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Study of velocities of dissipative Bragg solitons beyond the slowly-varying amplitude approximation

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Abstract. **The properties of dissipative solitons propagating in an active nonlinear fibre with a Bragg grating are studied without using the slowly-varying amplitude approximation. It is shown that dissipative solitons with close initial velocities acquire during their propagation certain values from a discrete set of velocities.**

Keywords: *dissipative solitons, Bragg grating, slowly-varying amplitude approximation.*

1. Introduction

Bragg gratings are widely used in modern optics and laser technology to provide frequency-selective transmission or reflection of light. They are widely used in single-mode optical fibres in modern fibreoptic communication systems [1 – 3]. Although at present fibre Bragg grating (FBGs) are often used in the linear regime, they are undoubtedly promising for essentially nonlinear regimes, including regimes of Bragg solitons – stable localised structures of highly intense laser radiation [2, 3].

Dissipative optical solitons, i.e. stable localised light structures in homogeneous nonlinear-optical media and systems with sources and energy losses, have properties interesting both for science and applications, for example, information processing [3 – 7]. Dissipative solitons in a passive one-dimensional photon-crystal film excited by external radiation have been considered recently in paper [8]. Dissipative solitons in active FBGs with the homogeneous amplification and absorption distribution in the longitudinal direction have been studied in [9] by using the slowly-varying amplitude approximation (SVAA). It was found that in this case Bragg solitons represent a one-parametric family, i.e. their velocity can be arbitrary in a certain range and detuning (frequency) – discrete. Some studies have been performed for immobile dissipative Bragg solitons without the SVAA. It has been found that in this case they are localised near the maxima of the refractive index grating [10 – 11]. In this paper, we discuss some other results obtained without the SVAA for propagating dissipative Bragg solitons.

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2. Theoretical model and initial relations

Consider the longitudinal propagation of counterpropagating waves in a single-mode fibre with a FBG and nonlinear (depending on the laser radiation intensity) amplification and absorption. We assume that the longitudinal distribution of the linear amplification and absorption is homogeneous. This model can be also used in the case of the inhomogeneous distribution of amplification and absorption for the periodic change in the fibre segments with amplification and absorption, if the lengths of these segments are small enough. The saturation of amplification and absorption is described by their power expansion taking into account the terms up to the fifth power in the field amplitudes. The inertialless saturation approximation is valid for the continuous regime and for pulses, whose duration exceeds the relaxation time of the medium. The wave equation for the electric field strength E can be written in the form:

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = 0, \qquad (1)$$

where $D(z,t) = E(z,t) + 4\pi[\chi_1 E(z,t) + \chi_3 E^3(z,t) + \chi_5 E^5(z,t)]$ is the electric inductance of the medium; χ_1, χ_3 and χ_5 are the coefficients of the linear and cubic polarisabilities and the polarisability of the fifth order, respectively, which in the general case are complex and their imaginary parts are responsible for the dissipative mechanisms of absorption and amplification. Let us assume that $E(z,t) = \mathrm{Re}\{[A(z,t)] \times \exp(-\mathrm{i}\omega_0 t)\}$, where ω_0 is the central radiation frequency. By using the approximation of a slowly-varying amplitude in time (but not along the longitudinal coordinate z), we obtain the equation for the change in the complex field amplitude $A(z,t)$:

$$\frac{\partial^2 A}{\partial z^2} - \frac{1}{c^2}\left\{[\varepsilon_0 + \varepsilon_1\cos(2\beta_{\mathrm{B}} z)]\left(-\omega_0^2 A - 2\mathrm{i}\omega_0\frac{\partial A}{\partial t}\right)\right.$$

$$+\varepsilon_2\left[-4\mathrm{i}\omega_0|A|^2\frac{\partial A}{\partial t} - 2\mathrm{i}\omega_0 A^2\frac{\partial A^*}{\partial t} - \omega_0^2|A|^2 A\right] +$$

$$+\varepsilon_3\left[-6i\omega_0|A|^4\frac{\partial A}{\partial t}-4i\omega_0|A|^2A^2\frac{\partial A^*}{\partial t}-\omega_0^2|A|^4A\right]\Bigg\}=0. \quad (2)$$

Here, the asterisk denotes a complex conjugate quantity; c is the speed of light in vacuum; $\varepsilon_0 = 1 + 4\pi\chi_1$ is the linear dielectric constant; the coefficient ε_1 is responsible for the small modulation depth of the linear dielectric constant, while ε_2 and ε_3 are propotional to the polarisability coefficients of the third and fifth orders, respectively ($\varepsilon_2 = 3\pi\chi_3$, $\varepsilon_3 = 5\pi\chi_5/2$); $\beta_B = \pi/\Lambda$ is the Bragg wave number; and Λ is the period of the Bragg grating.

3. Propagating dissipative Bragg solitons in the slowly-varying amplitude approximation

Within the framework of the SVAA over the longitudinal coordinate z, the general solution of Eqn (2) has the form

$$E(z,t)=\frac{1}{2}[A_f(z,t)\exp(i\beta_B z)+A_b(z,t)\exp(-i\beta_B z)]$$
$$\times\exp(-i\omega_0 t)+\text{c.c.}, \quad (3)$$

where $A_{f,b}(z,t)$ are envelopes of the waves propagating forward and backward along the z axis, respectively. Then, we obtain from (2) a system of equations for coupled modes

$$\frac{1}{V_g}\frac{\partial A_f}{\partial t}+\frac{\partial A_f}{\partial z}=i\chi A_b+[i(\delta_1+i\delta_2)+i\Gamma(|A_f|^2+2|A_b|^2)$$
$$+iS(|A_f|^4+3|A_b|^4+6|A_f|^2|A_b|^2)]A_f,$$
$$(4)$$
$$\frac{1}{V_g}\frac{\partial A_b}{\partial t}-\frac{\partial A_b}{\partial z}=i\chi A_f+[i(\delta_1+i\delta_2)+i\Gamma(|A_b|^2$$
$$+2|A_f|^2)+iS(|A_b|^4+3|A_f|^4+6|A_f|^2|A_b|^2)]A_b,$$

where V_g is the group velocity in the fibre in the absence of a FBG; $\delta_1 = (\omega_0 - \omega_B)/V_g$ is the detuning from the Bragg frequency ω_B; $\delta_2 = 2\pi\text{Im}(\chi_1)\omega_0/[c\text{Re}(\varepsilon_0)]$ is the parameter of linear absorption or amplification; $\chi = \omega_0\varepsilon_1/[4c\times(\text{Re}(\varepsilon_0))^{1/2}]$ is the coupling coefficient caused by the grating, which appears upon interaction between the forward and backward waves; $\Gamma \equiv \gamma_1 + i\gamma_2 = \omega_0\varepsilon_2/[2c(\text{Re}(\varepsilon_0))^{1/2}]$ is the third-order nonlinearity parameter; $S \equiv s_1 + is_2 = \omega_0\varepsilon_3/[2c\times(\text{Re}(\varepsilon_0))^{1/2}]$ is the fifth-order nonlinearity parameter. In the general case, Γ and S are complex quantities and their imaginary parts are responsible for the dissipative mechanisms of absorption and amplification. System of coupled modes (4) has been studied and its numerical soliton solution $A_{f,b}(z,t)$ has been found in [9]. A dissipative Bragg soliton propagating at a relative velocity of $\upsilon = 0.024$ is shown in Fig. 1. Figure 1a shows the intensity profiles $I_{f,b}(z) = |A_{f,b}|^2$ of the forward and backward waves. Here and in Fig. 2, the electric field amplitudes are normalised as $A_n = A(\gamma_1/\chi)^{1/2}$ (index n is omitted). The empiric relation $(I_f - I_b)/(I_f + I_b) \approx \upsilon$ is fulfilled for the intensities in the centre of the soliton. Figure 1b shows the phase difference $\Phi(z)$ of the forward and backward waves for the same propagating dissipative soliton. Note that its relative velocity υ in this approximation can be chosen arbitrarily, and for this velocity, other parameters of the medium being fixed, the frequency detuning of the dissipative Bragg soliton is the required quantity with a discrete set of possible values. Thus, within the framework of the SVAA, Bragg dissipative solitons represent a one-parametric family, unlike the two-parametric family of conservative (without amplification and absorption of the electric field) Bragg solitons, because both the velocity and detuning for a conservative soliton can be simultaneously arbitrary quantities [3].

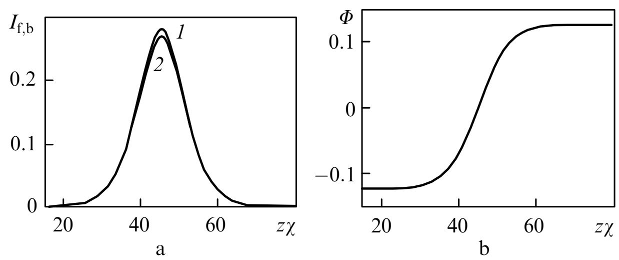

Figure 1. Profiles of intensities $I_{f,b}(z)$ (a) of the forward (*1*) and backward (*2*) waves and phase differences $\Phi(z)$ (b) of the forward and backward waves of a propagating dissipative soliton for $\delta_2 = 0.022$, $\gamma_1 = 2$, $\gamma_2 = -0.15$, $s_1 = -0.15$, $s_2 = 0.15$, $\chi = 100$, $\delta_1 = 99.18431$ and $\upsilon = 0.024$.

4. Propagating dissipative Bragg solitons beyond the slowly-varying amplitude approximation

In the previous section, Bragg solitons have been studied within the framework of the SVAA. This approximation is also used to describe the properties of conservative Bragg solitons [3]. The use of this approximation is valid for solitons whose longitudinal length exceeds the modulation period (and, correspondingly, the light wavelength). However, in the case of shorter Bragg solitons, noticeable quantitative deviations from the predictions of the standard method of coupled modes are possible. Of more interest are qualitatively new effects whose analogue is absent when the standard approximation of coupled modes is used. If we consider an immobile Bragg soliton, it can be located in any place of the FBG within the framework of the SVAA because in this approximation all these positions are equivalent. However, it is obvious that the longitudinal modulation of the refractive index makes different positions of the soliton inequivalent (in a infinite FBG only the symmetry with respect to the displacement per grating period should be preserved).

It was shown in [10, 11] that beyond the SVAA, the stable longitudinal positions of immobile fibre Bragg solitons are located near the maxima of the refractive index grating. Note that in a wide-aperture cavity with a FBG the centres of stable immobile solitons are also localised in the transverse direction in the maxima of the FBG of the refractive index [12]. Thus, we can consider the refractive index grating for a soliton as a periodic potential $U(z)$ for a mechanical particle. The stable immobile soliton is located in the maxima of the refractive index grating, while the immobile particle – in the minima of the potential field. The particle deflected from its equilibrium position begins to oscillate near this position. The same effect can be observed for slow solitons, which are initially in the equilibrium position. This result can be obtained by solving numerically Eqn (2). As initial values for $t_0 = 0$, the numerically known solution of equations of coupled modes was written in the form

$$A(z, t_0) = A_{\mathrm{f}}(z)\exp(\mathrm{i}\beta_{\mathrm{B}}z) + A_{\mathrm{b}}(z)\exp(-\mathrm{i}\beta_{\mathrm{B}}z), \qquad (5)$$

where $A_{\mathrm{f,b}}(z)$ are envelopes of the forward and backward waves obtained within the framework of the SVAA.

At rather large initial perturbations of the particle, i.e. when its initial velocity is high enough, the particle overcomes the first potential barrier and moves ahead. We have found that faster dissipative Bragg solitons, which can overcome the first period of the refractive index grating, propagate in the grating with a weak velocity modulation. Figure 2 shows the time evolution of this soliton obtained by solving numerically (2) by using the soliton solution for $A_{\mathrm{f,b}}(z)$ within the framework of the SVAA; in this case, it was assumed that $\upsilon = 0.02$ [this parameter is necessary to calculate the initial value from expression (5)]. Note that the value of $|A(z,t)|^2$ is modulated in time upon propagation of a soliton in the grating and, in general, this structure stably repeats at a considerable time interval $\Delta t = 0.15$ (the normalised time $t_{\mathrm{n}} = \chi V t$). Figure 2 illustrates only a fragment of the soliton development for $\Delta t = 0.008$.

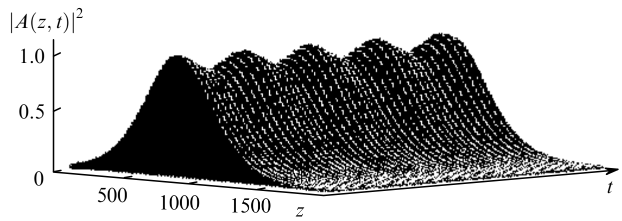

Figure 2. Time evolution of a dissipative Bragg soliton.

Figure 3 shows the displacement of the centres of these more fast dissipative Bragg solitons in time, where dark and light point correspond, respectively, to the displacements of soliton centres whose initial velocities υ of envelopes are 0.02 and 0.024 within the framework of the SVAA. It is obvious that at the beginning, the time displacement of the soliton centre with the initial velocity $\upsilon = 0.02$ has an oscillating character (this means that the instant soliton velocity also oscillates during its propagation), but later this soliton propagates with a nearly constant velocity. The slope of the solid straight line shows that the acquired velocity is 0.026, unlike the initial value of 0.02. If the soliton starts with the velocity $\upsilon = 0.024$ (light points), its centre moves somewhat faster than the centre of a soliton with the initial velocity $\upsilon = 0.02$. However, in some time the latter also acquires the average velocity 0.026.

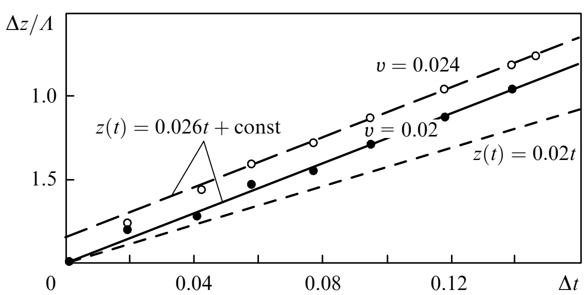

Figure 3. Displacement of centres of dissipative Bragg solitons capable of overcoming the first period of the refractive index grating.

An analogous analysis of the propagation of solitons with different initial velocities shows that propagating solitons with close initial velocities acquire definite values form a discrete set of velocities $\upsilon = 0,\ 0.015,\ 0.026, \ldots$.

5. Conclusions

Thus, we have obtained qualitatively new results beyond the slowly-varying amplitude approximation, which are absent within the framework of this approximation. The most important result is the discreteness of the set of average propagation velocities of dissipative Bragg solitons and localisation of an immobile soliton in the FBG near the maximum of the refractive index.

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