requirement that the maximum of the local gain g achieved at the point

$$z_0 = z_{\mathrm{BEC}} + \frac{V(z_{\mathrm{BEC}})\tau}{\eta}\ln(1+\eta), \tag{46}$$

should be positive determined the critical concentration of isomers at the input to region V

$$\left.\frac{n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})}{n(z_{\mathrm{BEC}})}\right|_{\mathrm{crit}} = \frac{\chi}{\sigma\eta}\,(1+\eta)^{1/\eta+1}, \tag{47}$$

below which even local amplification is impossible.

For $\eta = 1$, we have $n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})/n(z_{\mathrm{BEC}})|_{\mathrm{crit}} \approx 6\times10^{-3}$ and 2×10^{-2} for silver and cesium, respectively.

The asymptotic behaviour of the current value of the stimulated emission cross section causes the shift of the coordinate of the amplification onset to $z_1 > z_{\mathrm{BEC}}$ (laser lethargy), while the depletion of the isomer concentration due to spontaneous decay gives the coordinate $z_2 > z_0 > z_1 > z_{\mathrm{BEC}}$ of the gain disappearance. These characteristic points $z_{1,2}$ are determined as the roots of the equation

$$\frac{1}{\eta}\exp\left(-\frac{z_{1,2}-z_0}{V(z_{\mathrm{BEC}})\tau}\right)\left[\eta+1-\exp\left(-\eta\,\frac{z_{1,2}-z_0}{V(z_{\mathrm{BEC}})\tau}\right)\right]$$
$$= \left[\frac{n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})}{n(z_{\mathrm{BEC}})}\right]^{-1}\left[\frac{n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})}{n(z_{\mathrm{BEC}})}\right]_{\mathrm{crit}}. \tag{48}$$

It follows from this that the length L of the region of interaction of isomers with the field can be reasonably limited by the inequalities

$$z_1 \leqslant L + z_{\mathrm{BEC}} \leqslant z_2. \tag{49}$$

The total gain G over the entire length of the interaction region is determined by the expression

$$\ln G = \sigma n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})V(z_{\mathrm{BEC}})\tau\left[1-\exp\left(-\frac{L}{V(z_{\mathrm{BEC}})\tau}\right)\right]$$
$$-\frac{\sigma n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})V(z_{\mathrm{BEC}})\tau}{1+\eta}$$
$$\times\left[1-\exp\left(-\frac{1+\eta}{V(z_{\mathrm{BEC}})\tau}\,L\right)\right] - \chi n(z_{\mathrm{BEC}})L. \tag{50}$$

The requirement of the single-pass gain $G \geqslant 1$ specifies the relative threshold concentration of isomers at the input to amplification region V:

$$\left.\frac{n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})}{n(z_{\mathrm{BEC}})}\right|_{\mathrm{thr}} \geqslant \frac{\chi L}{\sigma V(z_{\mathrm{BEC}})\tau}$$
$$\times\left\{\left[1-\exp\left(-\frac{L}{V(z_{\mathrm{BEC}})\tau}\right)\right]\right.$$
$$\left.-\frac{1}{1+\eta}\left[1-\exp\left(-\frac{1+\eta}{V(z_{\mathrm{BEC}})\tau}\,L\right)\right]\right\}^{-1}. \tag{51}$$

In this case, the ratio $L/V(z_{\mathrm{BEC}})\tau > 1$ satisfying inequality (44) can be found from the condition that the maximum gain $G_{\max}$ is achieved for $L = L_{\max}$ determined as the second root of equation (48) ($L_{\max} = x_2 - z_{\mathrm{BEC}}$).

Figure 2 shows schematically the dependences $g(z)$ and $G(z)$ demonstrating transitions from the initial absorption to amplification and then to saturation at characteristic points z_1, z_2, and z_0.

For silver and cesium, $L_{\max}/V(z_{\mathrm{BEC}})\tau = 5.3$ and 4.1, respectively, i.e. $L_{\max} = 6.48\times10^2$ and 3.9×10^4 cm, and the relative threshold concentrations of the isomer are $[n^*_{\mathrm{BEC}}(z_{\mathrm{BEC}})/n(z_{\mathrm{BEC}})]_{\mathrm{thr}} = 0.016$ and 0.0425, respectively, which exceeds, of course, critical concentration (47). As a result, the maximum total gain (50) is $G_{\max} = 1.0023$ for

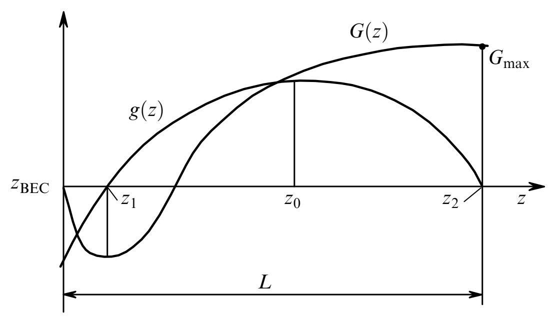

Figure 2. Amplification of the gamma-ray flux.

$n(z_{BEC}) = 1.8 \times 10^{14}$ cm^{-3} for silver and 1.0013 for $n(z_{BEC}) = 1.3 \times 10^{13}$ cm^{-3} for cesium.

The optimal lengths L_{max} of the region of interaction between isomers and the photon field obtained above, which correspond to the maximal gain G_{max}, may be inconvenient for experiments due to a long length. This difficulty can be easily removed by decreasing the length L down to $L < L_{max}$ at the expense of a small decrease in the total gain because amplification near $L \approx L_{max}$ saturates and occurs at the concentration of isomeric nuclei almost exhausted due to spontaneous decay.

4.5.9. Critical remarks

Thus, numerical estimates within the framework of assumptions made above do not exclude the principal possibility of the amplification of a flux of gamma-ray photons emitted from metastable states of isomeric nuclei despite the existence of strong negative factors that can prohibit amplification. The estimated values of the total gain only slightly exceed unity, in accordance with the value of the test parameter G_A (27). The use of the so-called quantum conveyer for atoms solves the problem of the atomic-beam compression, which is simultaneously a key problem for achieving the super-threshold concentration of isomers and overcoming the critical conditions of the BEC formation without rapid decreasing the gas temperature.

Here, however, an important remark should be made. The gain was estimated by using the concept of the Bose condensation of free atoms of the continuous spectrum and corresponding expressions for the concentration of the Bose fraction $n_{BEC} \equiv f_1(T/T_0)$ (29) and the critical temperature $T_0 \equiv f_2(n)$ (30). However, the atoms in the scenario with the quantum conveyer are not free but are located in a potential well with discrete states, and it is assumed in [129] that this does not lead to principal differences, introducing only some quantitative corrections to the estimates. As a matter of fact, this question is much more complicated, because the passage from free atoms to atoms in a quantum trap with the discrete spectrum of states causes substantial changes even in the form of functions f_1 and f_2. For example, in a completely closed trap these functions have the form [136]

$$n_{BEC} \equiv f_1\left(\frac{T}{T_0}\right) = n\left[1 - \left(\frac{T}{T_0}\right)^3\right], \qquad (52)$$

$$T_0 \equiv f_2(n) = 0.94\,\frac{E_{00}}{k_B}\,(nv)^{1/3}, \qquad (53)$$

which considerably differs from (29) and (30) (v is the effective volume of a quantum trap). In the case of a quantum conveyer with a partially closed trap with the transverse confinement and free longitudinal movement, the functions f_1 and f_2 should differ both from (29), (30) and (52), (53), which requires a separate detailed analysis, whose results can differ noticeably from the results obtained above.

Recall once more that the question of the fundamental value of the lifetime Θ_{BEC} of atoms in a BEC remains open and requires theoretical and experimental studies. Therefore, it seems that it is not time yet to make full-scale experimental attempts to observe stimulated emission of gamma rays by long-lived isomers contained in a BEC. However, it is undoubtedly necessary to study theoretically and experimentally various elements of the experimental scenario proposed above, such as the quantum conveyer for atoms, the phase transition to a BEC at a constant temperature and increasing gas concentration in a partially closed quantum trap, the theoretical determination of the form of functions $f_1(T/T_0)$ and $f_2(n)$, the more substantiated choice of coefficients η in (45), etc.

These studies are also important because it is interesting to observe the stimulated emission of nuclei contained in a BEC not only in long-lived nuclear isomers but also in atoms, for example, from the metastable $2\,^3S_1$ state of atomic helium emitting $\sim$ 20-eV VUV photons. This experimental configuration was simulated in [143, 152, 153].

5. Conclusions

5.1 Comparison and classification of different NGL concepts

The main conclusion of the discussion presented in the paper confirms the initial thesis that the central inherent conflict of any concept of a nuclear gamma-ray laser is the antagonism between the necessity to accumulate the critical amount N_A of acting excited nuclei (see section 4.1), which is sufficient for obtaining the acceptable quantum amplification, and the requirement to narrow the gamma-ray emission line down to its natural radiative width to obtain the maximum stimulated emission cross section. In this case, a considerable number N_A of nuclei should be achieved even in the absence of resonance absorption of gamma-ray photons by unexcited nuclei and when the real population inversion is not necessary.

The fundamental contradiction between these factors is that perturbations caused by pumping of the required high intensity and at the same time the limited spectral density of available pump sources force us to use isotopes with the relatively long lifetime τ of the upper level of the laser transition, whereas the longer is the lifetime τ, the more difficult is to approach the natural radiative width $\Delta\omega_\gamma$ of the gamma-ray line.

The attempts to find a compromise between these two conflicting factors, discussed in sections 3.2, 4.2, 4.3, 4.4, and 4.5, can hardly be considered successful from the point of view of the development of a NGL as a practical device.* Nevertheless, the three of them (sections 4.2, 4.3, and 4.5) can be distinguished as promising for the development, in particular, in the experimental aspect. To compare the positive and negative features of these (and other possible) NGL concepts, it is convenient to introduce their simple classification.

For this purpose, based on expression (1) for the local gain g [but, unlike estimates in section 4.1, not neglecting some parameters in (1)], we derive the expression for the total number of nuclei acting in the amplification channel with the cross section ΔS:

$$N_A \equiv n_2 L \Delta S = \frac{2\pi}{\beta}\,\ln G_A\,\frac{\Delta S}{\lambda^2}\,(1 - \rho_1 - \rho_2)^{-1}, \qquad (54)$$

* Here, we do not consider experimental papers [154, 155] in which photon counts of the doubled energy were observed and in fact only the attempt was made to verify the fulfilment of the Einstein radiation laws in nuclear physics.

where $G_A = \exp(gL)$ is the acceptable total exponential single-pass gain in a medium of length L and the coefficients

$$\rho_1 \equiv \frac{n_1}{n_2}\frac{2J_2+1}{2J_1+1}, \quad (55)$$

and

$$\rho_2 \equiv \frac{\chi}{\sigma\beta}\frac{n}{n_2} \quad (56)$$

characterise the population inversion and the relative level of nonresonance photon losses in a medium with the total concentration n of nuclei (atoms) of all types, respectively, and $\rho_1 + \rho_2 < 1$ above the amplification threshold.

The coefficients ρ_1 and ρ_2 and ratios $\Delta S/\lambda^2$ and β determine the quality of the NGL design if we assume that the quality parameter $Q \equiv N_A^{-1}$ characterising the NGL is higher, the smaller is the total number N_A of acting nuclei for the given value of G_A, i.e. the closer is β to unity and the smaller are other coefficients.

The introduced coefficients ρ_1 and ρ_2 allow us to classify and compare the NGL schemes considered above.

(i) In a condensed amplifying medium with the real inversion, the coefficients ρ_1 and ρ_2 are large enough and their sum will only slightly differ from unity. Thus, the Mössbauer NGL scheme (section 3.2) requires the presence of the real inversion because of the spectral coincidence of the nuclear gamma-ray emission and absorption lines. Therefore, ρ_1 cannot be very small. The number of acting nuclei is only a small fraction of the total number of atoms of a single-crystal matrix ($n \gg n_2$), and therefore ρ_2 also cannot be small even when for the small ratio $\chi/\sigma\beta \ll 1$, which is typical for many cases.

(ii) In a medium with the inversionless amplification (section 4.3), $\rho_1 = 0$, but ρ_2, as in scheme 1, remains not small, because it is assumed at present that the inversionless amplification with $\beta \to 1$ can be achieved only for the zero-phonon Mössbauer line in a condensed medium.

(iii) In the hidden inversion NGL scheme (section 4.2) and in a hypothetical scheme with nuclei in a BEC (section 4.5), the real inversion is not required, i.e. $\rho_1 = 0$, and the amplifying medium is a strongly rarefied gas with $n/n_2 \sim 1$, so that $\rho_2 \ll 1$.

As a result, by estimating the quality parameter Q by the minimal inverse value N_A of the total number of acting nuclei, we can rank different NGL schemes from the lowest parameter Q_i (scheme i) to the highest parameter Q_{iii} (schemes ii and iii): $Q_i \ll Q_{ii} \ll Q_{iii}$. One can see, however, from section 5.2 that this ranking is not complete.

Aside from such a formal comparison, it is useful to summarise the main features of different NGL concepts.

The NGL concept with deeply cooled ensembles of free nuclei with the hidden inversion (section 4.2) is attractive first of all because it attempts to reconcile fundamentally inherent contradictions preventing the performance of full-scale experiments on the observation of stimulated nuclear emission of gamma rays of the laser type.

The stimulated emission of gamma rays occurs in fact as the stimulated coherent backward scattering of pump photons under the condition of the hidden population inversion, which does not require the lasting accumulation of excited nuclei; thus, the requirement $\beta \to 1$ can be fulfilled for excited nuclear states with the emission line with not too small natural width. In this case, the pump does not produce perturbations due to a high transparency of the rarefied amplifying medium.

Other positive features of this NGL concept are that the production of deeply cooled beams of neutral atoms, their confinement in extended quantum traps, the generation of pump X-rays of high spectral density, etc. have been achieved in many experiments, and the list of possible nuclide candidates is quite extensive [156].

However, although the possibility of observing the stimulated emission of nuclei is undoubtedly attractive, unfortunately the complexity of a cumbersome experimental setup including the unique relativistic X-ray source of the last generation (parameters of possible X-ray sources are analysed in [157]) presents a contrast to the quantitative insignificance of the expected result (the low quantum yield), which makes problematic the development of simple NGLs, which could be transformed from a device for studies to a reliable routine laboratory tool.

Of the general physical interest is the inversionless NGL concept (section 4.3), which combines the Mössbauer spectroscopy with laser methods for obtaining coherent atomic states. The overcoming of the fundamental inconsistency of the NGL problem pointed out above is based, first, on the use of the zero-phonon Mössbauer gamma line of the natural width with $\beta \to 1$ and, second, on the theoretical and experimental advances in the development of lasers with inversionless amplification appearing due to the preparation of coherent nonabsorbing states of the lower level of the laser transition. This phenomenon applied to nuclear states would mean the elimination of the resonance absorption of gamma-ray photons by unexcited nuclei and, correspondingly, the creation of the prerequisites (as in the previous scheme, although for different reason) for the stimulated emission of gamma rays without the real nuclear population inversion, i.e. formally speaking, for any arbitrarily small but finite population of the upper level of the nuclear laser transition.

Unfortunately, as pointed out in section 4.1, this remarkable fact does not eliminate the necessity in the accumulation of a considerable amount of excited nuclei for maintaining the acceptable value of the total gain G and, hence, in the efficient pumping. The concept of the inversionless amplification of gamma rays can be considered in a certain sense as the development of the scheme of the Mössbauer NGL (section 3) because it is assumed that the nonabsorbing coherent nuclear states are produced under the conditions of the Mössbauer experiment. Therefore, as in section 3.2, anxieties remain that pumping of any type can be incompatible with the preservation of the zero-phonon gamma-ray line (for example, pumping can heat a matrix up to the inadmissible temperature about 100 K [109]). In addition, the maintenance of the Mössbauer condition $\beta \to 1$ is complicated by manipulations with sublevels with the aim of producing a nonabsorbing coherent state.

Nevertheless, these and other possible anxieties by no means can be considered as the absolute prohibition for the development of NGLs with inversionless amplification.

The NGL concept with the BEC gain medium (section 4.5) differs from all other NGL schemes by the insufficient theoretical development. This concept represents an attractive but hypothetical picture rather than a ready working material. The central assumption of this concept is that BEC atoms with overlapping wave functions, which form a *megaatom*, are radically deprived of individual motions, so that the inhomogeneous broadening of gamma-ray lines of nuclei contained in the BEC atoms is so suppressed that the condition $\beta \to 1$ is fulfilled even for isomeric states with long lifetimes τ and, hence, with extremely narrow lines of the natural radiative width. In this case, the fundamental contradiction of the NGL problem is solved because the condition $\beta \to 1$ specifies the maximum value of the total stimulated emission cross section, while long lifetimes τ solve the problem of the efficient nonperturbing or even preliminary pumping. Of course, many complex problems remain unsolved in this case as well (for example, the elimination of experimental difficulties related to the *laser lethargy* at long lifetimes), but they are not already fundamental.

Unfortunately, however, the proposed hypothesis has neither theoretical nor experimental proofs at present. The attractiveness of its possible consequences stimulates a thorough and extensive study of this problem, first of all, the elucidation of the concept of the natural lifetime of BEC atoms (the quantum coherence time) Θ_{BEC}.

The interesting assumption [158] about the possibility to control efficiently the coupling of nuclei with high-multiplicity quantum transitions and the electromagnetic field of specially formed photon beams having the corresponding angular momenta adequate to the transition multiplicity also requires further studies. It can be expected that such 'matching' of the field with quantum transitions will open up the possibility to control radiative processes in nuclei, in particular, to use long-lived isomers with strongly forbidden spontaneous transitions of high multiplicities.

One can see from the comparison of different NGL versions that, despite the different levels of their development and experimental demand and disadvantages inherent in each of them, we can conclude that concepts proposed in sections 4.2, 4.3, and 4.5 should be undoubtedly further studied, without preferring any of them.

Nevertheless, it is important to emphasise that the success of a full-scale experiment on the first observation of the stimulated emission of gamma-ray photons by nuclei by using any of the imperfect schemes proposed above would be an invaluable stimulus for the general progress of quantum nucleonics.

5.2 Uncertainty relation and the 'NGL hyperbola'

It has long been known by numerous examples that any perfect constructive and technological solution of a problem is not only useful in practice but also causes the esthetical satisfaction. However, neither of the NGL concepts considered above possesses this quality, in particular, due to cumbersome and intricate methods of their pumping. Therefore, a search for new NGL concepts will be undoubtedly continued.

The main inherent conflict of the NGL pointed out above is in fact determined by the fundamental restriction imposed by the uncertainty relation $\Delta\omega_\gamma\tau \geqslant 2\pi$ for the excited-state lifetime τ and the natural linewidth $\Delta\omega_\gamma/2\pi$ of the gamma-ray emission line. Indeed, by multiplying both sides of the inequality $\Delta\omega_\gamma \geqslant 2\pi/\tau$ by the total number of acting nuclei $N_{\mathrm{A}} = Q^{-1}$ (54) in the volume of the amplifying medium with the cross section ΔS, we obtain the relation

$$P\tau \geqslant \frac{2\pi}{\beta}\ln G_{\mathrm{A}}\frac{\Delta S}{\lambda^2}(1-\rho_1-\rho_2)^{-1} \equiv Q^{-1}, \tag{57}$$

where G_{A} is the total gain required for the NGL operation; $P = N_{\mathrm{A}}/\tau$ is the minimal pump rate, i.e. the number of excited nuclei produced in the medium volume per unit time by the pump of any type to compensate for their spontaneous decay and maintaining the relation $N_{\mathrm{A}} = \mathrm{const}$ by assuming that the influence of stimulated decay on the population of the upper level of the laser transition is negligibly small. The quantity Q^{-1} in (57) is a fixed free quality parameter of the device, which is determined by the restricted possibilities of an experimenter trying to reduce the ratio $\Delta S/\lambda^2$ and coefficients ρ_1 and ρ_2 and increase $\beta \to 1$.

Thus, expression (57) can be treated as a family of fundamental 'NGL hyperbolas' $P \geqslant Q^{-1}/\tau$ with the family parameter Q, which are the direct consequence of the uncertainty relation $\Delta\omega_\gamma\tau \approx 2\pi$.

Consider some illustrative examples with a rather arbitrary choice of values: $P\tau \geqslant 5\times10^9$ if $G_{\mathrm{A}} = 1.5$, $\Delta S/\lambda^2 = 10^8$, $\beta \to 1$, and $\rho_1+\rho_2 = 0.95$, i.e. $P \geqslant 5\times10^{15}\ \mathrm{s}^{-1}$ for $\tau = 10^{-6}$ s (Mössbauer schemes of different types, including the inversionless scheme); $P\tau \geqslant 2.5\times10^8$ for $\rho_1+\rho_2 \ll 1$ and the same other parameters, i.e. $P \geqslant 2.5\times10^{17}\ \mathrm{s}^{-1}$ for $\tau = 10^{-9}$ s (scheme with deeply cooled free nuclei with the hidden inversion), $P \geqslant 2.5\times10^6\ \mathrm{s}^{-1}$ for $\tau = 100$ s (hypothetical scheme with BEC nuclei), etc. One can see from these examples that the ranking of different NGL schemes by the pump level can noticeably differ from that presented in section 5.1, where the value of the lifetime was ignored.

As a result, the NGL schemes considered above and the field for searching new NGL concepts are located near the hyperbola $P \geqslant Q^{-1}/\tau$ (57).

5.3 Autonomous NGL?

By moving further along the right branch of hyperbola (57), we can suggest tentatively that probably the most adequate solution of the NGL problem is the development of an autonomous NGL without external pumping, in which a coherent beam of gamma-ray quanta is a direct product of the nuclear reaction of the release of the intrinsic nuclear energy, for example, the energy of metastable states of long-lived isomers. In this case, the central problem of the NGL is only the elimination of the excess broadening of the gamma-ray line by narrowing it down to the extremely small natural width ($\beta \to 1$) inherent in long-lived states. The methods for solving of the latter problem still remain problematic (see, in particular, section 4.5).

Although the autonomous NGL is quite attractive, its specific feature, which can be considered partially as negative, is that due to the very low probability of spontaneous emission, the generation can be initiated even at a high enough gain only after the seeding of photons into modes of the medium. In this case, the problem of the stability of the stationary generation of such an NGL after switching off seeding photons appears. The matter is that [159] the condition of stationary generation is not the equality of the total gain to the total photon losses, as usually accepted: losses always exceed the gain by a small dimensionless parameter $\mu \ll 1$ (the difference $1-\mu$ is equal to the ratio of the gain to losses); the deficit in the inflow of stimulated photons appearing for this reason is compensated by the supply of spontaneous photons to the generating modes of the medium. (By the way, the finiteness of the small parameter μ characterising the contribution of spontaneous photons to the laser radiation prevents, along with other reasons, the achievement of its complete coherence.)

Therefore, the switching off of the flow of external photons in the autonomous NGL considered here means in fact that the small but finite parameter μ should be directed to zero ($\mu \rightarrow 0$). The stability of such a transient process is not obvious.

5.4 Problems parallel to the NGL development problem

The study of radiative processes in nuclei, especially induced processes, is undoubtedly of interest aside from the NGL development problem. An illustrative example is the possibility (if the hypothesis about the coherent properties of a BEC is confirmed) of observing extremely narrow gamma-ray line of the natural linewidth, for which the coherence length is many orders of magnitude larger than that for Mössbauer gamma lines [160].

Of interest is also an aspect of the NGL problem such as the energy production problem [161 – 164]. The understanding of the fact that in a gamma-ray laser based on long-lived metastable isomers the exothermal chain nuclear reaction proceeds was reflected in the very first proposals [8] entitled 'A source of coherent gamma rays based on the chain reaction of induced nuclear transitions' (with a direct indication to one of the possible applications: 'a nuclear reactor as the energy source') and 'On the possibility of the chain reaction of the induced radiative transitions in excited nuclei'.

Indeed, the energy density stored in a number of isomers is ~ 100 MJ g^{-1}, which is approximately two orders of magnitude lower than the energy content of fissionable materials, but three orders of magnitude greater than the caloric value of the hydrocarbon fuel. The advantages and disadvantages of this intermediate position determine the place of the radiative nuclear chain reaction in the hierarchy of energy production. The main argument in favour of this position is, of course, of the ecological nature, namely, the absence of the long-lived radioactive waste.

Finally, as a contrast to the pragmatic problem of the NGL development, we can recall the exotic problem, lying beyond laboratory studies, such as the possible role of natural processes of the induced emission of gamma quanta in cosmological and astrophysical phenomena ('cosmic gamma-ray lasers') [165].

5.5 'Never say never'

Thus, however, will a nuclear gamma-ray laser or any other device emitting stimulated nuclear gamma-ray radiation be created one day (and when)?

The position of a prophet is usually dangerous, while the prediction of the scientific progress is at least an inconsiderate occupation. Here are several remarkable prophecies made by experts, who were undoubtedly quite qualified in their fields of science (taken from the book 'Specialists are talking', USA).

William Thomson (1895): 'Flying in an apparatus heavier than air is impossible'.

William Thomson (1897): 'Wireless telegraph has no future'.

William Thomson (1900): 'X-rays is only a joke'.

Ernest Rutherford (1919): 'Nuclear physics never will have any practical applications'.

Oliver Heaviside (1893): 'Transmission of electromagnetic waves through metal tubes (i.e. waveguides) is impossible'.

In the light of the above said and looking back to the Greats, it is reasonable to refrain from the answer to the question formulated above, but by following the wise rule 'never say never', we can risk to make the statement: *any natural phenomenon considered speculatively, which does not contradict to the fundamental laws of nature, can exist and will be observed experimentally sooner of later if the real demand in it appears*. This requires, in V.L. Ginzburg's words, only (!) the ideological 'breakthrough; (see section 1.1).

And finally, the words of the known physicist Dyson in Princeton: 'The best way to learn about the future is to stay alive as long as you can and see what happens' can serve as the optimistic conclusion of all the above-said and the parting words of the author to readers*.

*Dyson F J. – Institut for Advance Study, Princeton, NY 08540; e-mail: dyson@ias.edu.

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